Four-Wave Mixing and Many-Particle Effects in Semiconductors

Rolf Binder

College of Optical Sciences and Department of Physics
The University of Arizona

review article

Many-body and correlation effects in semiconductors

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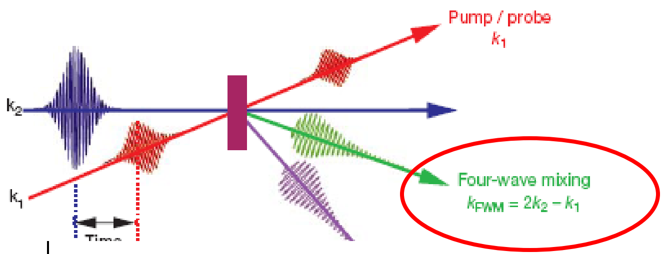
Box 1

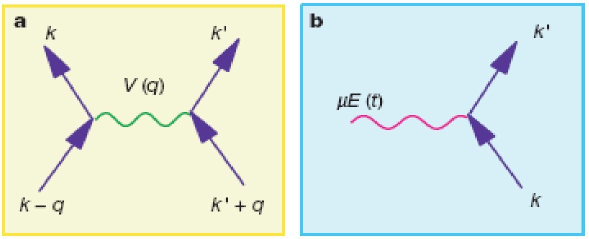
Principle of time-resolved nonlinear optical spectroscopy experiments



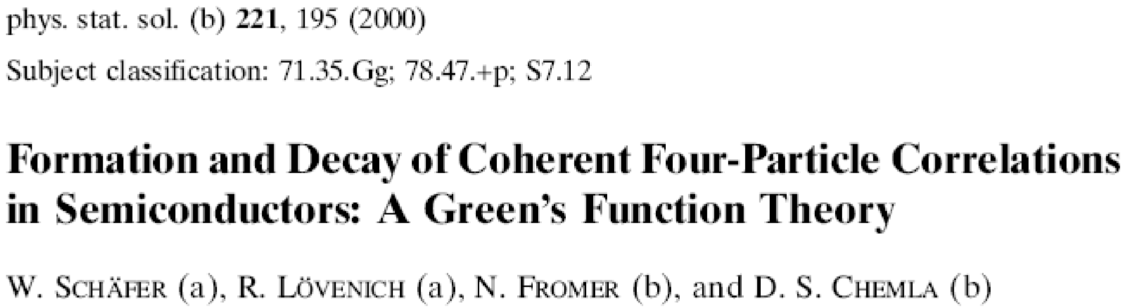
Box 1 Figure Schematic of a generic time-reso

Box 2

Many-body effects in light–semiconductor interaction



Box 2 Figure Feynman diagrams for (**a**) Coulomb interaction between carriers, and (**b**) coupling with electromagnetic field.



phys. stat. sol. (b) **221**, 195 (2000)

Subject classification: 71.35.Gg; 78.47.+p; S7.12

Formation and Decay of Coherent Four-Particle Correlations in Semiconductors: A Green's Function Theory

W. SCHÄFER (a), R. LÖVENICH (a), N. FROMER (b), and D. S. CHEMLA (b)

A lot of diagrams...

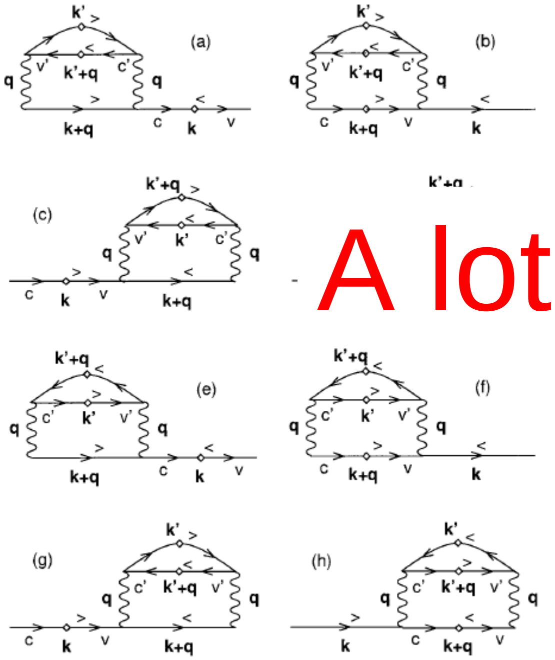


Fig. 1. Diagrammatic representation of the RPA polarization scattering diagrams in second-order Born approximation

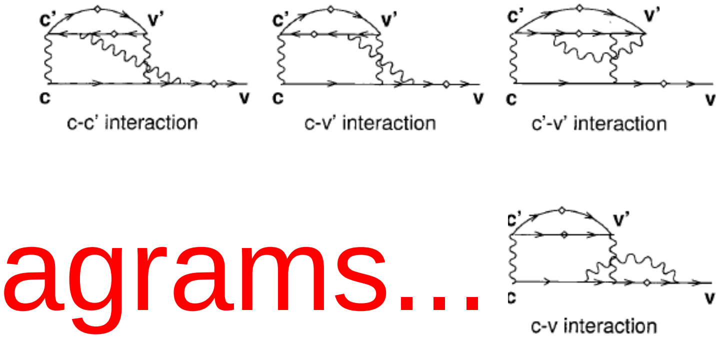


Fig. 2. Third-order contributions resulting from the RPA polarization scattering diagram a) of Fig. 1

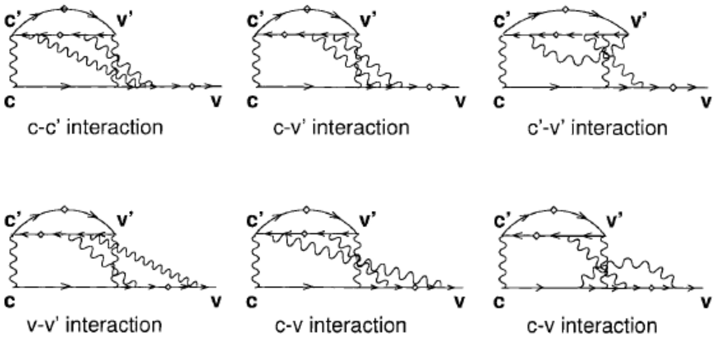


Fig. 3. Fourth-order contributions resulting from the second diagram of Fig. 2

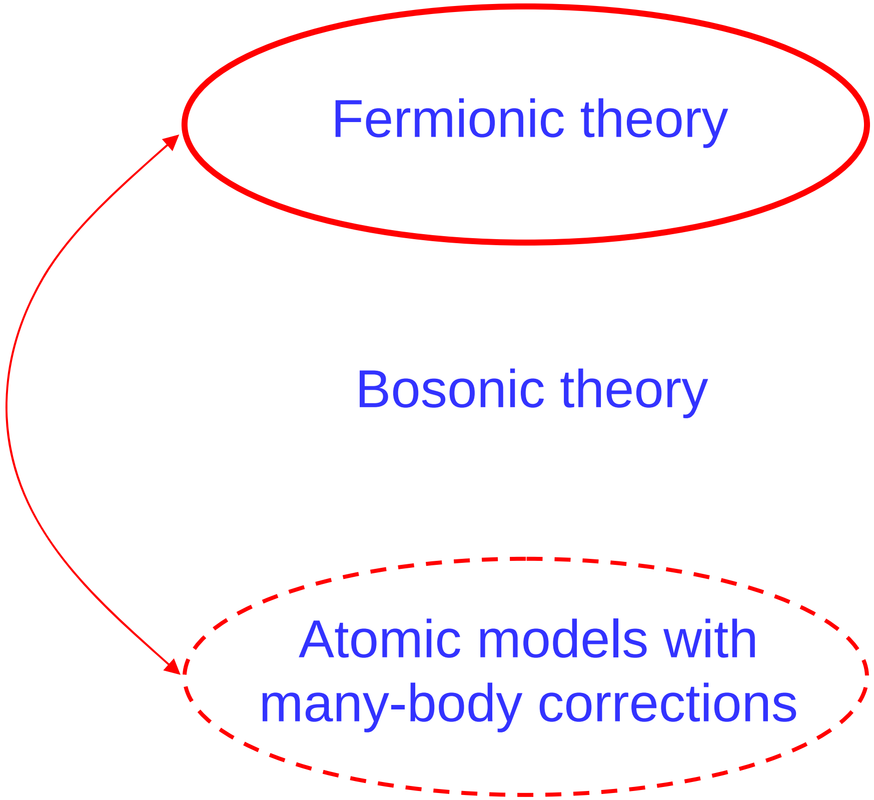
This tutorial is about

using Feynman diagrams and four-wave mixing to study many-particle correlations in semiconductors

It is **not** about

photon echo, holography, phase conjugation, third-harmonic generation, ...

Three theoretical approaches



Fermionic theory

Bosonic theory

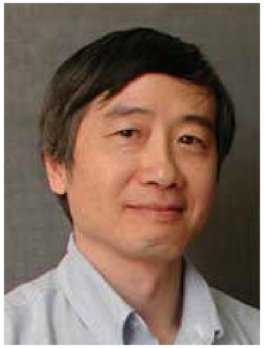
Atomic models with
many-body corrections

First:

Acknowledgements

Special thanks to

Nai H. Kwong
College of Optical Sciences
University of Arizona



...and to (past and present) graduate students and postdocs

Ilya Rumyantsev (now Synopsis)
Zhenshan Yang (now Texas A&M Univ.)
Ryu Takayama (now Canon)
Dan Nguyen (now NP Photonics)
Stefan Schumacher (now Heriot-Watt Univ.)
Greg Rupper
Baijie Gu

ERATO, AFOSR, DARPA, JSOP

Outline:

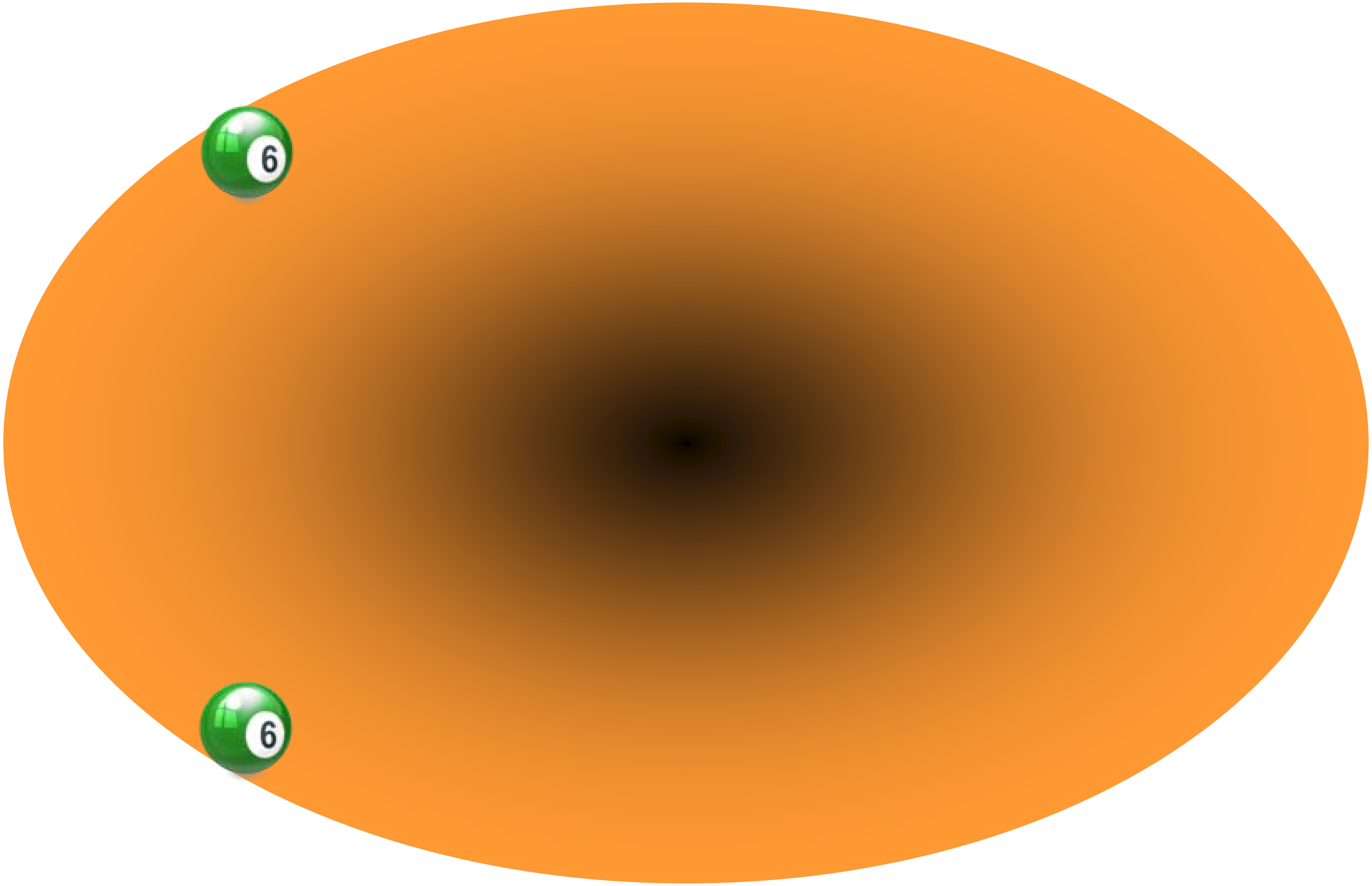
- Introduction
- Many-particle theory and Green's functions
- Third-order optical response
- Correlations beyond third order
- Few-level systems
- FWM instabilities (time permitting)

Next:

Weekend experiences with perturbative and non-perturbative two-particle correlations

Two-particle interaction

Two-particle correlations: bound states



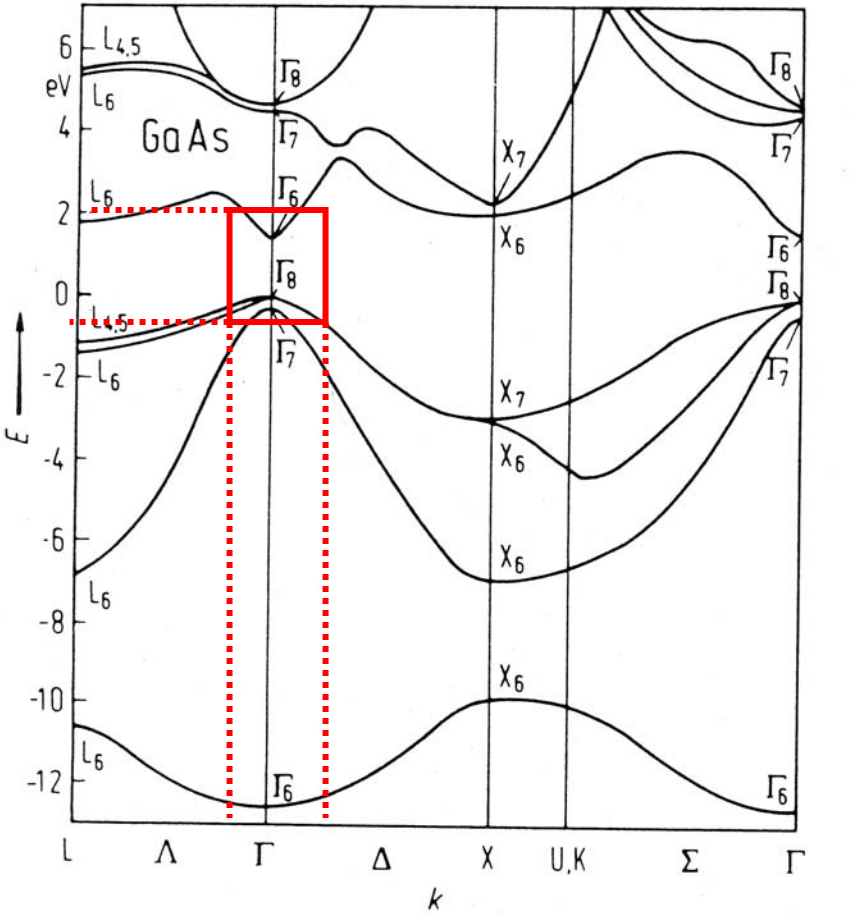
Two-particle correlation: continuum states

Next:

Introduction to semiconductors

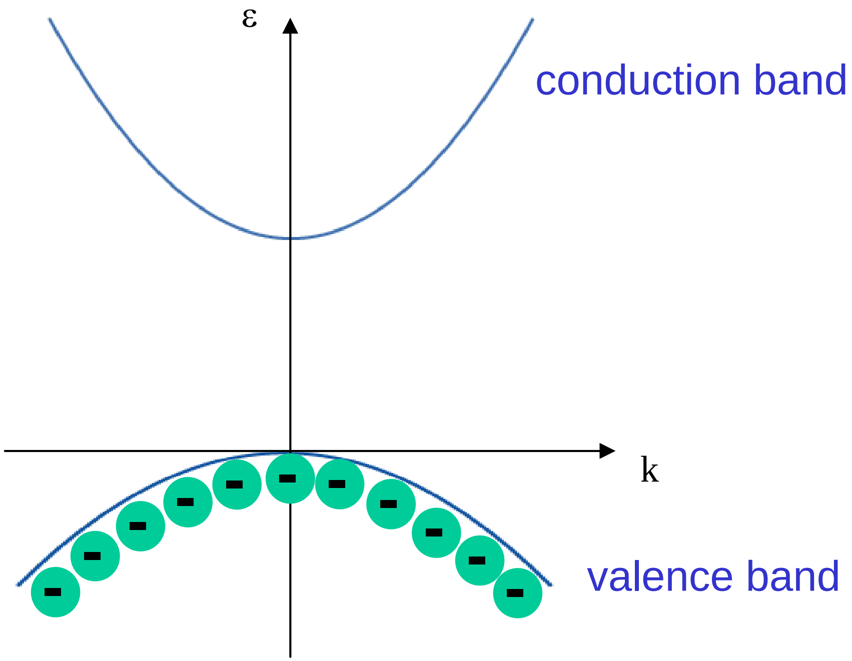
GaAs bandstructure

Optical excitation typically close to Γ point (k=0)

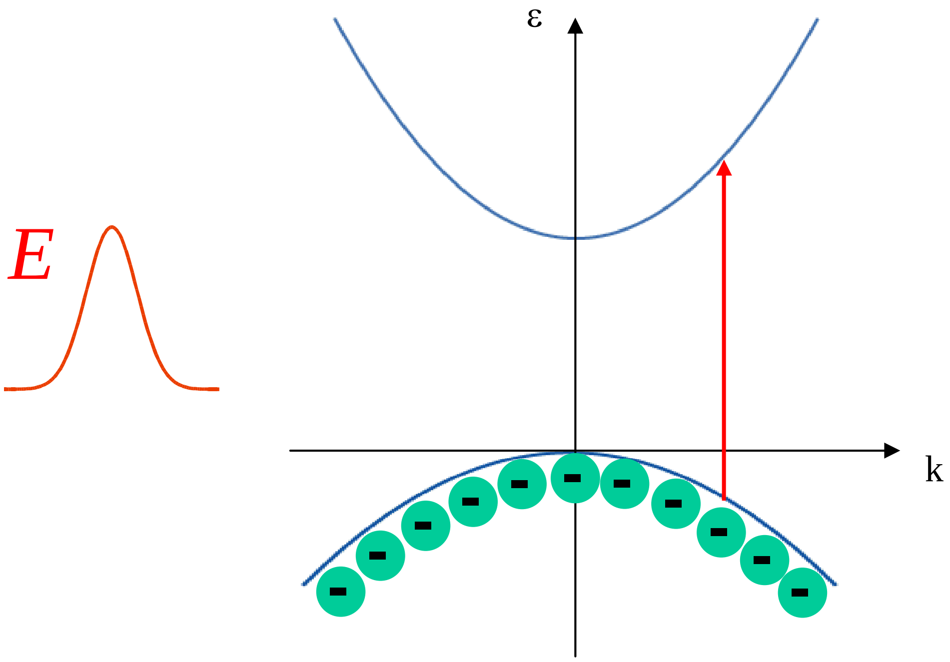


Parabolic Bandstructure near k=0

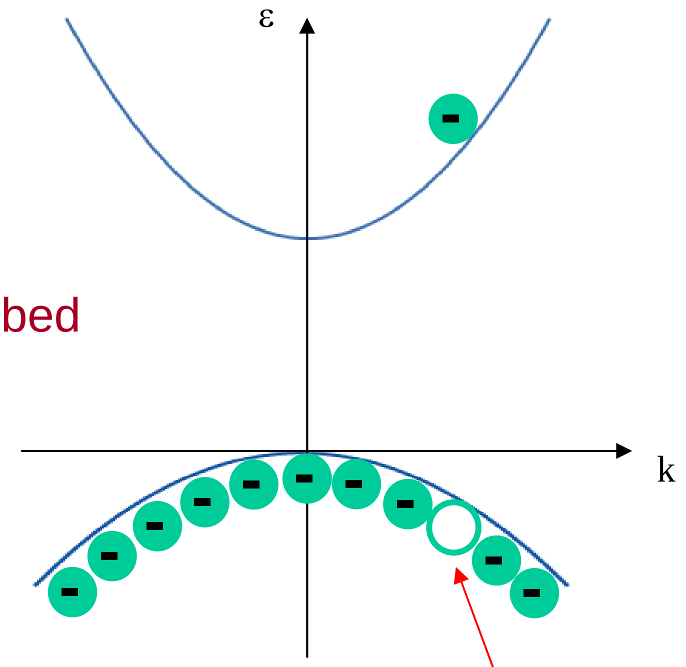
Ground state:
full valence band



Optical excitation



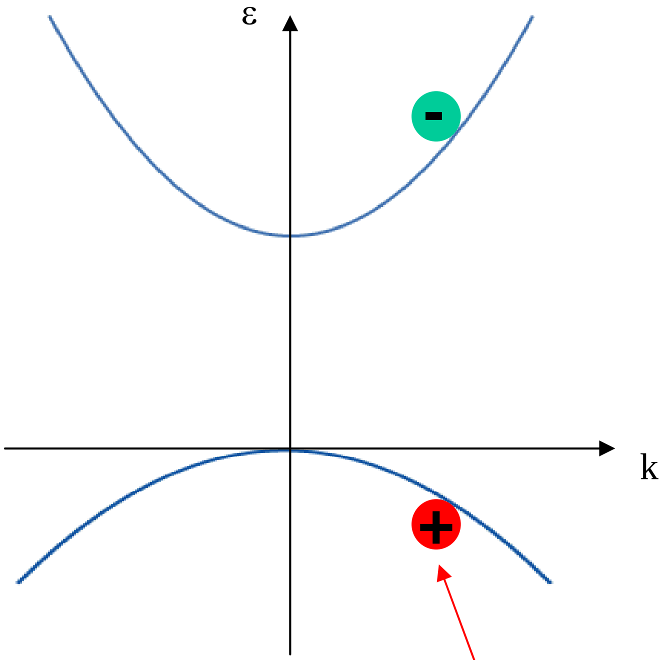
Optical excitation



light absorbed

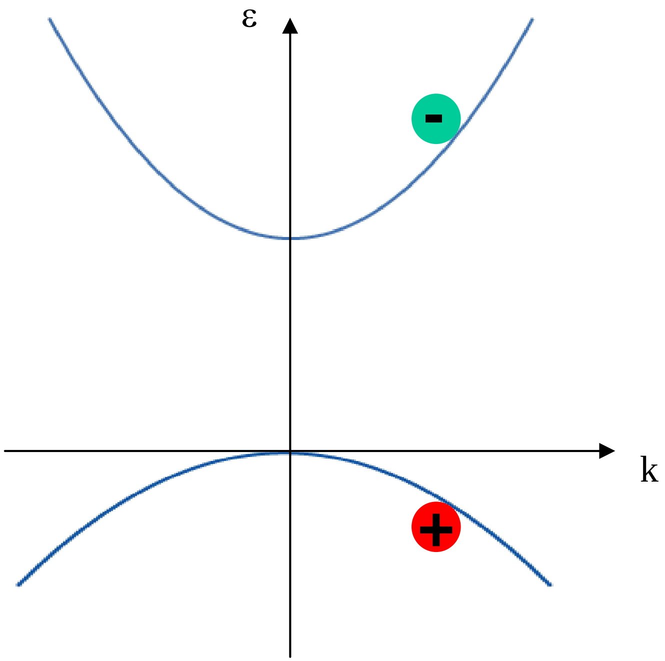
“hole” in valence band

Hole = positively charged “particle”

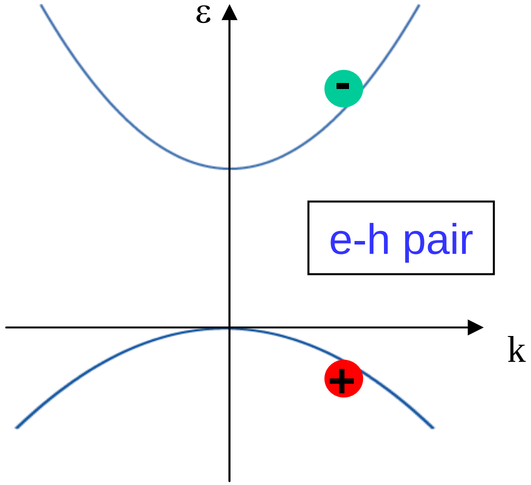


“hole” in valence band

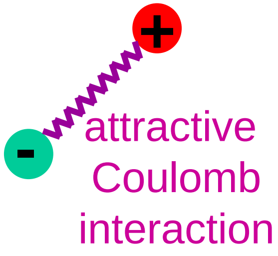
Hole = positively charged “particle”



Excitons

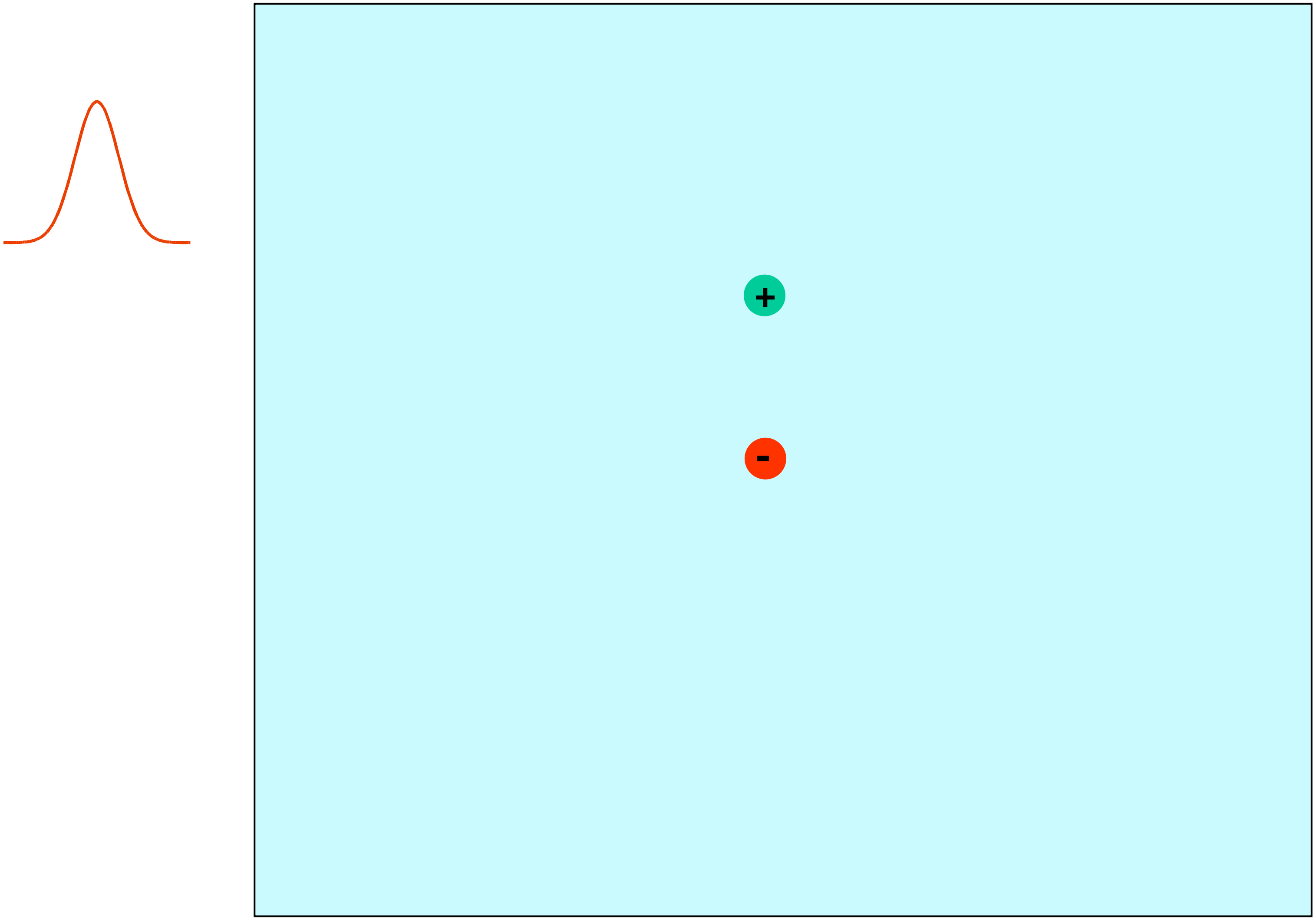


real space:



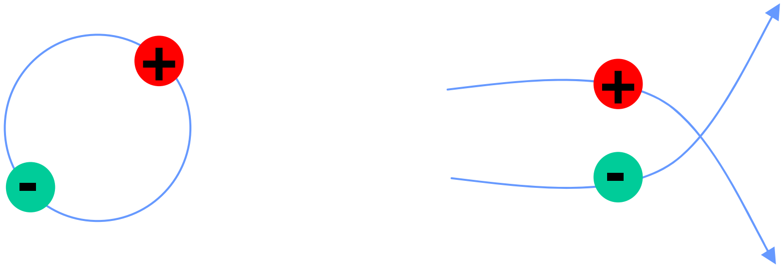
- Bound states (Rydberg law) $\varepsilon_n = E_g - \frac{E_b}{n^2}$
- Continuum states

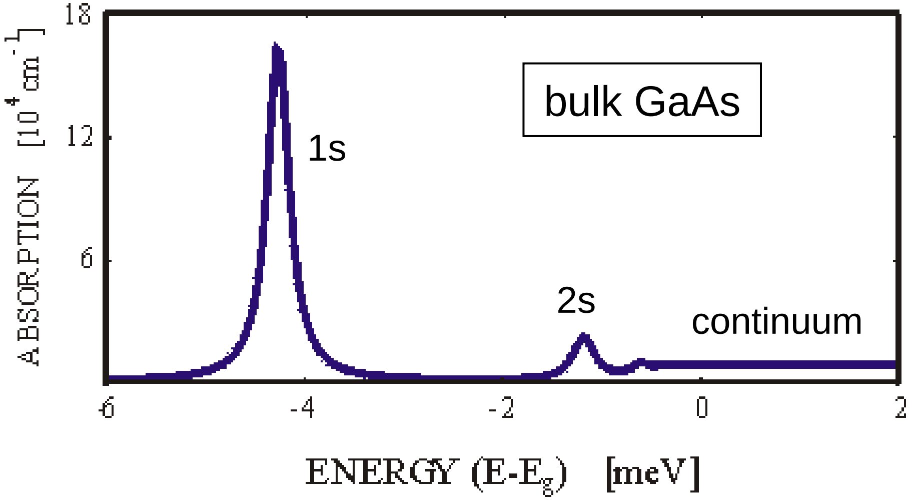
Excitons



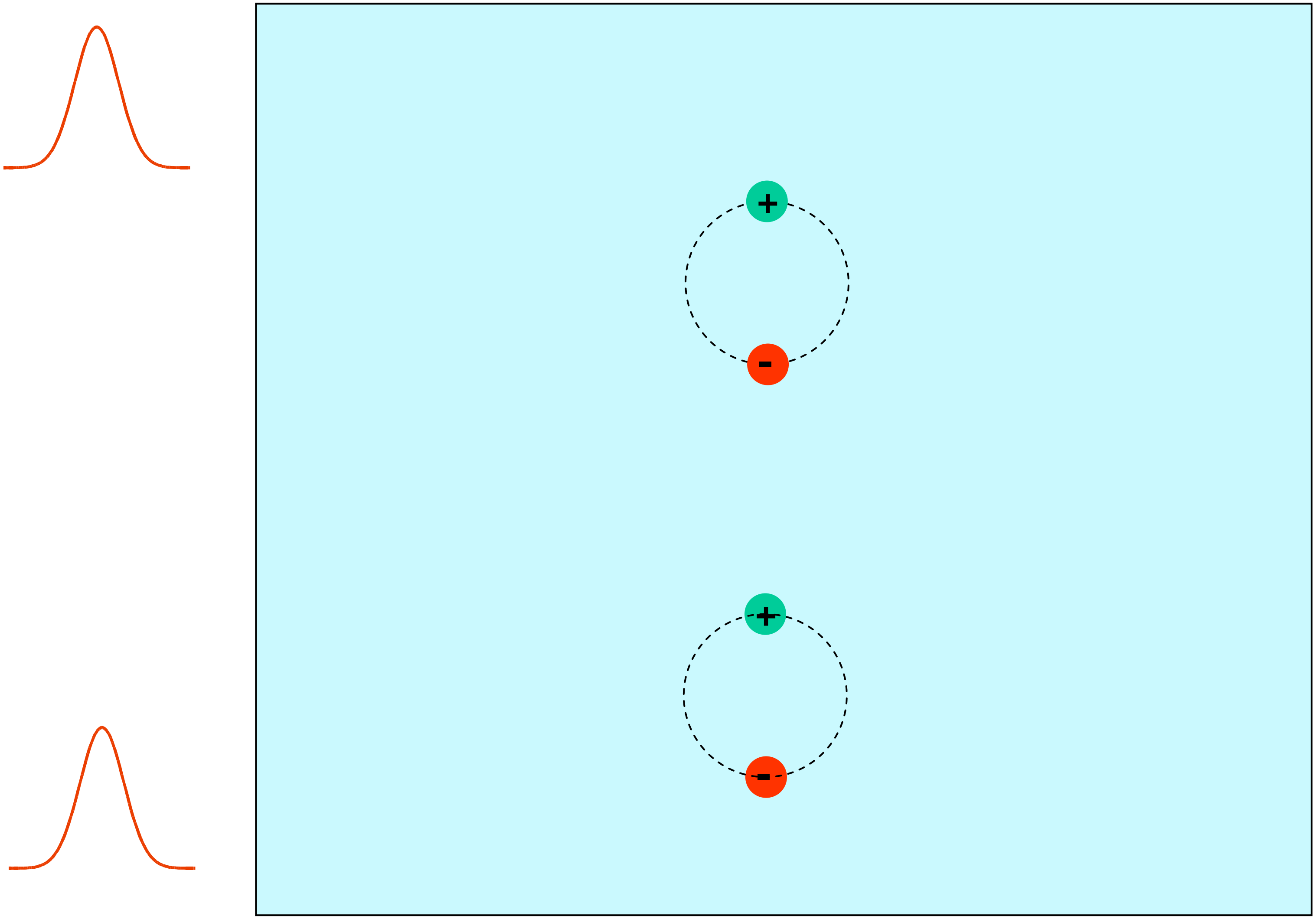
Excitons

real space:





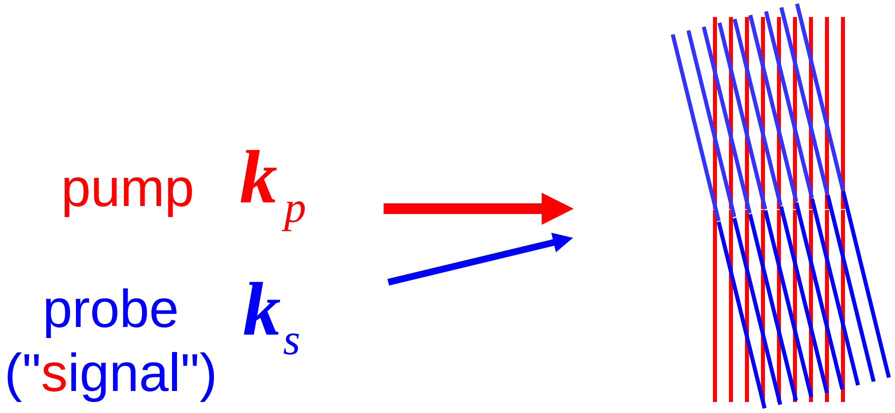
Nonlinear excitation: two-exciton states



Next:

The concept of four-wave mixing (FWM)

Wave mixing and self diffraction

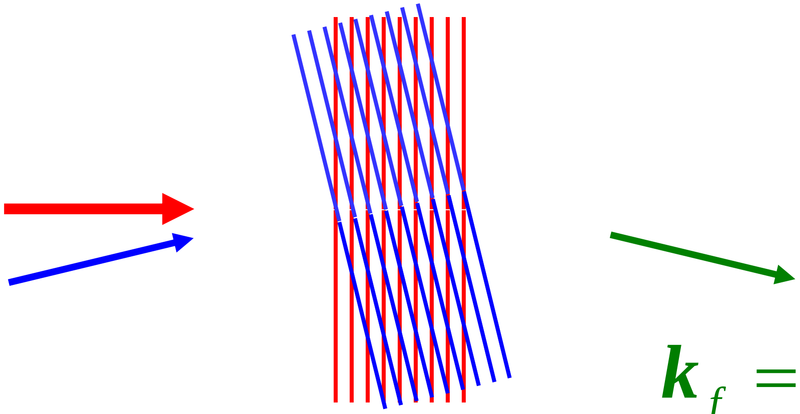


pump $\boldsymbol{k}_p$

probe $\boldsymbol{k}_s$
("signal")

Grating with spatial wave vector $\Delta\boldsymbol{k}_f = \boldsymbol{k}_p - \boldsymbol{k}_s$

Forward four-wave mixing



pump $\boldsymbol{k}_p$

probe $\boldsymbol{k}_s$
("signal")

$$\boldsymbol{k}_f = \boldsymbol{k}_p + \Delta\boldsymbol{k}$$
$$= 2\boldsymbol{k}_p - \boldsymbol{k}_s$$

diffracted light

$$\left\{ \frac{\varepsilon_b}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \right\} \boldsymbol{E} = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{P}$$

Third-order response:
(schematically; integrals over space and time suppressed)

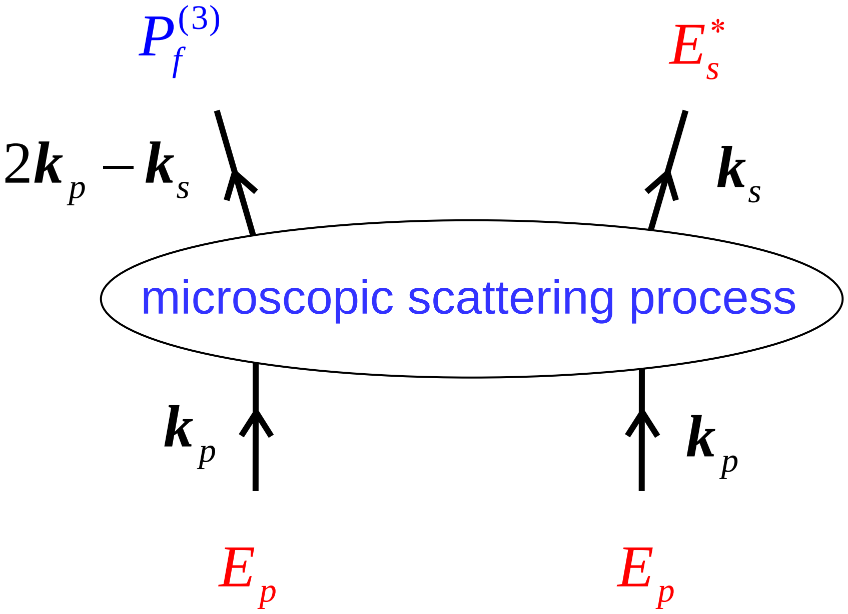
$$P \;=\; \enclose{circle}{\chi^{(3)}}\; E^* \;\; E \;\; E$$

spatial dependence: $e^{i\mathbf{k}_f\mathbf{r}}$ $\qquad$ $e^{-i\mathbf{k}_s\mathbf{r}}$ $e^{i\mathbf{k}_p\mathbf{r}}$ $e^{i\mathbf{k}_p\mathbf{r}}$

information about excitonic correlations

Strategy in this talk: present P as Feynman diagrams

Four-wave mixing



$P_f^{(3)}$

E_s^*

$2\boldsymbol{k}_p - \boldsymbol{k}_s$

$\boldsymbol{k}_s$

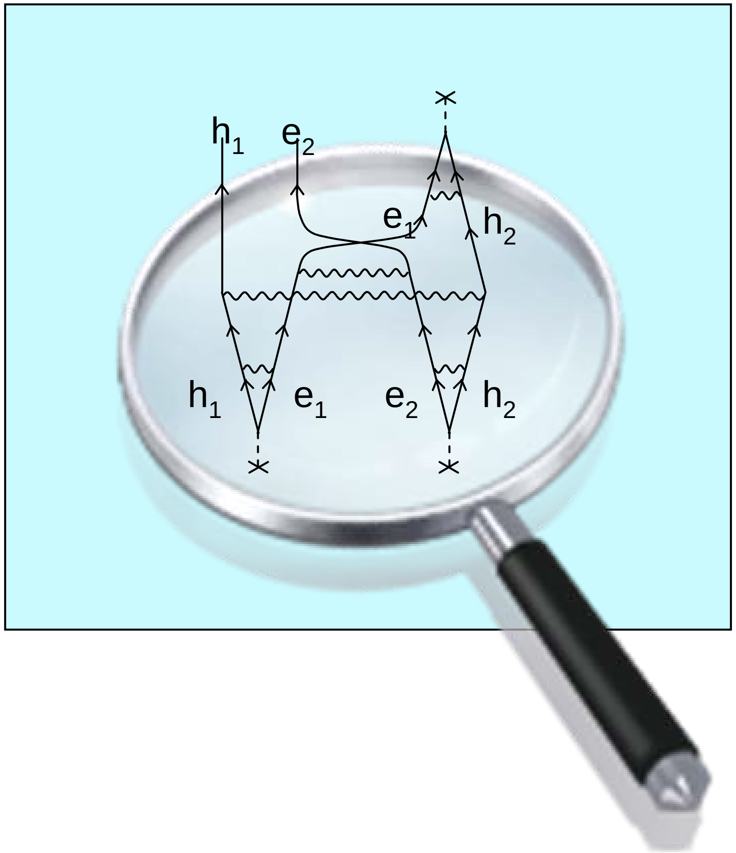
microscopic scattering process

$\boldsymbol{k}_p$

$\boldsymbol{k}_p$

E_p

E_p


h1
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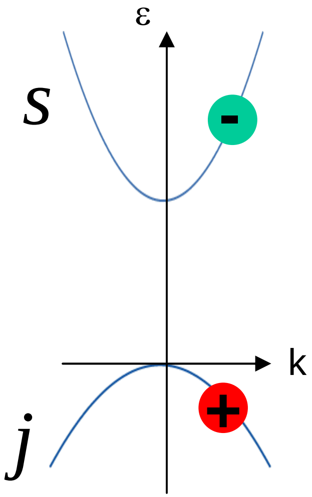
Next:

Introduction to many-particle theory



electrons	$a_{sk}^{\dagger}$	a_{sk}
holes	$a_{jk}^{\dagger}$	a_{jk}
	creation operators	annihilation operators

Fermionic semiconductor Hamiltonian

$$H = H_{band} + H_{Coulomb} + H_{light-coupling}$$

$$H_{band} = \sum_{s\mathbf{k}} \varepsilon^{e}_{s\mathbf{k}} \underbrace{a^{\dagger}_{s\mathbf{k}}\ a_{s\mathbf{k}}} + \sum_{j\mathbf{k}} \varepsilon^{h}_{j\mathbf{k}} \underbrace{a^{\dagger}_{j\mathbf{k}}\ a_{j\mathbf{k}}}$$

e and h occupation number operators

$$H_{Coulomb} = \frac{1}{2} \sum_{\substack{\text{all}\\ \text{indices}}} V^{c}_{q} \Big[\underbrace{a^{\dagger}_{s,\mathbf{k}+\mathbf{q}}\ a^{\dagger}_{s',\mathbf{k}-\mathbf{q}}\ a_{s',\mathbf{k}'}\ a_{s,\mathbf{k}}}_{\text{e-e}} + \underbrace{a^{\dagger}_{j,\mathbf{k}+\mathbf{q}}\ a^{\dagger}_{j',\mathbf{k}-\mathbf{q}}\ a_{j',\mathbf{k}'}\ a_{j,\mathbf{k}}}_{\text{h-h}} + 2\underbrace{a^{\dagger}_{s,\mathbf{k}+\mathbf{q}}\ a^{\dagger}_{j,\mathbf{k}-\mathbf{q}}\ a_{j,\mathbf{k}'}\ a_{s,\mathbf{k}}}_{\text{e-h interaction}} \Big]$$

$$H_{light-coupling} = \sum_{\substack{\text{all}\\ \text{indices}}} \Big[\vec{\mu}^{*}_{sj} \cdot \vec{E}^{*}(t)\ \underbrace{a_{s,\mathbf{k}}\ a_{j,-\mathbf{k}}} + \vec{\mu}_{sj} \cdot \vec{E}(t)\ \underbrace{a^{\dagger}_{s,\mathbf{k}}\ a^{\dagger}_{j,-\mathbf{k}}} \Big]$$

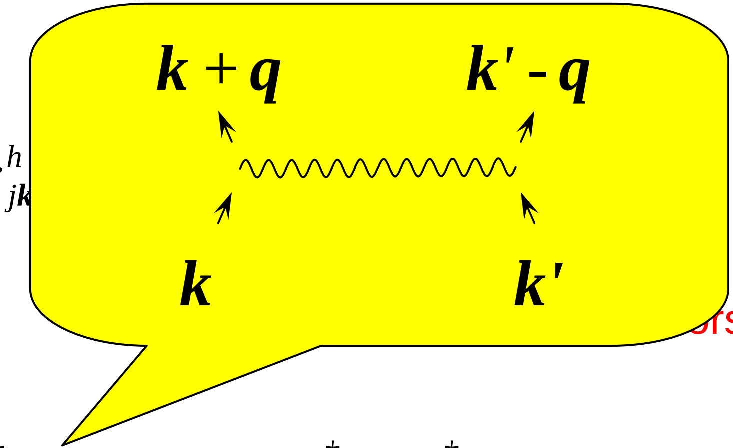
electron-hole pair annihilation and creation operators

Fermionic semiconductor Hamiltonian

$$H = H_{band} + H_{Coulomb} + H_{light-coupling}$$

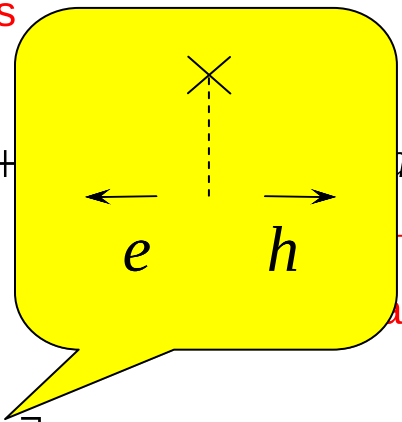
$$H_{band} = \sum_{s\mathbf{k}} \varepsilon^{e}_{s\mathbf{k}} \underbrace{a^{\dagger}_{s\mathbf{k}}\ a_{s\mathbf{k}}}_{\text{e}} + \sum_{j\mathbf{k}} \varepsilon^{h}_{j\mathbf{k}}$$

e and ors



$$H_{Coulomb} = \frac{1}{2} \sum_{\substack{\text{all}\\ \text{indices}}} V^{c}_{q} \Big[\underbrace{a^{\dagger}_{s,\mathbf{k}+\mathbf{q}}\ a^{\dagger}_{s',\mathbf{k}-\mathbf{q}}\ a_{s',\mathbf{k}'}\ a_{s,\mathbf{k}}}_{\text{e-e}} + \underbrace{a^{\dagger}_{j,\mathbf{k}+\mathbf{q}}\ a^{\dagger}_{j',\mathbf{k}-\mathbf{q}}\ a_{j',\mathbf{k}'}\ a_{j,\mathbf{k}}}_{\text{h-h}} + \quad {}_{s,\mathbf{k}} \Big]$$

action



$$H_{light-coupling} = \sum_{\substack{\text{all}\\ \text{indices}}} \Big[\vec{\mu}^{*}_{sj} \cdot \vec{E}^{*}(t)\ \underbrace{a_{s,\mathbf{k}}\ a_{j,-\mathbf{k}}} + \vec{\mu}_{sj} \cdot \vec{E}(t)\ \underbrace{a^{\dagger}_{s,\mathbf{k}}\ a^{\dagger}_{j,-\mathbf{k}}} \Big]$$

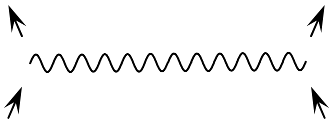
electron-hole pair annihilation and creation operators

$$H = H_{band} + H_{Coulomb} + H_{light-coupling}$$

Show only basic structure:

$$H_{band} = \varepsilon \, a^{\dagger} \, a$$

$$H_{Coulomb} = V^{c} \, a^{\dagger} \, a^{\dagger} \, a \;\; a$$



$$H_{light-coupling} = E^{*} \, a \;\; a \; + E \, a^{\dagger} \, a^{\dagger}$$

Next:

About expectation values and Green's functions

Expectation values

$$p_{eh}(t)=\langle a_h(t)\, a_e(t)\rangle$$

interband polarization

$$\left.\begin{array}{l} f_e\,(t)=\langle a_e^{\dagger}(t)a_e(t)\rangle \\ \\ f_h(t)=\langle a_h^{\dagger}(t)a_h(t)\rangle \end{array}\right\}$$

occupation functions

Two-time functions

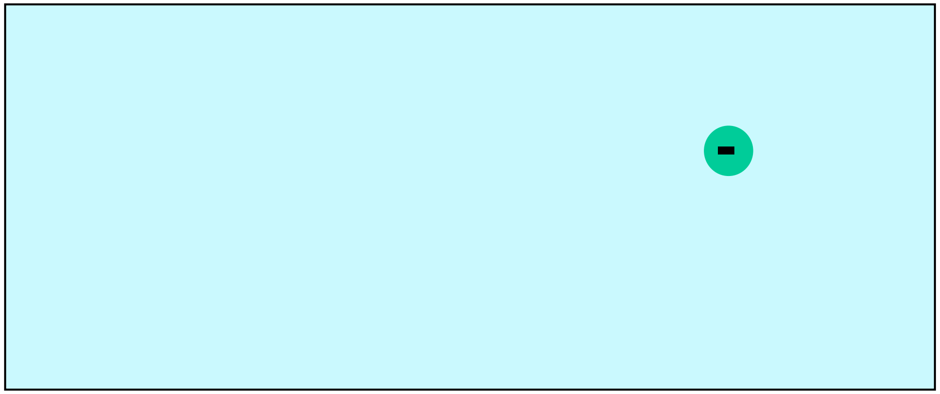
$$p_{eh}(t)=\langle a_h(t)\, a_e(t')\rangle\Big|_{t=t'}$$

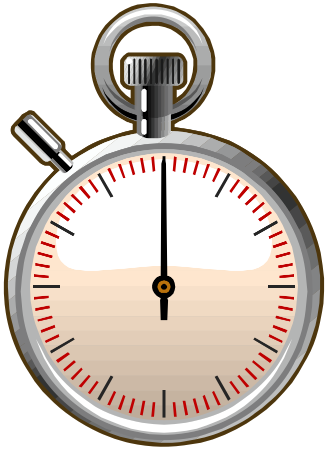
$$f_e(t)=\langle a_e^{\dagger}(t)\, a_e(t')\rangle\Big|_{t=t'}$$

Particle propagators

$$\langle\, a(t)\;\; a^{\dagger}(t')\,\rangle$$

t later than t'

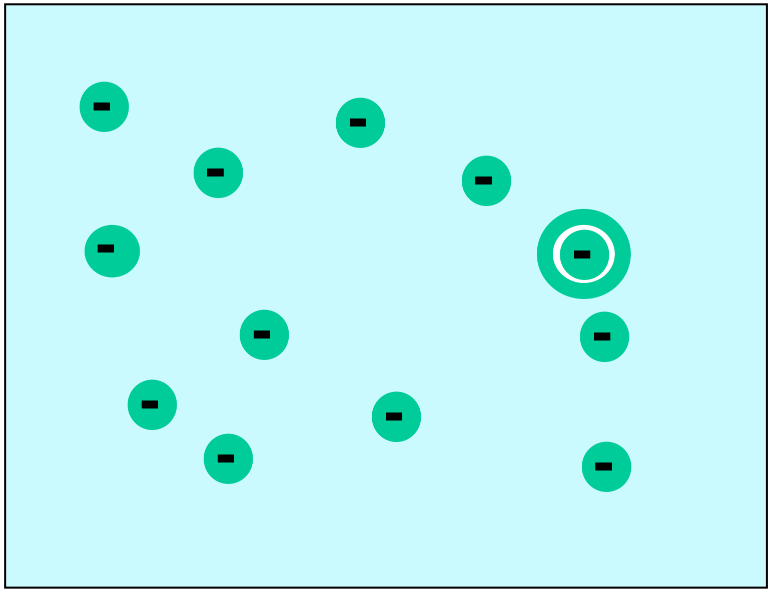


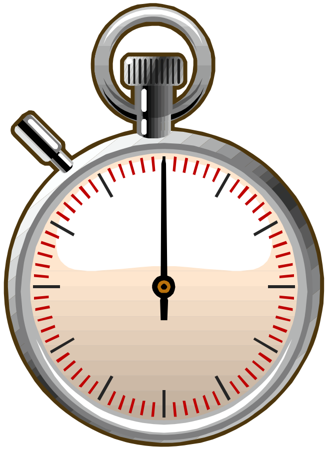


carries information about particle energy

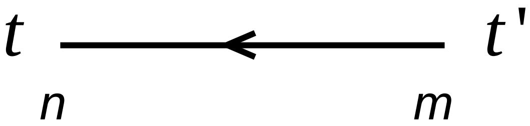
Hole propagators

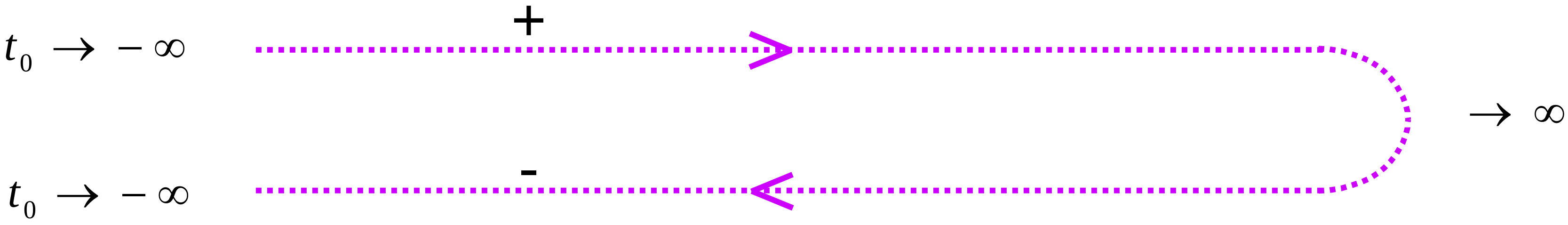
$$\langle\, a^{\dagger}(t')\, a(t)\, \rangle \qquad t' \text{ later than } t$$





$$iG_{nm}(t,t') = \langle \hat{T}_c \, a_n(t) \, a_m^{\dagger}(t') \rangle$$

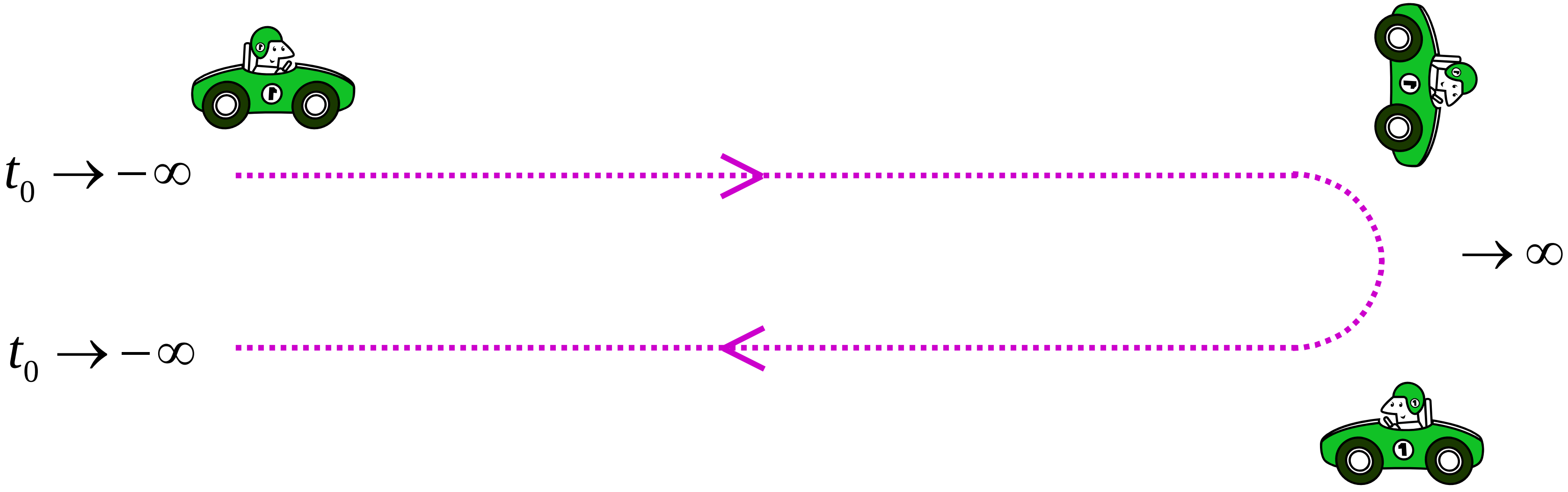




if t' later on contour than t: $iG_{nm}(t,t') = -\langle a_m^{\dagger}(t') \, a_n(t) \rangle$

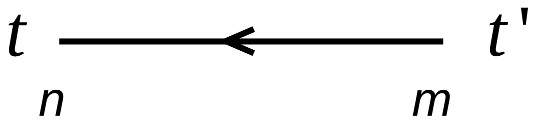
if t' earlier on contour than t: $iG_{nm}(t,t') = +\langle a_n(t) \, a_m^{\dagger}(t') \rangle$

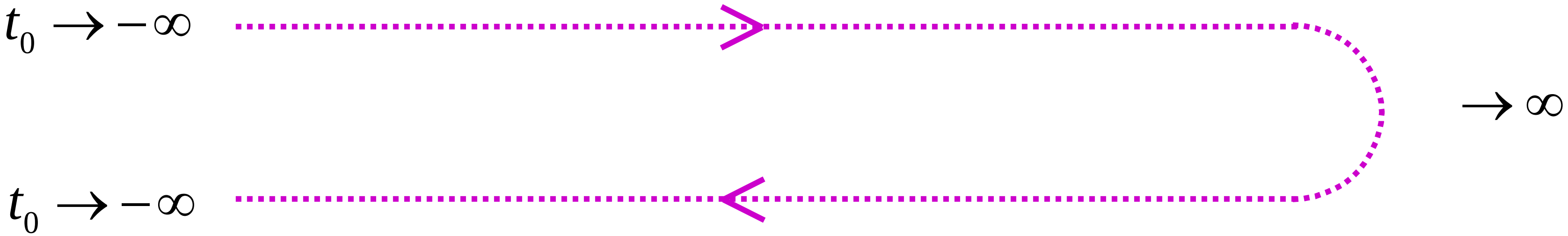
Schwinger, J. Math. Phys. 2, 407 (1961); Keldysh, Sov. Phys. JETP 20, 235 (1965)



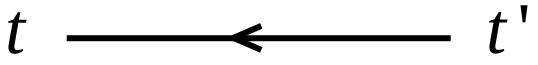
All one-particle Green's functions have "time arrow"

$$iG_{nm}(t,t') = \langle \hat{T}_c\, a_n(t)\, a_m^{\dagger}(t')\rangle$$



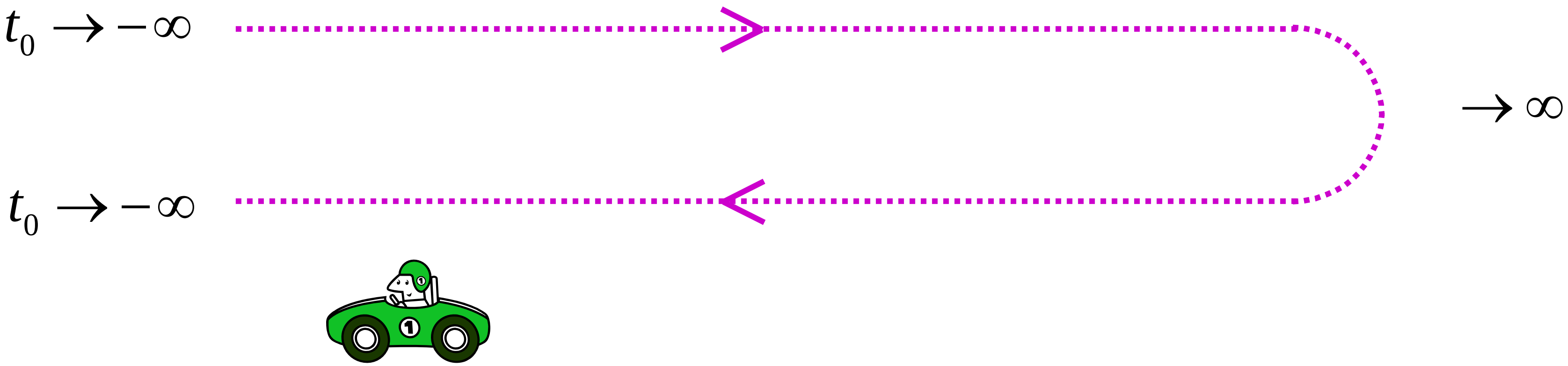


Usual propagators (non density-type)

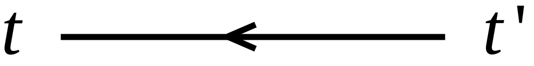


arrow points **forward** in time

$$iG_{nm}(t,t') = \langle a_n(t) a_m^\dagger(t') \rangle$$



"Density-type"

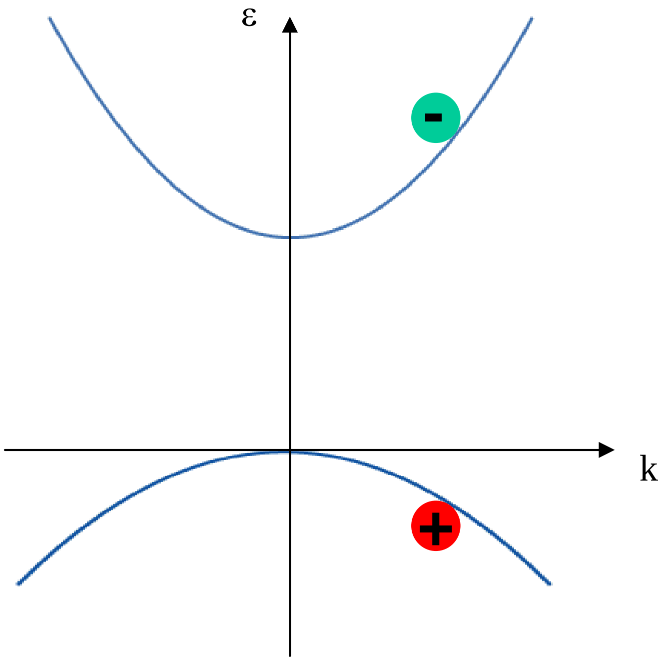


arrow points **backward** in time

$$iG_{nm}(t,t') = \langle a_m^\dagger(t')a_n(t)\rangle$$

Next:

A diagram tool box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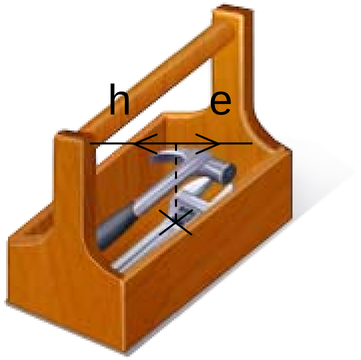
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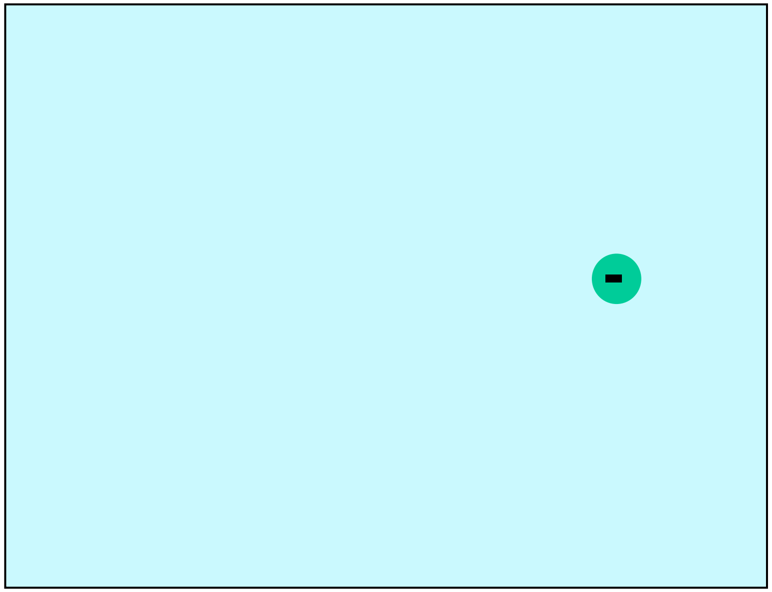
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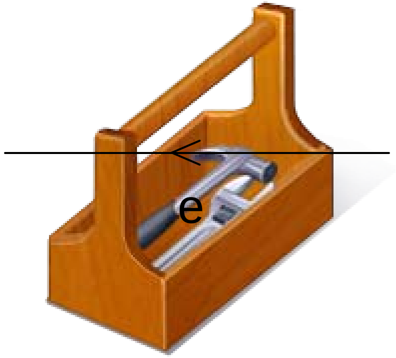
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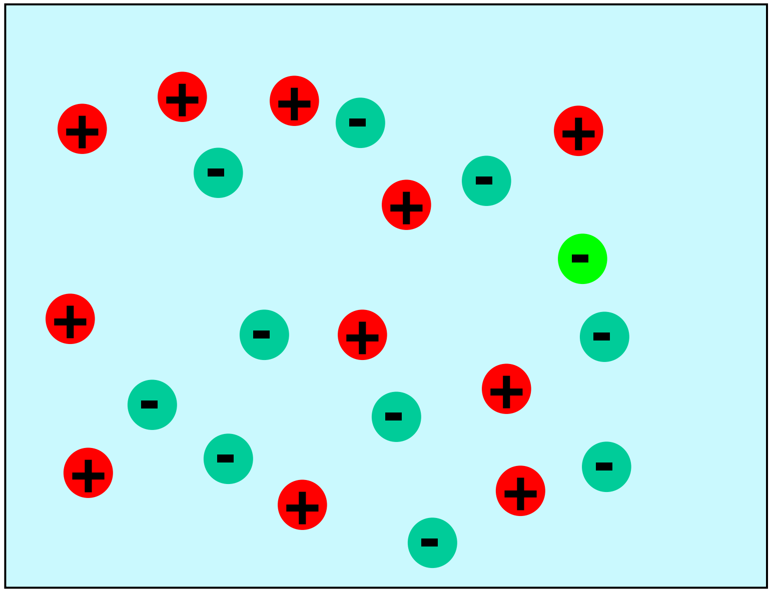

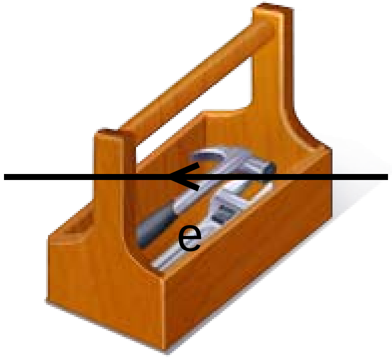
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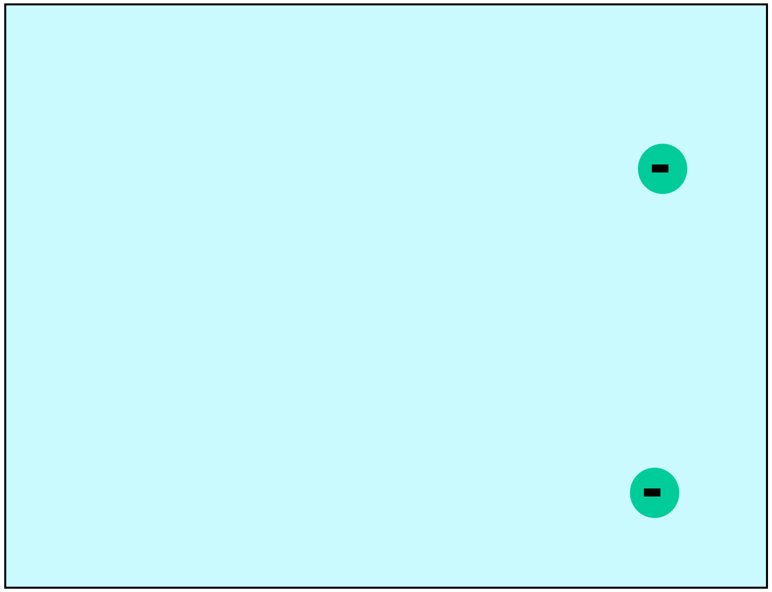
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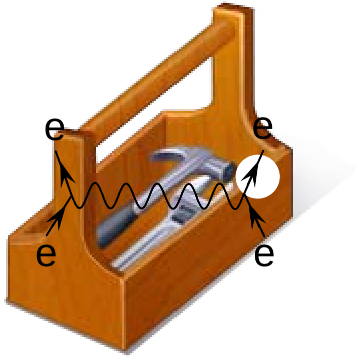
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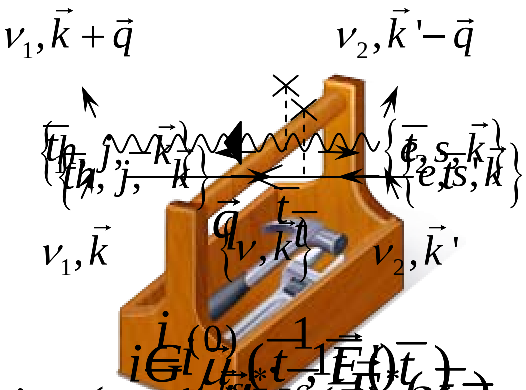
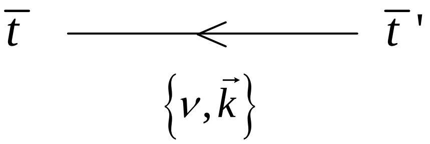
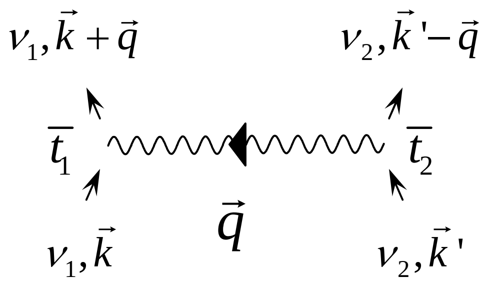


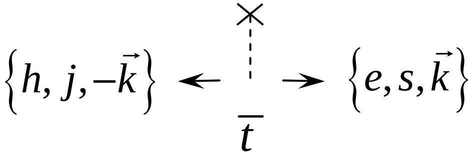
$$\frac{-i}{\hbar}\,\mathrm{sign}(\nu_1,\nu_2)\,V(\vec{q})\,\delta(t_1-t_2)$$

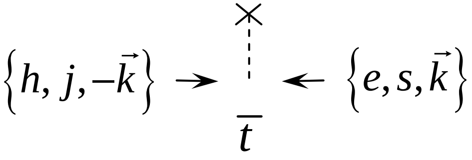


$$iG^{(0)}_{\nu,\vec{k}}(\bar{t},\bar{t}\,')$$



$$\frac{-i}{\hbar}\,\mathrm{sign}(\nu_1,\nu_2)\,V^c(\vec{q})\,\delta(\bar{t}_1-\bar{t}_2)$$





$$\frac{i}{\hbar}\,\vec{\mu}_{sj}\cdot\frac{1}{2}\vec{E}(\bar{t})$$

$$\frac{-i}{\hbar}\,\vec{\mu}^{*}_{sj}\cdot\frac{1}{2}\vec{E}^{*}(\bar{t})$$

Next:

More about Green's functions, propagators and Feynman diagrams

The idea:

$$\langle \hat{T}_c\, a_n(t)\, a_m^\dagger(t')\rangle$$

represent as perturbation series
via Feynman diagrams

$$\langle \hat{T}_c\, a_n(t)\, a_m^\dagger(t')\rangle\Big|_{t'=t+\varepsilon}$$

obtain expectation values
from equal-time limit

Perturbation theory: $H = H_0 + H'$

$$\langle \hat{T}_c \, a_n(t) \, a_m^\dagger(t') \rangle = \langle \hat{T}_c \, e^{-\frac{i}{\hbar}\int_c dt'' H_I'(t'')} a_{I,n}(t) \, a_{I,m}^\dagger(t') \rangle$$

full Green's function (propagators)

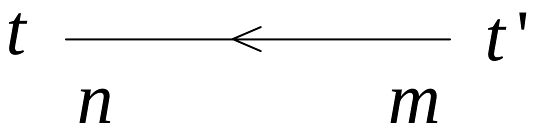
free particle operators

Feynman diagrams: expand exponential and factorize:

$$e^{ix} = 1 + ix - \frac{1}{2}x^2 + \cdots$$

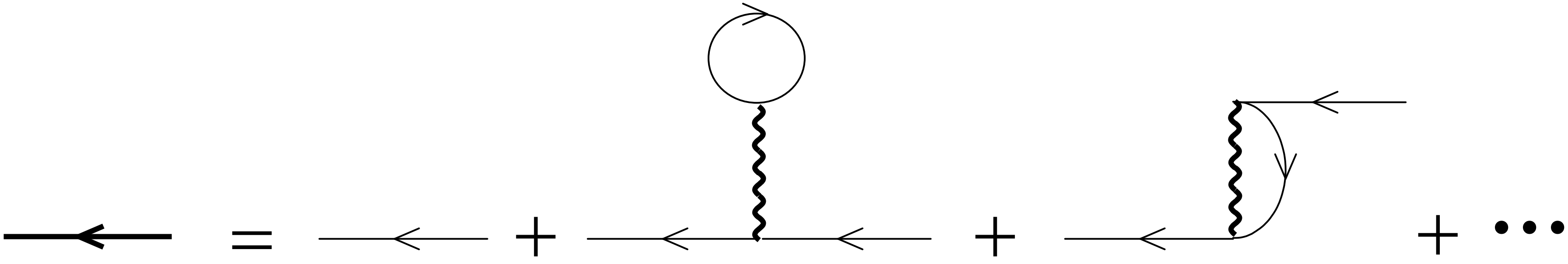
$$\langle a_{I,i} \, a_{I,j} a_{I,k}^\dagger \, a_{I,l}^\dagger \rangle = \langle a_{I,i} \, a_{I,l}^\dagger \rangle \langle a_{I,j} \, a_{I,k}^\dagger \rangle - \langle a_{I,i} \, a_{I,k}^\dagger \rangle \langle a_{I,j} \, a_{I,l}^\dagger \rangle$$

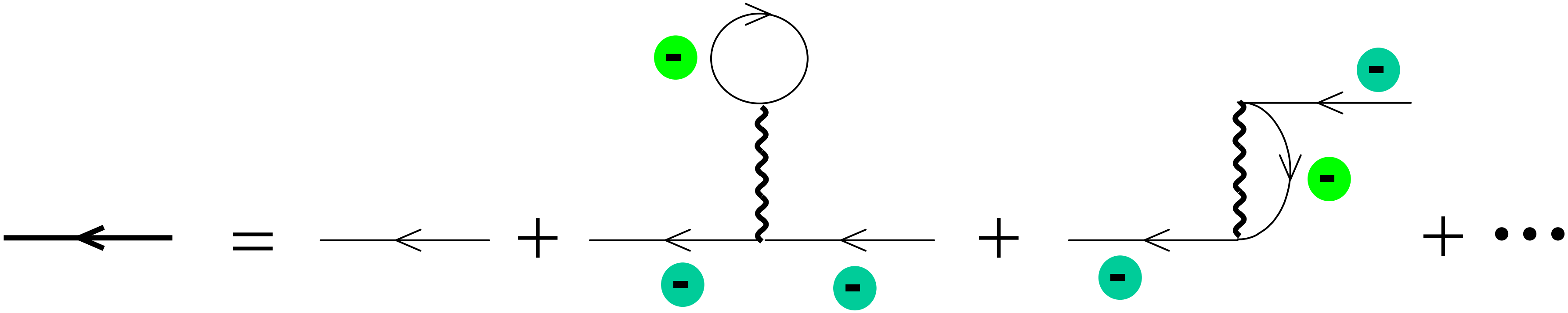
$H' = 0:$



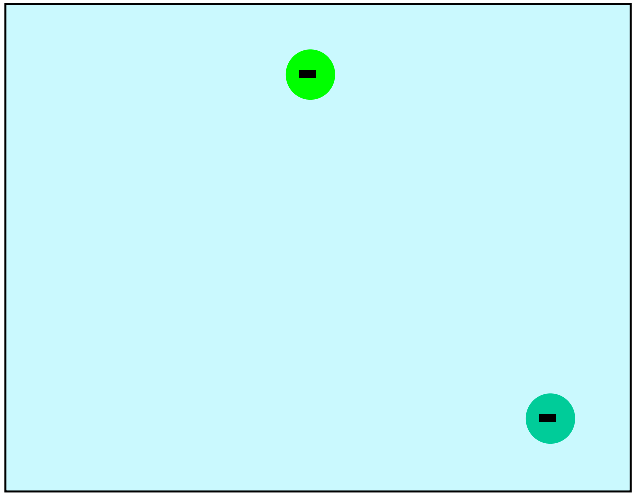
$$iG^{(0)}_{nm}(t,t') = \langle \hat{T}_c \; a_{I,n}(t) \; a^{\dagger}_{I,m}(t') \rangle$$

$H' \neq 0:$

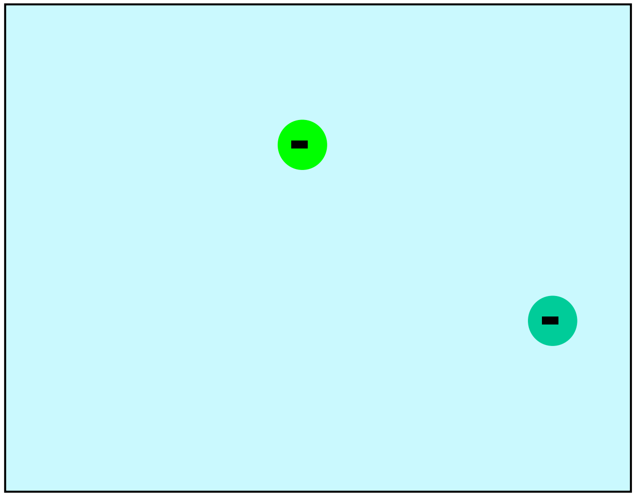




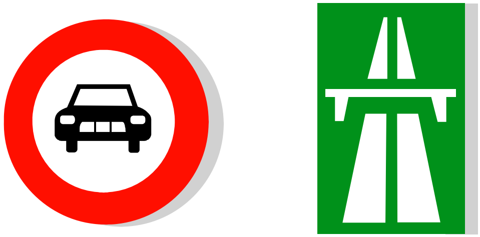
direct Coulomb interaction



exchange Coulomb interaction



Rules and Regulations
(short version)



1. Draw all topologically distinct connected diagrams with two external points

2. Sum over all internal indices

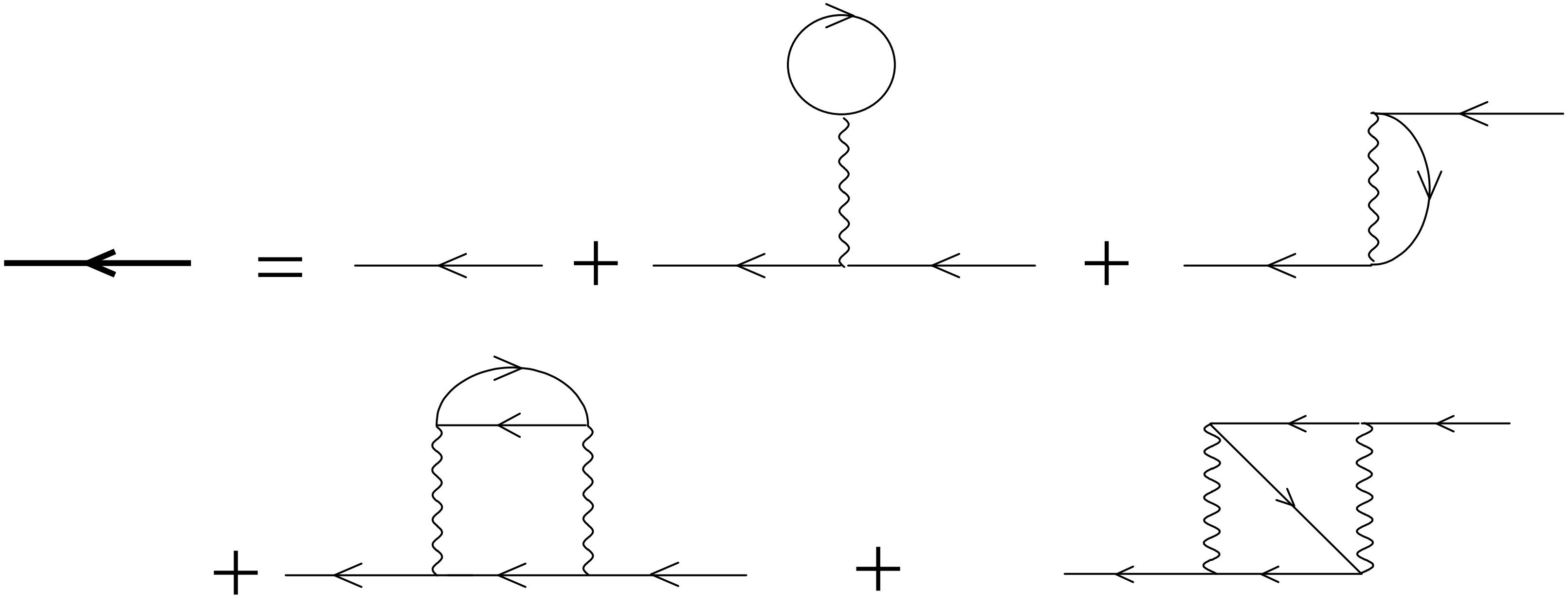
3. Attach an additional factor of (-1) for each closed Fermion loop

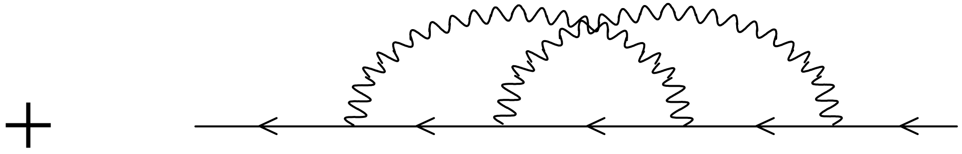
---

Complete rules:

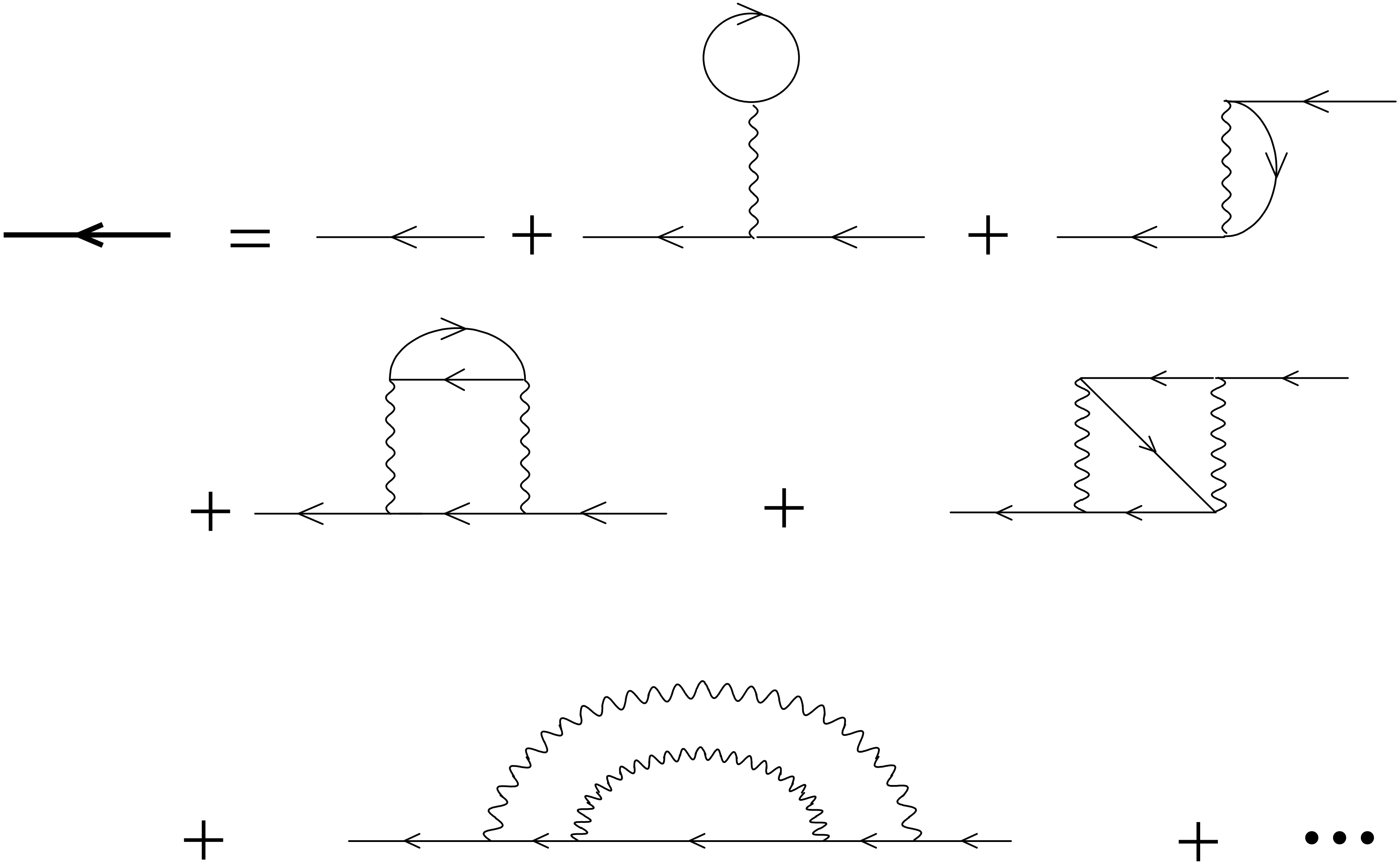
- Thermodynamic equilibrium: Fetter, Walecka, *Quantum Theory of Many Particle Systems*
- Optically excited semiconductors: Kwong, Binder, Phys. Rev. B 61, 8341 (2000)
- Semiconductors with quantized light: Kwong, Rupper, Binder, Phys. Rev. B 79, 155205 (2009)

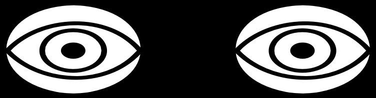
Wick's theorem: sum up all different graphs



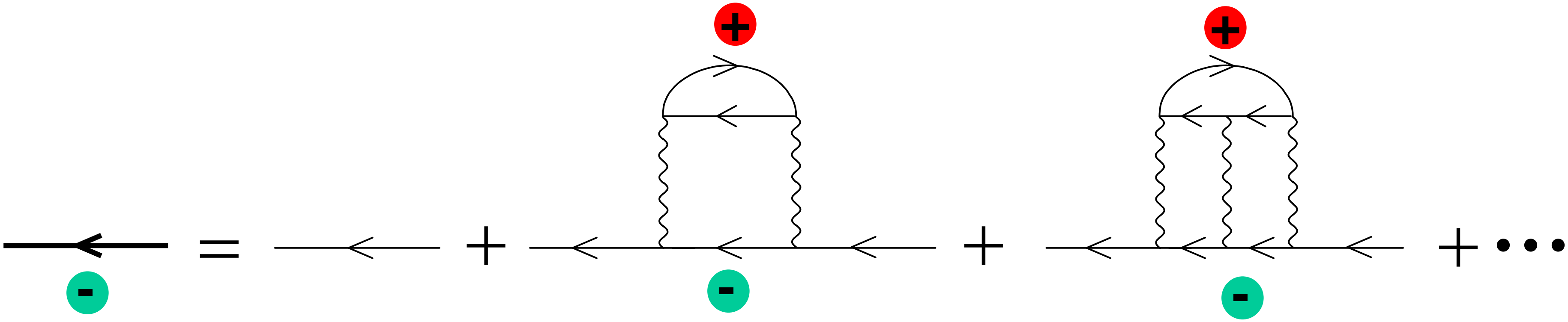


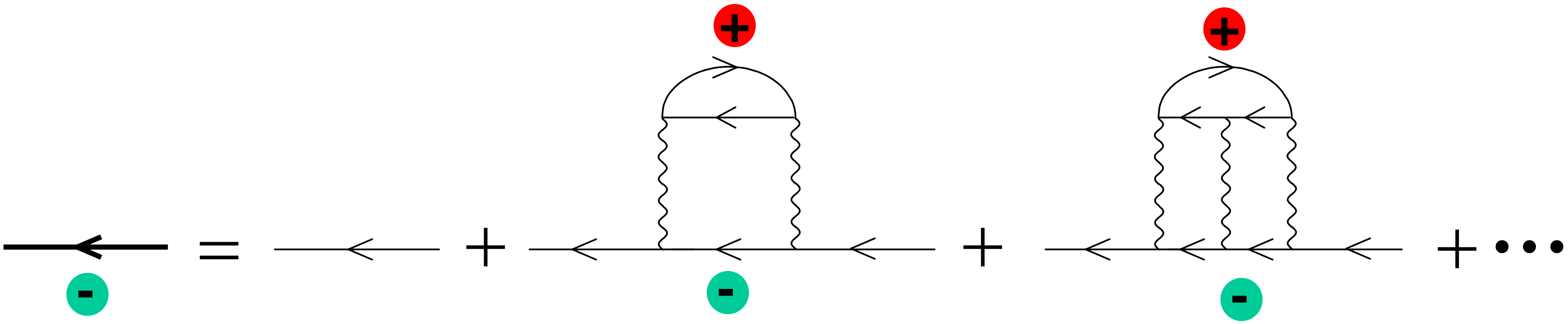
Wick's theorem: sum up all different graphs

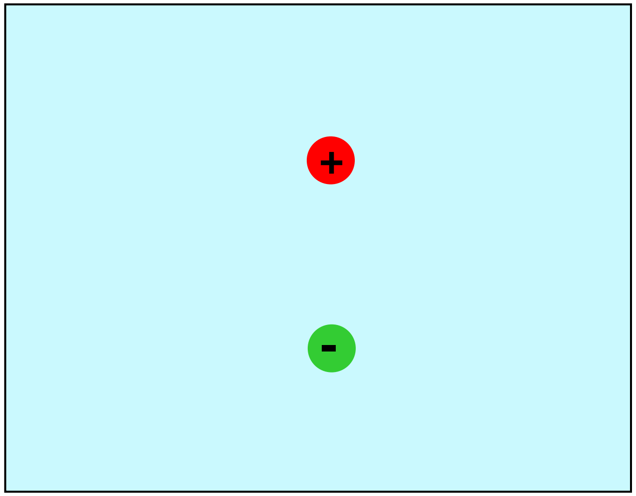




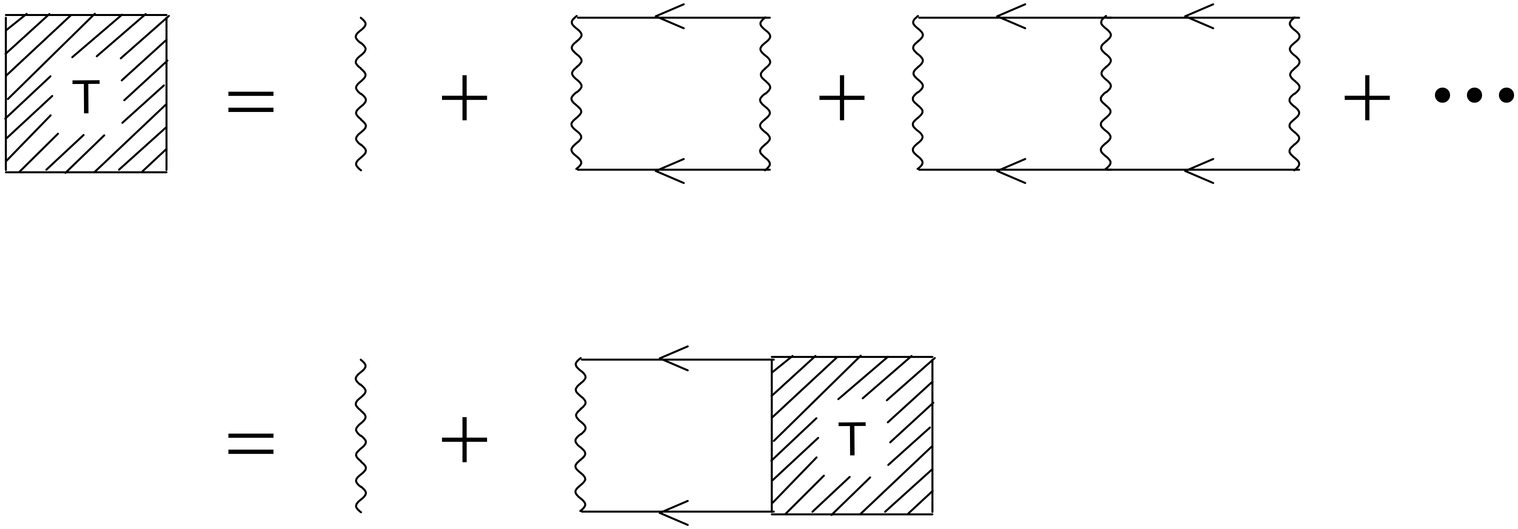
All of them???







Ladder diagrams



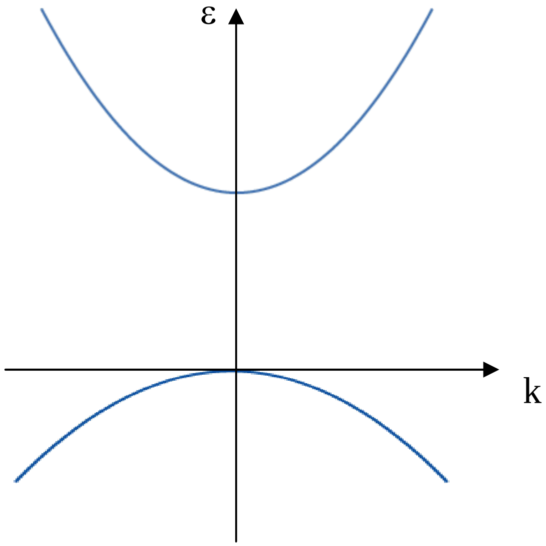
describes non-perturbative Coulomb correlation (including possible **bound states**)

Next:

Third-order nonlinear optical response and excitonic diagrams

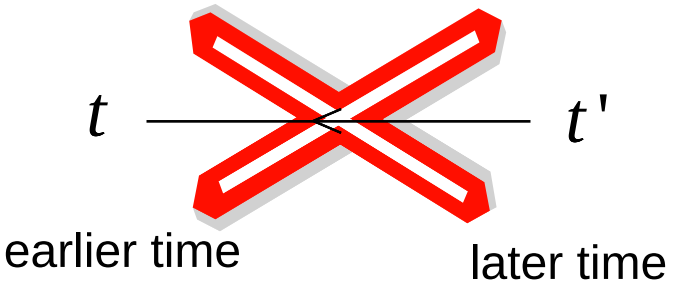
Dynamics-controlled truncation (DCT)

(Axt, Stahl, Z. Phys. B 93, 205 (1994))

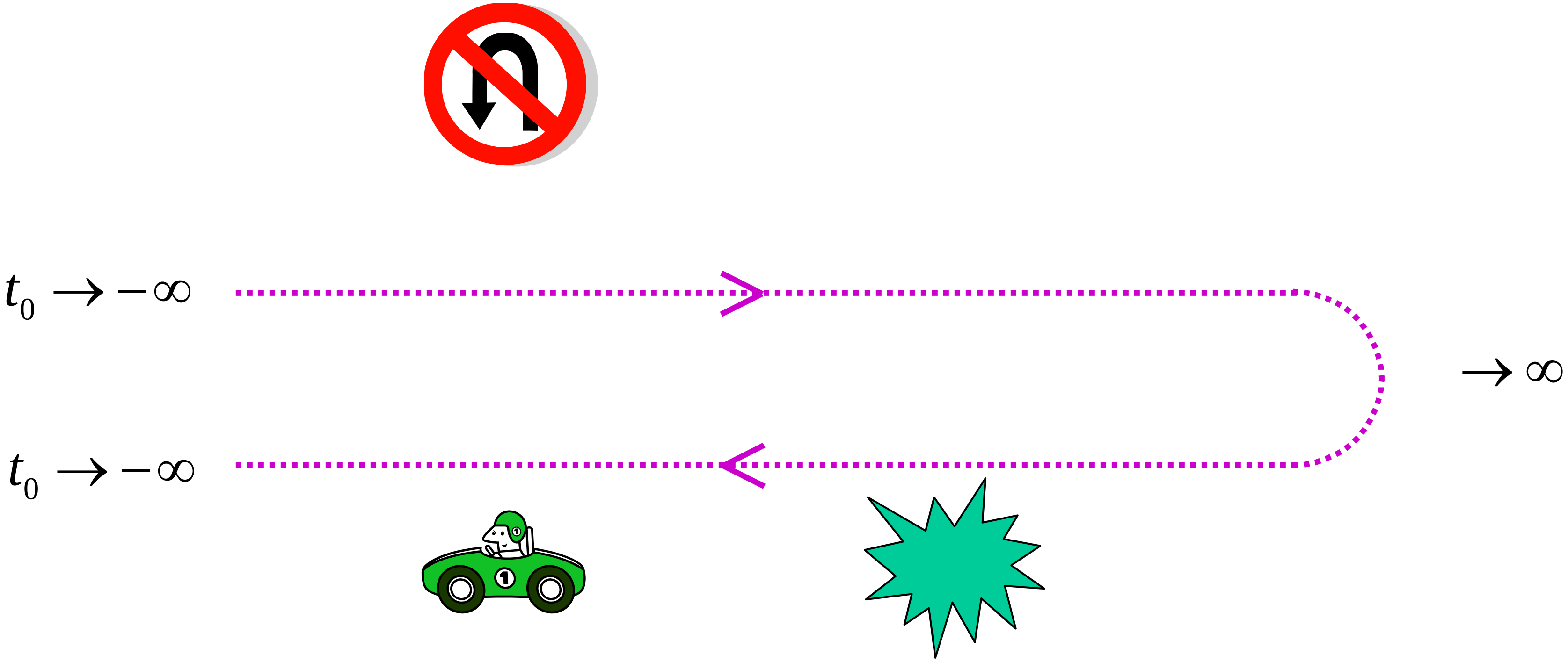


Without optical excitation, no density-like propagators:

$$\langle\, a_I^\dagger(t') a_I(t) \,\rangle$$



Green's function approach to DCT: Kwong, Binder, Phys. Rev. B 61, 8341 (2000)



$t_0 \to -\infty$

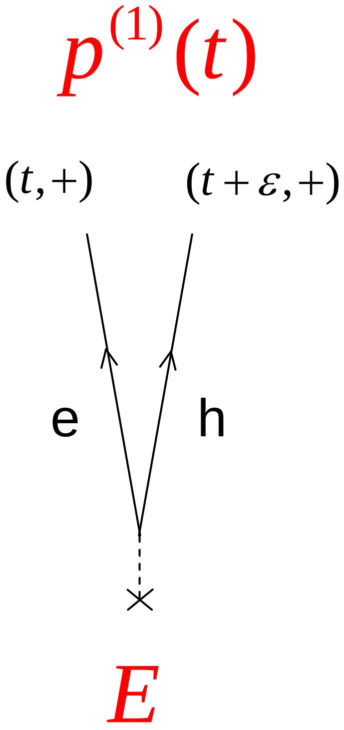
$\to \infty$

$t_0 \to -\infty$

DCT: "Don't Counterpropagate in Time"

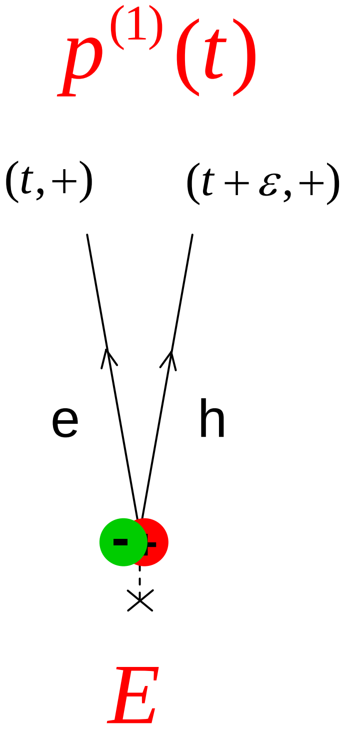
First-order polarization

$$\langle a_h(t)\, a_e(t) \rangle$$

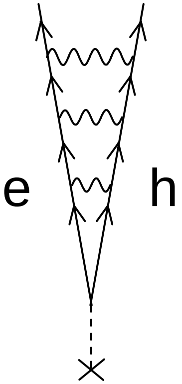


First-order polarization

$$\langle\, a_h(t)\; a_e(t)\,\rangle$$



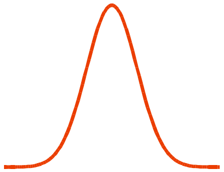
sum up ladder diagrams: excitons

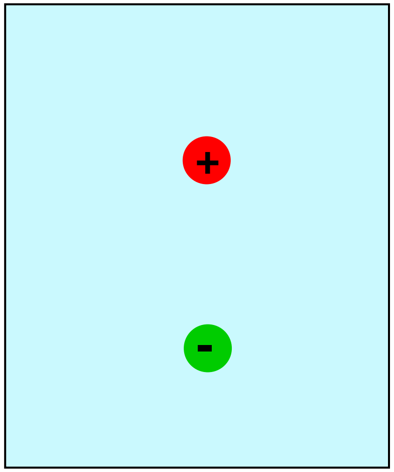


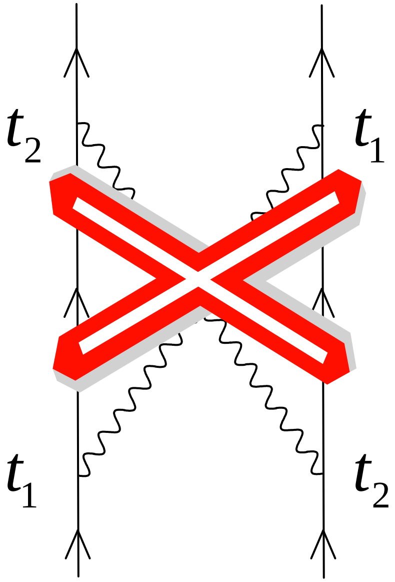
In 1s approximation: $p_{eh}(\boldsymbol{k},\boldsymbol{q},t) \cong \phi_{1s}(\boldsymbol{k})\; p(\boldsymbol{q},t)$

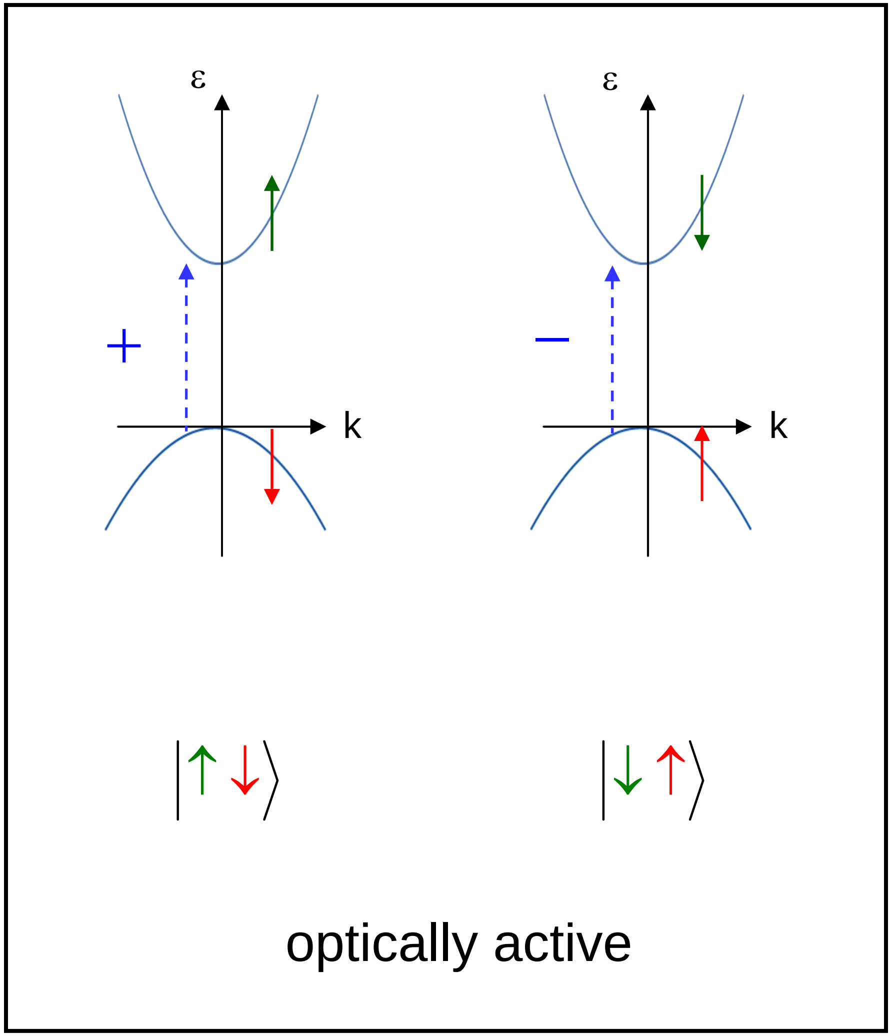
relative e-h momentum

center-of-mass momentum $\boldsymbol{q} \cong 0$

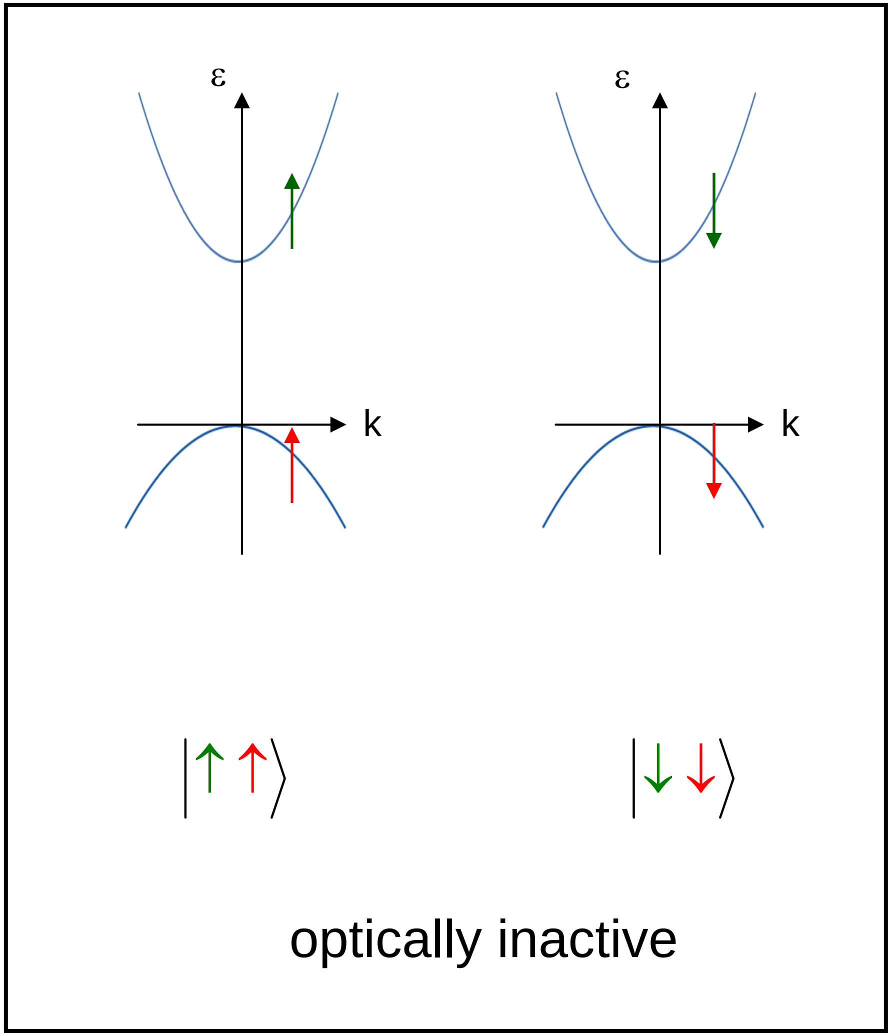





t_2
t_1
t_1
t_2



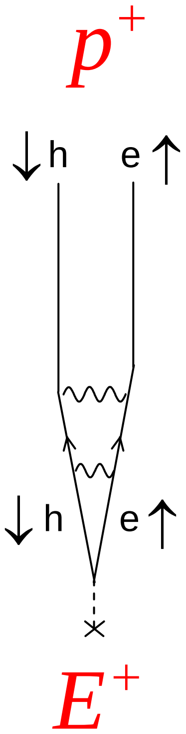
optically active



optically inactive

notation: | electron spin, hole spin⟩

First order excitonic response



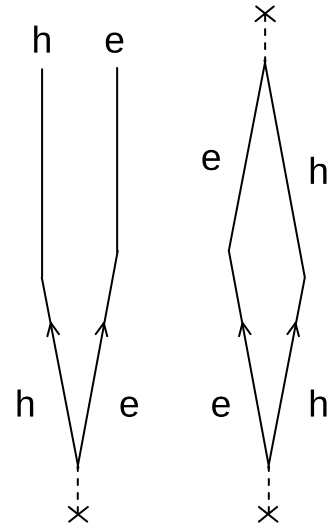
In 1s approximation:

$$i\hbar\,\dot{p}^+ = (\varepsilon_x - i\gamma)\,p^+ - \phi^*(0)\,E^+$$

with $E^+ \equiv \boldsymbol{d}_{\mathrm{cv}}\boldsymbol{E}^+$

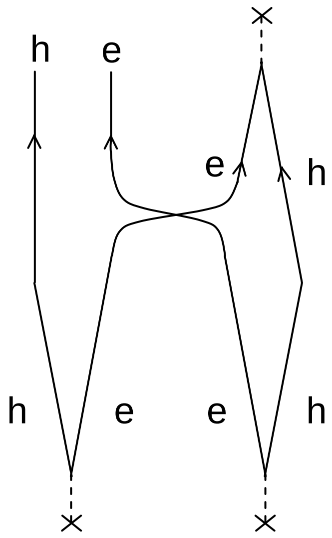
The two basic third-order diagrams

$\langle a_h a_e \rangle$



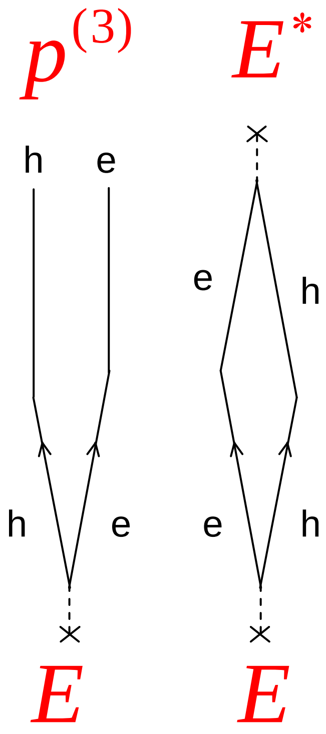
direct

$\langle a_h a_e \rangle$

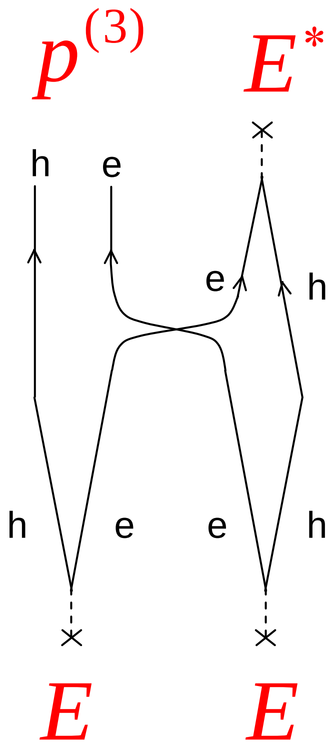


electron exchange

The two basic third-order diagrams



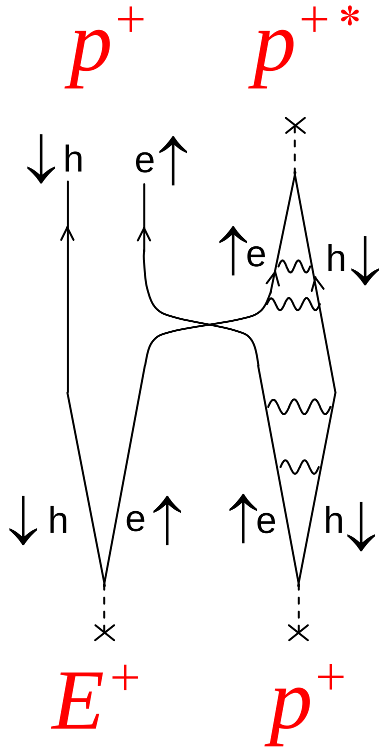
direct



electron exchange

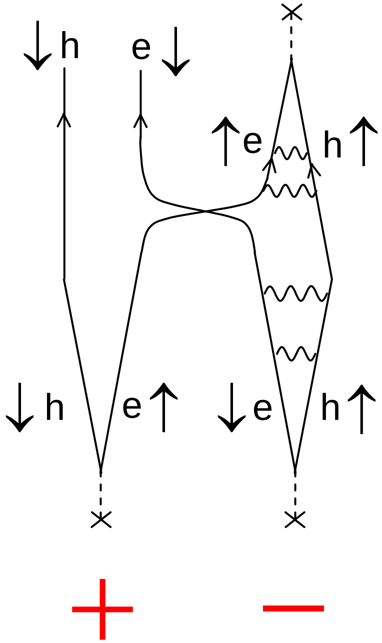
Without Coulomb interaction,
this diagram is disconnected
(does not contribute)

Phase-Space Filling (PSF)



$$i\hbar\dot{p}^{+}=(\varepsilon_x-i\gamma)p^{+}-\phi_{1s}^{*}(0)\,E^{+}+2A^{\mathrm{PSF}}\,|\,p^{+}\,|^{2}\,E^{+}$$

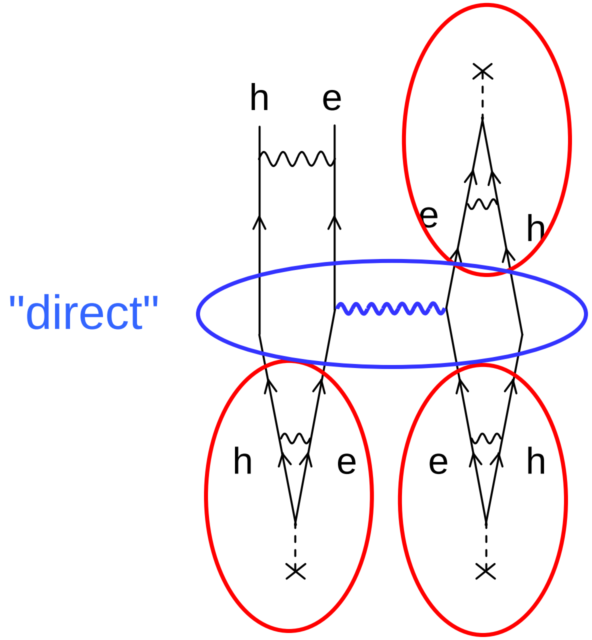
not possible:



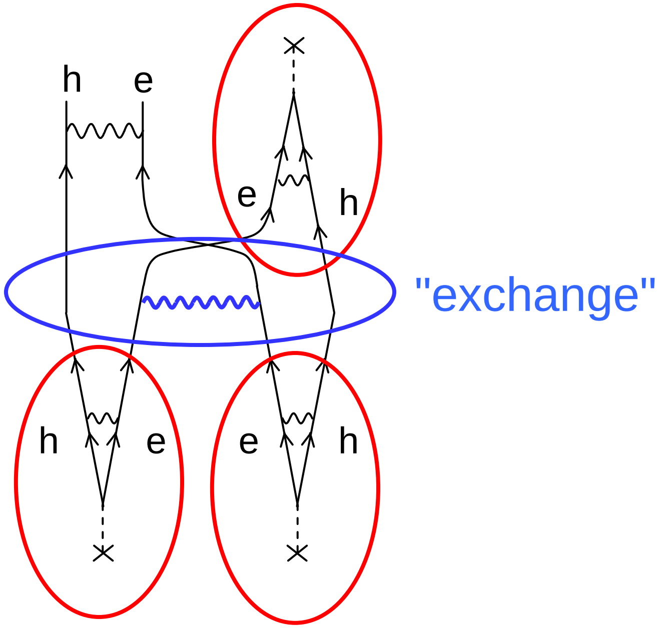
$|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$

not optically active

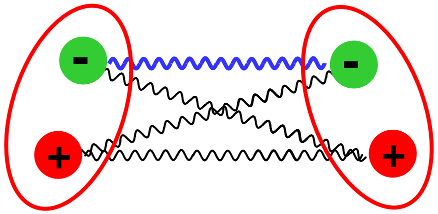
$$A^{\mathrm{PSF}}\, p^{+}\; p^{-}\; E^{+}$$



exciton-exciton interaction
(1 of 4 contributions)

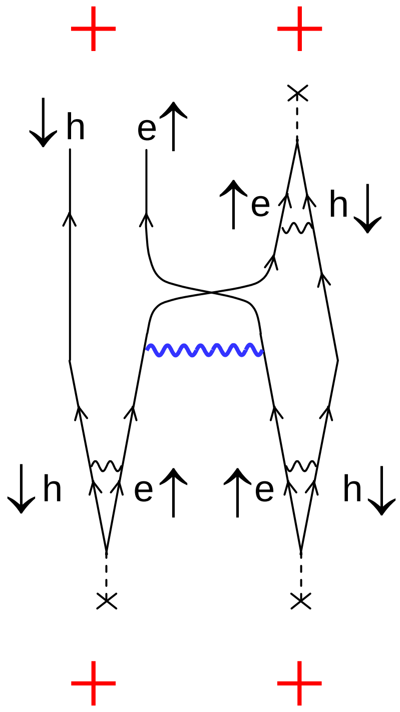


excitons
q $\cong 0$



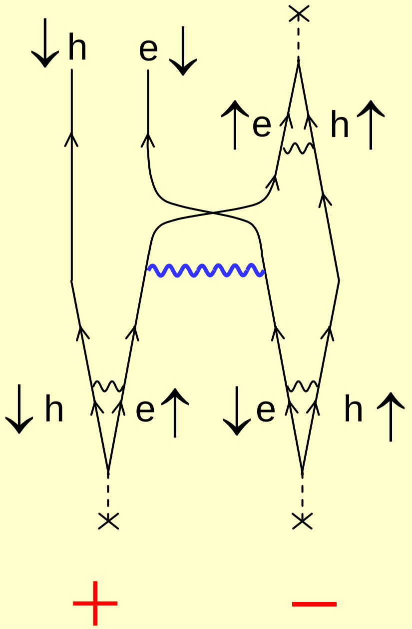
$$i\dot{p}\Big|_{HF} = (W^{dir} + W^{exc})\,|\,p\,|^2\;p$$

$$i\hbar\,\dot{p}^{+} = (\varepsilon_x - i\gamma)\,p^{+} - \text{(PSF term)} + V^{\mathrm{HF}}\,|\,p^{+}|^{2}\;p^{+}$$

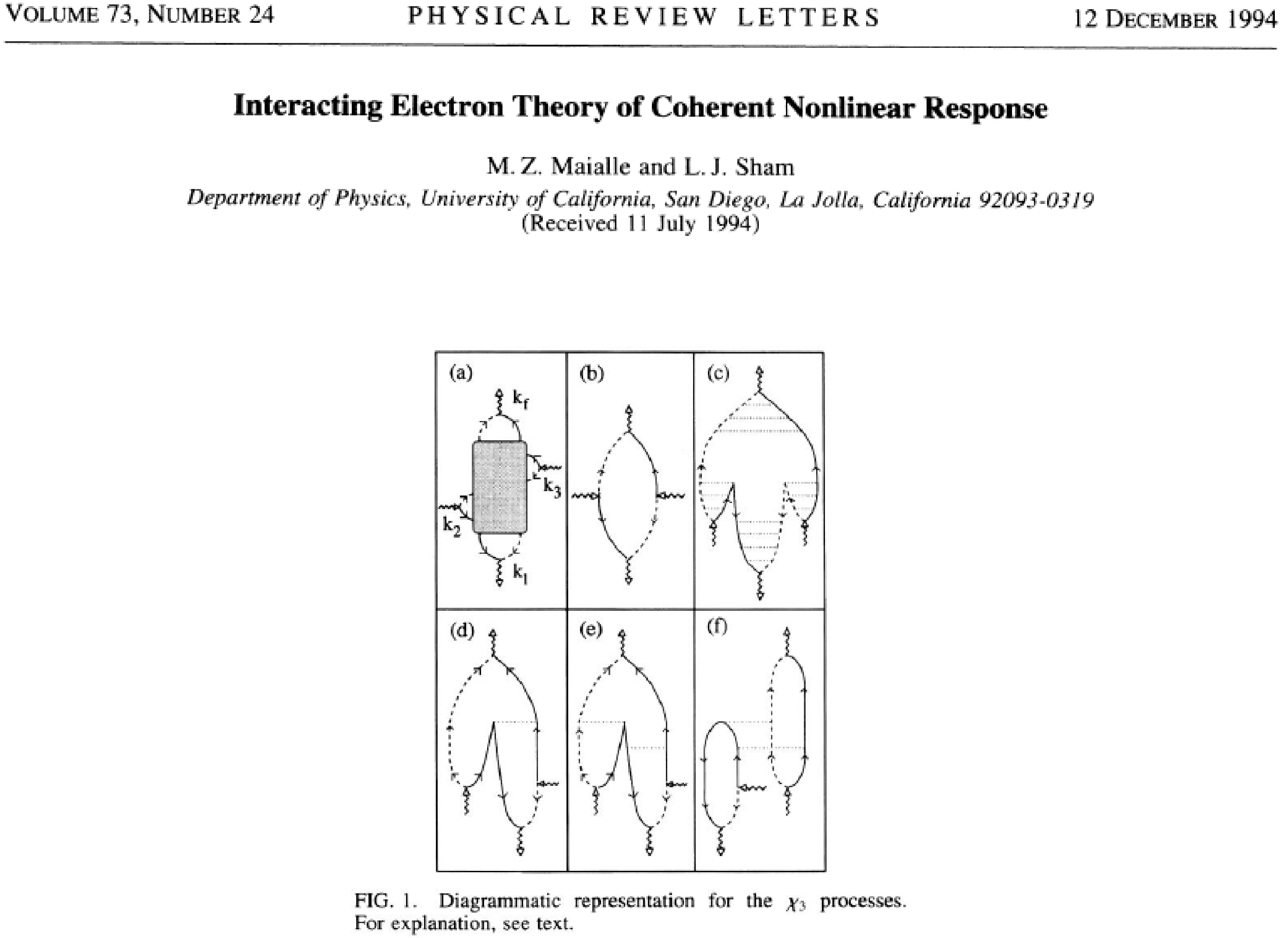


Hartree term zero, Fock term only ++

optically inactive



cannot be: $|\,p^{+}|^{2}\;p^{-}$



VOLUME 73, NUMBER 24 PHYSICAL REVIEW LETTERS 12 DECEMBER 1994

Interacting Electron Theory of Coherent Nonlinear Response

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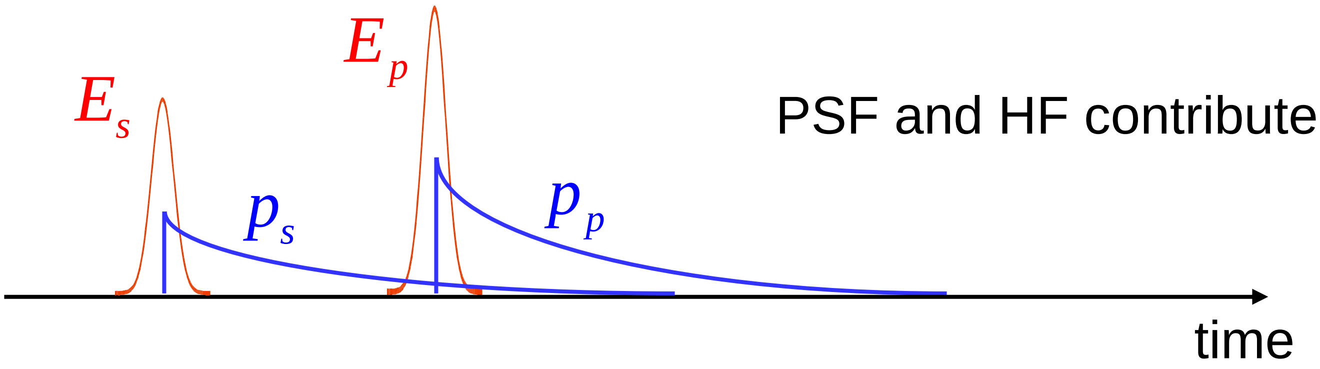
(Received 11 July 1994)

FIG. 1. Diagrammatic representation for the χ_3 processes. For explanation, see text.

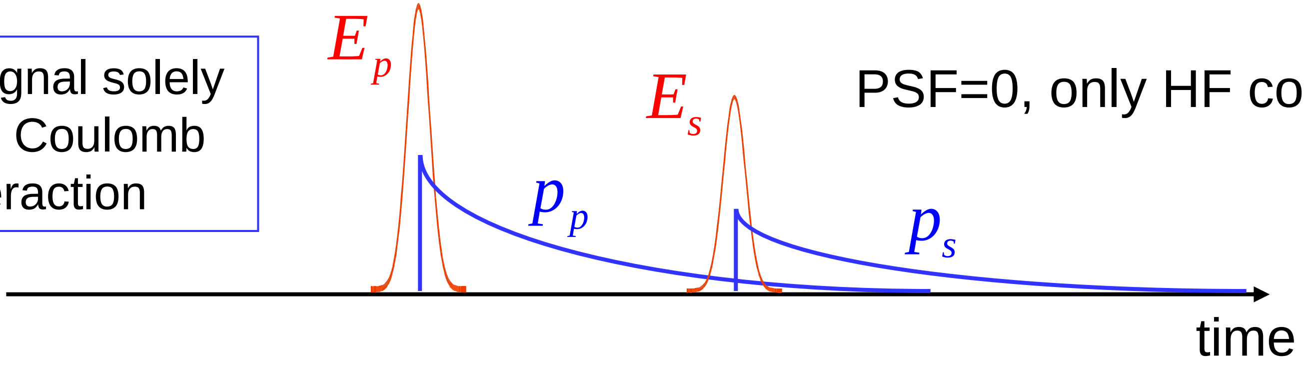
Phase-space filling vs Hartree-Fock in FWM

$$i\hbar \dot{p}_f = (\varepsilon_x - i\gamma) p_f + 2A^{\mathrm{PSF}} p_s^* p_p E_p + V^{\mathrm{HF}} p_s^* p_p p_p$$



PSF and HF contribute

FWM signal solely due to Coulomb interaction



PSF=0, only HF contributes

PHYSICAL REVIEW A VOLUME 42, NUMBER 9 1 NOVEMBER 1990

Line shape of time-resolved four-wave mixing

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(Received 8 May 1990)

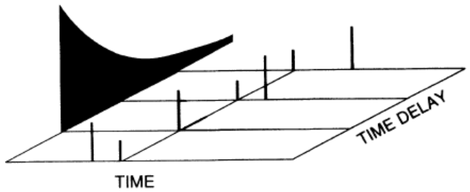

FIG. 5. Graphical representation of the predictions of the noninteracting two-level model of Ref. 3 for δ pulses. The temporal position of the two incident pulses is marked by the spikes. Pulse 2 is displayed with twice the strength of pulse 1. On the second line the two pulses overlap, which corresponds to zero time delay. The light areas represent the square of the magnitude of the third-order polarization as a function of time for various time delays. The full area on the left is the energy of the diffracted signal as a function of the time delay (i.e., the time integral of the light area at a given time delay).

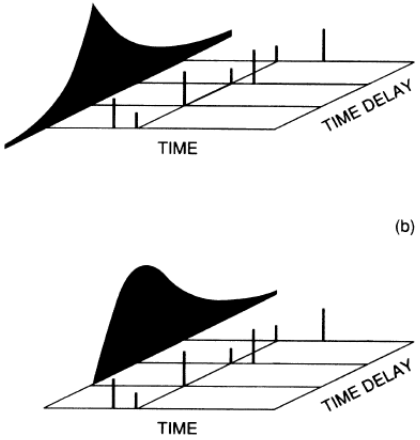

FIG. 7. Same as Fig. 5, but including local-field corrections. Results for (a) a homogeneously broadened line and (b) a strongly inhomogeneously broadened line are depicted. In contrast to the noninteracting two-level model, Fig. 5, one also finds a signal for negative time delay. This additional signal, however, is quite sensitive to inhomogeneous broadening and eventually disappears completely for a strongly inhomogeneously broadened line.

homogeneously broadened

inhomogeneously broadened

negative delay time: pump first

VOLUME 65, NUMBER 11 PHYSICAL REVIEW LETTERS 10 SEPTEMBER 1990

Effects of Coherent Polarization Interactions on Time-Resolved Degenerate Four-Wave Mixing

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(Received 13 April 1990)

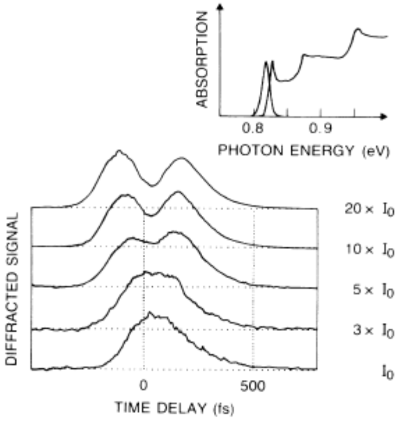


FIG. 3. DFWM signals in InGaAs/InAlAs vs time delay for a lattice temperature of 5 K, 10-meV laser detuning, and different excitation intensities. Inset: Sample absorption at 5 K and the pulse spectrum.

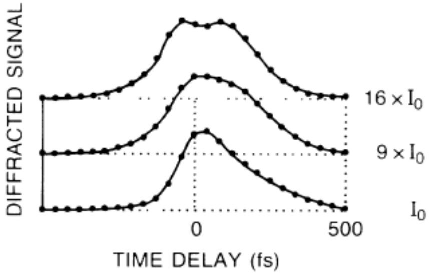
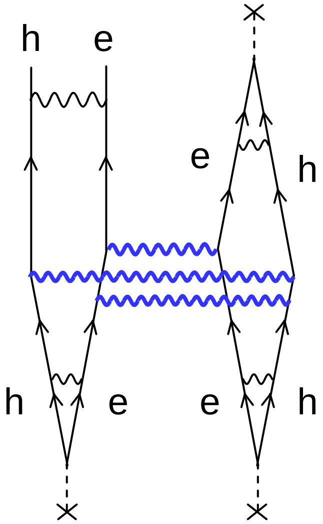


FIG. 4. Calculated DFWM signals vs time delay for different excitation intensities.

negative delay time: pump first

Beyond Hartree-Fock: Excitonic Correlation Functions



Excitonic correlation functions:

$$G^{++} \leftrightarrow \ \mathrm{sum}\left(W^{dir} - W^{exc}\right)$$

$$G^{+-} \leftrightarrow \ \mathrm{sum}\left(W^{dir} - W^{exc}\right)$$

$$+\,\mathrm{sum}\left(W^{dir} + W^{exc}\right)$$

$$i\hbar\dot{p}^{+} = (\varepsilon_x - i\gamma)p^{+} - [\phi_{1s}^{*}(0) - 2A^{\mathrm{PSF}} |p^{+}|^{2}] E^{+}$$

Phase-space filling

$$+V^{\mathrm{HF}} |p^{+}|^{2} p^{+}$$

Hartree-Fock
Coulomb interaction

$$\left.\begin{array}{l} +2\, p^{+*} \int_{-\infty}^{\infty} dt'\ G^{++}(t-t')\, p^{+}(t')\, p^{+}(t') \\ +\, p^{-*} \int_{-\infty}^{\infty} dt'\, G^{+-}(t-t')\, p^{+}(t')\, p^{-}(t') \end{array}\right\}$$

Time-retarded
two-exciton
correlations
(incl. biexciton)

Takayama, Kwong, Rumyantsev, Kuwata-Gonokami, Binder, Eur. Phys. J. B 25, 445 (2002)

Same equation for the coherent third order interband polarization:

- **Dynamics Controlled Truncation**
  Axt , Stahl, Z. Phys. B 93, 195 (1994)
- **Hubbard operators, force-force correlation function**
  Oestreich, Schoenhammer, Sham, Phys. Rev. B 58,12920 (1998)
- **Cumulant expansions**
  Meier, Koch, Phys. Rev. B 59, 13202 (1999);
  Hoyer, Kira, S.W. Koch, Phys. Rev. B 67, 155113 (2003)
- **Nonequilibrium Green's functions**
  Kwong, Binder, Phys. Rev. B 61, 8341 (2000)

see also: Schäfer, Wegener, *Semiconductor Optics and Transport Phenomena*
(Springer, Berlin, 2002)

Degenerate FWM
(all fields at frequency ω)

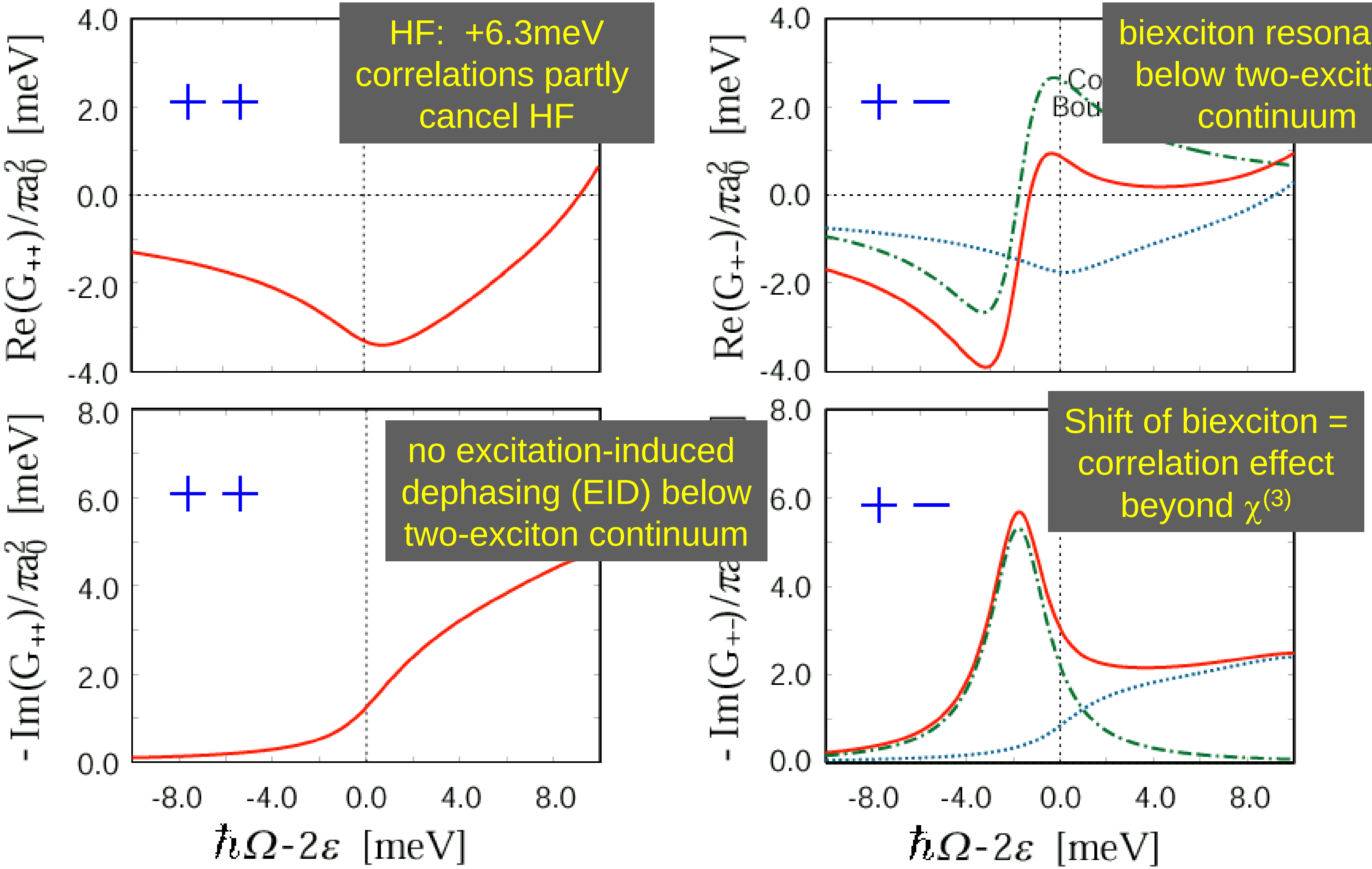
$$p_f^{\pm}(\omega) = \chi^{++}(\omega)\, E_{p\pm}^{2}(\omega)\, E_{s\pm}^{*}(\omega) + \chi^{+-}(\omega)\, E_{s\mp}^{*}(\omega) E_{p\pm}(\omega) E_{p\mp}(\omega)$$

$$\chi^{++}(\omega) \sim \left|\chi^{(1)}(\omega)\right|^{2} \left[\chi^{(1)}(\omega)\right] \left\{G^{PSF}(\omega) + V^{HF} + 2G^{++}(2\omega)\right\}$$

$$\chi^{+-}(\omega) \sim \left|\chi^{(1)}(\omega)\right|^{2} \left[\chi^{(1)}(\omega)\right] G^{+-}(2\omega)$$

with $\chi^{(1)}(\omega) \sim \dfrac{1}{\omega - \varepsilon_x + i\gamma}$ $\quad G^{PSF}(\omega) \sim 1/\chi^{(1)}(\omega)$

- Takayama, Kwong, Rumyantsev, Kuwata-Gonokami, Binder, JOSA-B 21, 2164 (2004)
- Kwong, Takayama, Rumyantsev, Kuwata-Gonokami, Binder, Phys. Rev. B 64, 045316 (2001)



HF: +6.3meV
correlations partly
cancel HF

biexciton resonance
below two-exciton
continuum

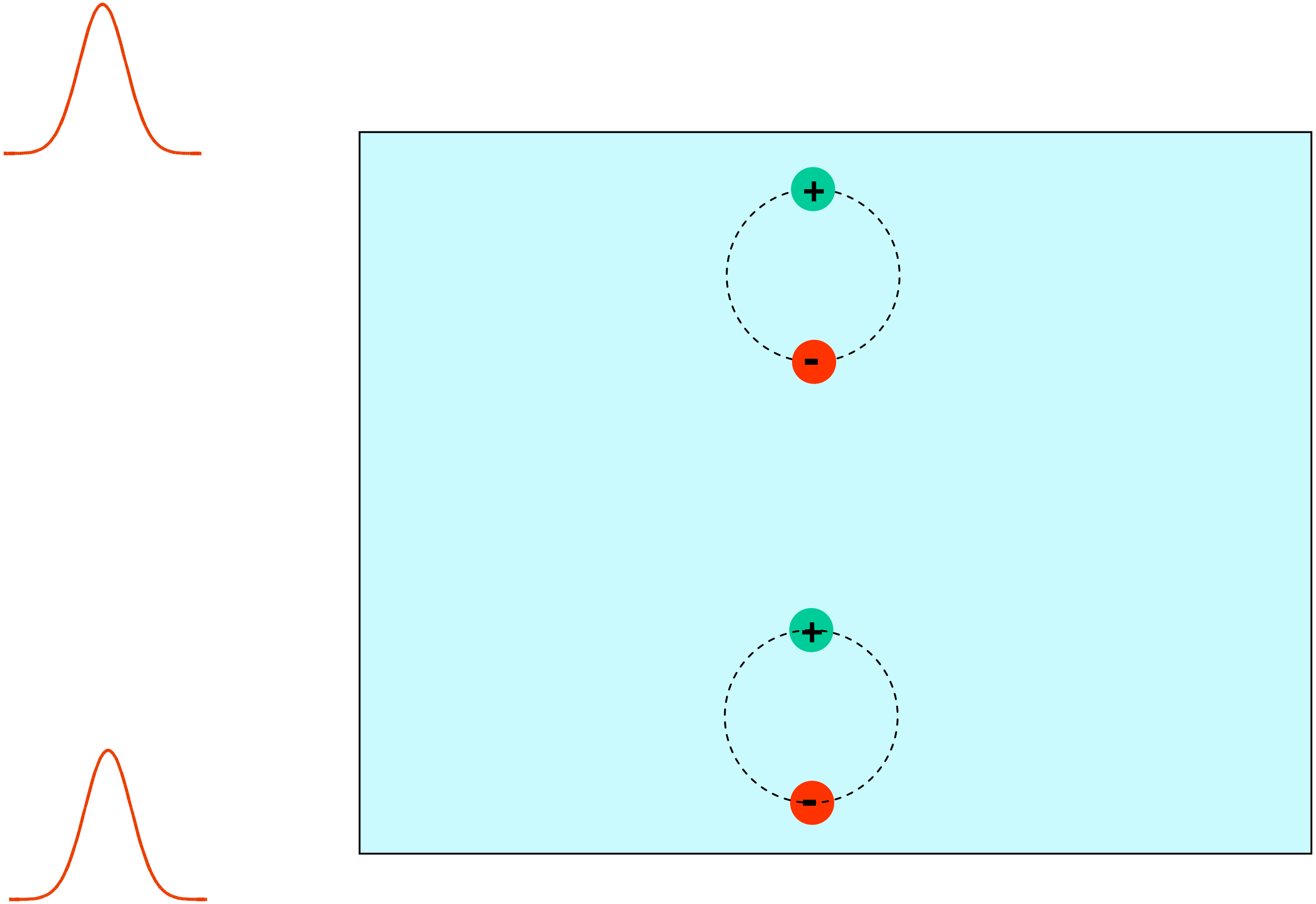
no excitation-induced
dephasing (EID) below
two-exciton continuum

Shift of biexciton =
correlation effect
beyond $\chi^{(3)}$

Takayama, Kwong, Rumyantsev, Kuwata-Gonokami, Binder, Eur. Phys. J. B 25, 445 (2002)

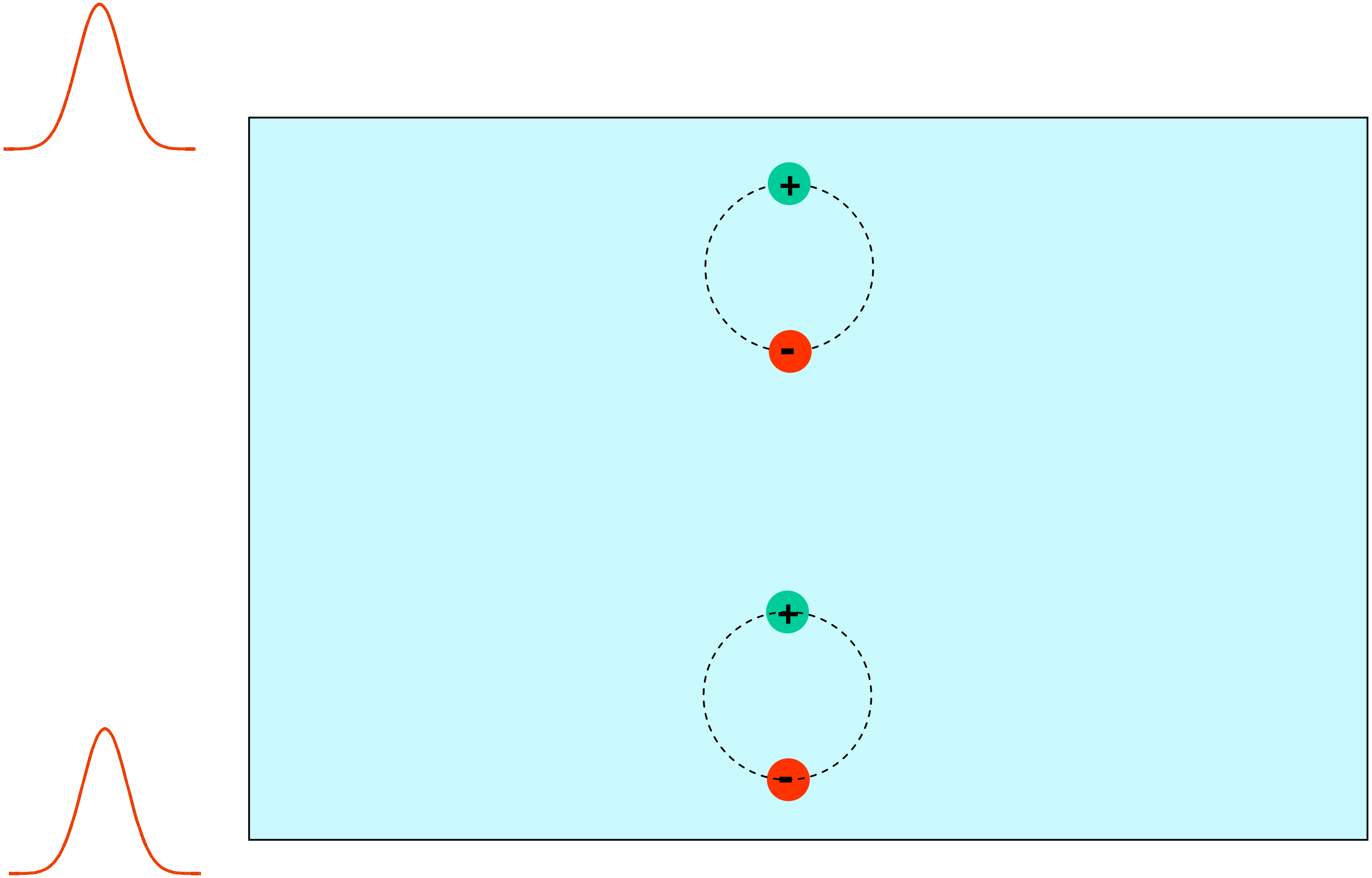
$$G^{+-}\ p^{+}\ p^{-}$$

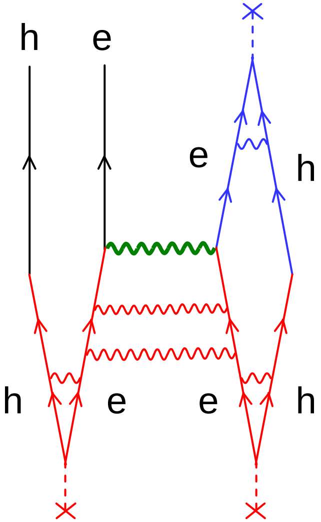
includes bound two-exciton states (biexciton)



$G^{++}\ p^{+}\ p^{+}$

only continuum states





$$i\frac{d}{dt}\langle\, a\, a\,\rangle\bigg|_{corr} = W\,\langle a^\dagger\, a^\dagger\rangle\langle a\; a\; a\; a\,\rangle_{corr}$$

Hierarchy of correlation functions

$$\langle\, a\, a\,\rangle \xleftrightarrow[W]{} \langle a\; a\; a\; a\,\rangle \xleftrightarrow[W]{} \langle a\; a\; a\; a\; a\; a\,\rangle \xleftrightarrow[W]{} \cdots$$

Hierarchy of correlation functions

$$\langle \underbrace{a_1^+ \cdots a_n^+}_{n} \; \underbrace{a_1 \cdots a_m}_{m} \rangle$$

Proven:

If initial e-h density zero, one has, exact to order of E,

- truncation of hierarchy
- factorization to yield closed set of equations of motion

Axt, Stahl, Z. Phys. B 93, 205 (1994)

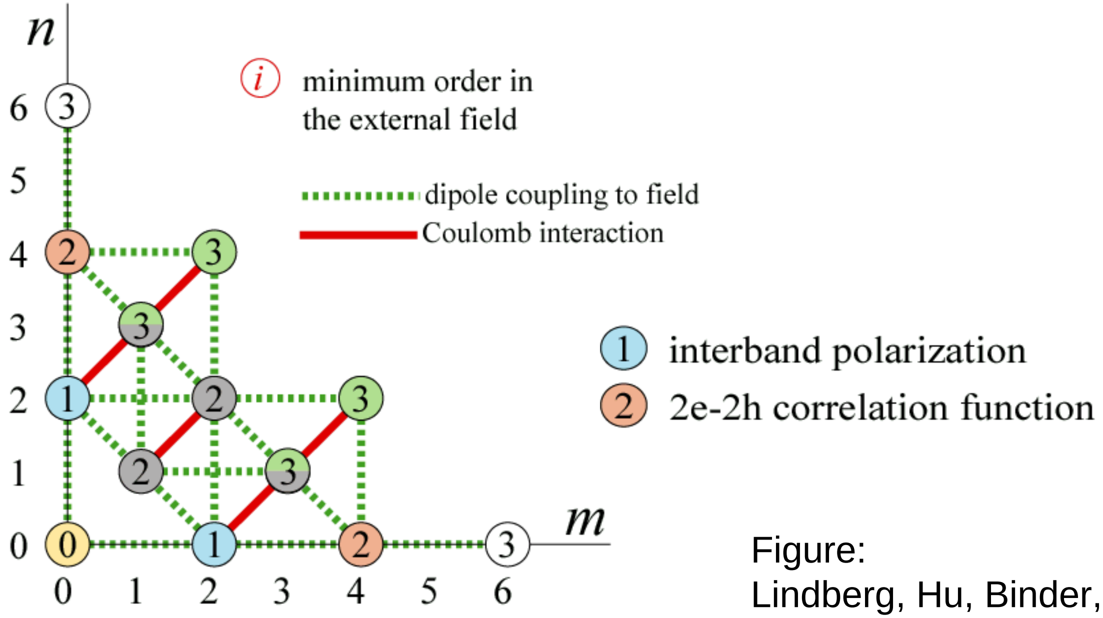


Figure:
Lindberg, Hu, Binder, Koch, Phys. Rev. B 50, 18060 (1994)

Next:

Some experimental FWM data

1016 J. Opt. Soc. Am. B/Vol. 13, No. 5/May 1996

Time-resolved measurements of the polarization state of four-wave mixing signals from GaAs multiple quantum wells

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J. G. Pellegrino

National Institute of Standards and Technology, Gaithersburg, Maryland 20899

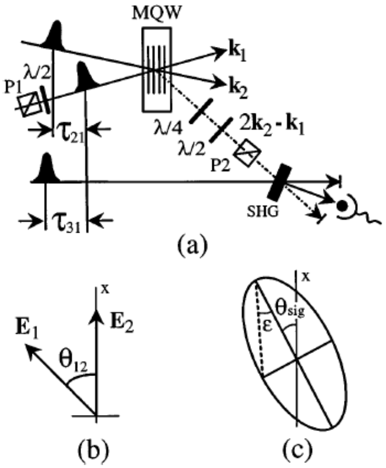


Fig. 1. Schematic of (a) the experimental geometry for time resolving the polarization state of the emitted FWM signal, where $\lambda/4$ denotes quarter-wave plate; $\lambda/2$, half-wave plate; SHG, second-harmonic-generation crystal; and P1 and P2, polarizers; (b) the orientation of the two linear input polarizations; and (c) the polarization ellipse showing the azimuthal angle θ_{sig} and the ellipticity angle ε.

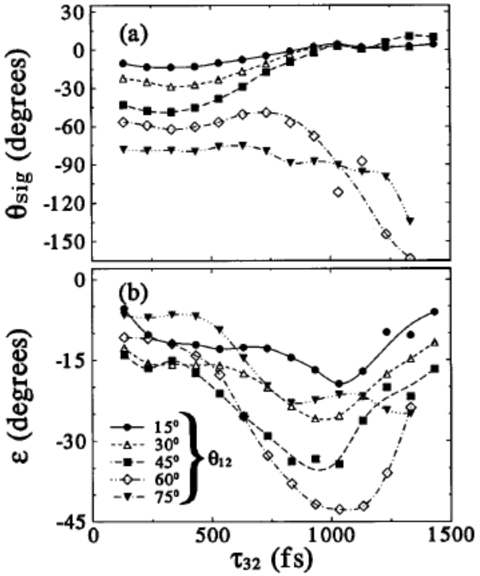


Fig. 2. Measurements of (a) the azimuthal angle θ_{sig} and (b) the ellipticity angle ε as a function of time for selected angles θ_{12} between the two input polarizations for a total peak fluence of 1.0 $\mu J/cm^2$. The curves connect the data points only as a rough guide for the eye.

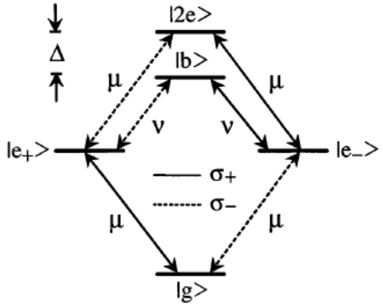
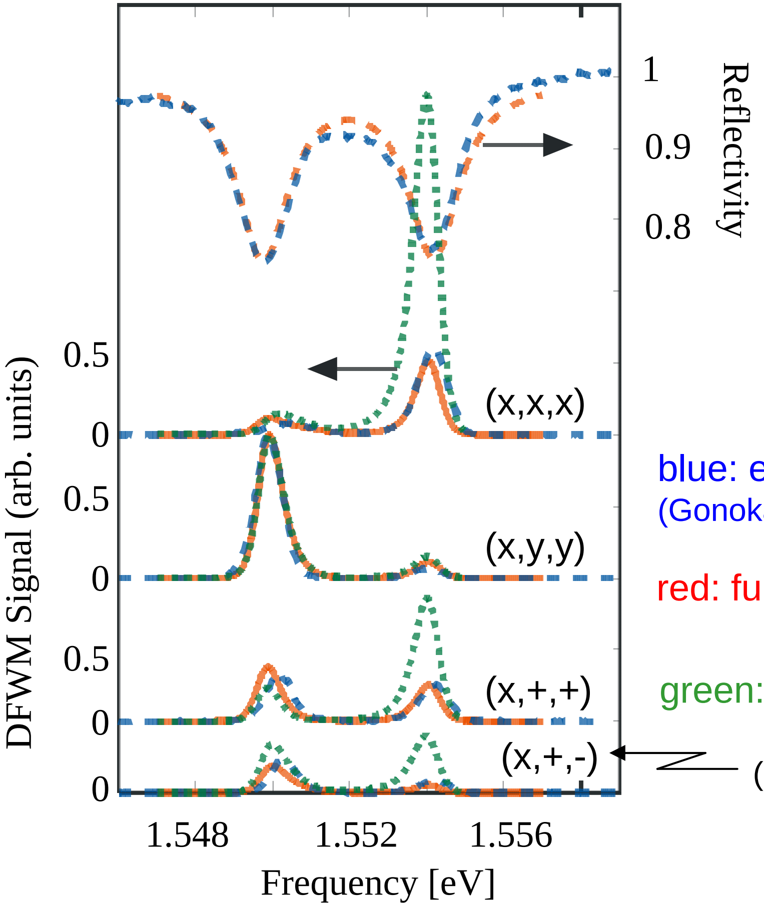


Fig. 6. Schematic of the five-level two-particle system describing the ground state $|g\rangle$, the single-exciton states $|e_\pm\rangle$, the biexciton state $|b\rangle$, and the unbound two-exciton state $|2e\rangle$. The solid (dashed) lines represent transitions coupled by σ_+ (σ_-) light, $\hbar\Delta$ is the biexciton binding energy, and μ and ν correspond to the exciton and the biexciton dipole matrix elements, respectively.

Identified biexciton, "local field" (HF), and EID

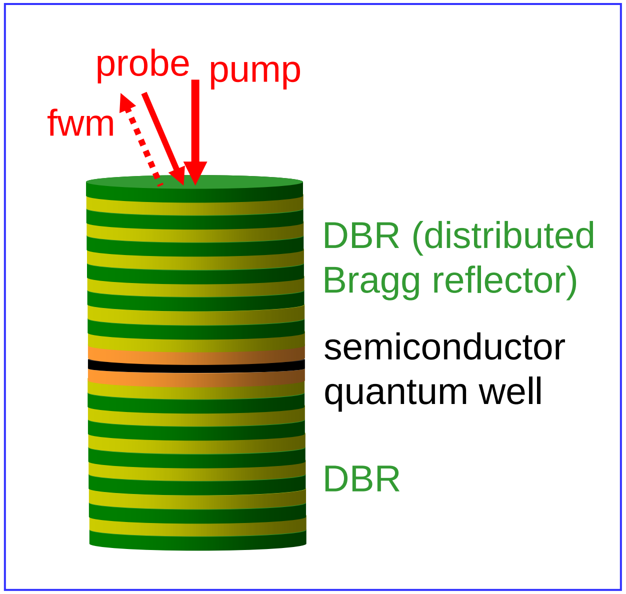
Signature of non-perturbative continuum correlations

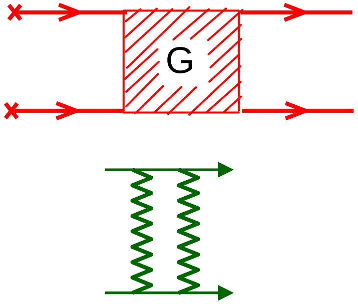


blue: experiment
(Gonokami et al., 1997)

red: full theory

green: 2nd Born





- Kwong, Takayama, Rumyantsev, Kuwata-Gonokami, Binder, Phys. Rev. Lett. 87, 27402 (2001)
- Kuwata-Gonokami, Inoue, Suzuura, Shirane, Shimano, Phys. Rev. Lett. 79, 1341 (1997)

VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

Coherent Dynamics of Excitonic Nonlinear Optical Response in the Nonperturbative Regime

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(Received 24 June 1998)

microcavity FWM

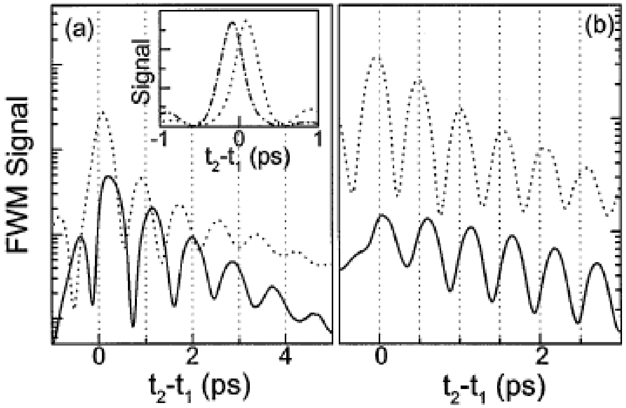

FIG. 1. Time-integrated FWM response for the upper (solid lines) and lower (dashed lines) cavity-polariton modes. (a) At zero exciton cavity detuning. (b) The cavity mode is 3 nm below the hh exciton line center. The inset in (a) also shows FWM responses near $t_2 - t_1 = 0$ along the directions of $2\mathbf{k}_2 - \mathbf{k}_1$ (dashed lines) and $2\mathbf{k}_1 - \mathbf{k}_2$ (dotted lines) for the lower cavity-polariton mode.

Experiment

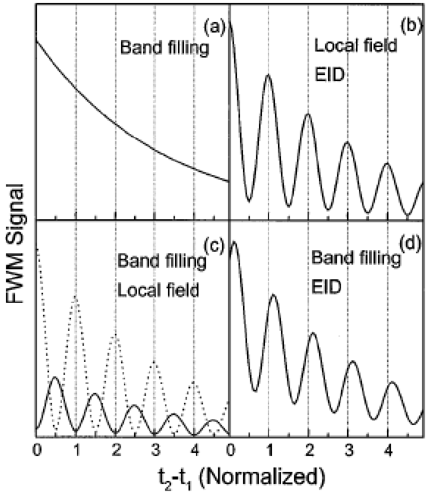

FIG. 3. Calculated time-integrated FWM response due to mechanisms indicated in the figure, where we have assumed $\delta = 0$, $\varepsilon/\Omega = -0.5$, $\sigma N_0/\Omega = 0.8$, $\gamma/\Omega = 0.05$, and $\Omega =$ 2.5 meV. The time delay is normalized to the oscillation period. Except for (c), FWM responses are identical for both cavity-polariton modes. For (c), the solid (dashed) line is for the upper (lower) cavity-polariton mode.

Theory

identified importance
of EID and local field

VOLUME 89, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 2002

Quantum Interference of Virtual and Real Amplitudes in a Semiconductor Exciton System

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(Received 7 March 2002; published 18 November 2002)

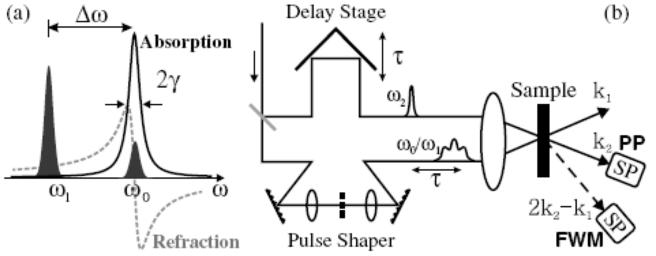
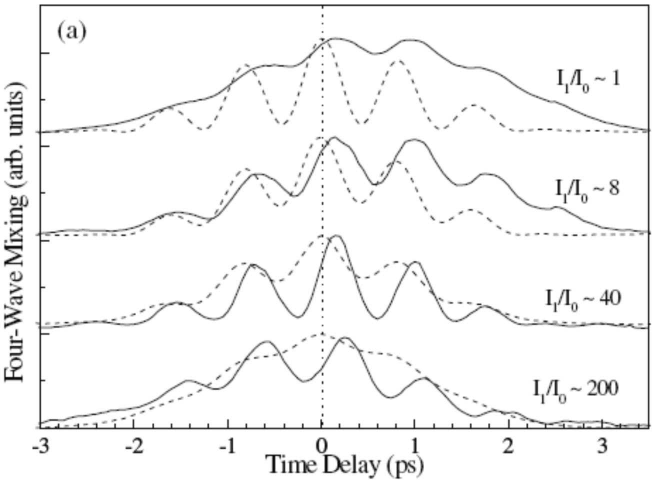


FIG. 1. (a) Schematic diagram of the experiments for a simple two-level system whose natural frequency and damping constant are ω_0 and γ, respectively. Real amplitude at ω_0 and a virtual one at ω_1 are simultaneously excited by two-color picosecond pulse. (b) Schematics of the experimental setup describing picosecond/femtosecond PP and FWM. Spectrally broad 30 fs pulses are used as a probe. SP stands for spectrometer.



two-color pump
(at and below exciton)

$$\Rightarrow G(\omega_1+\omega_2)$$

90º phase shift between FWM and cross correlation

Next:

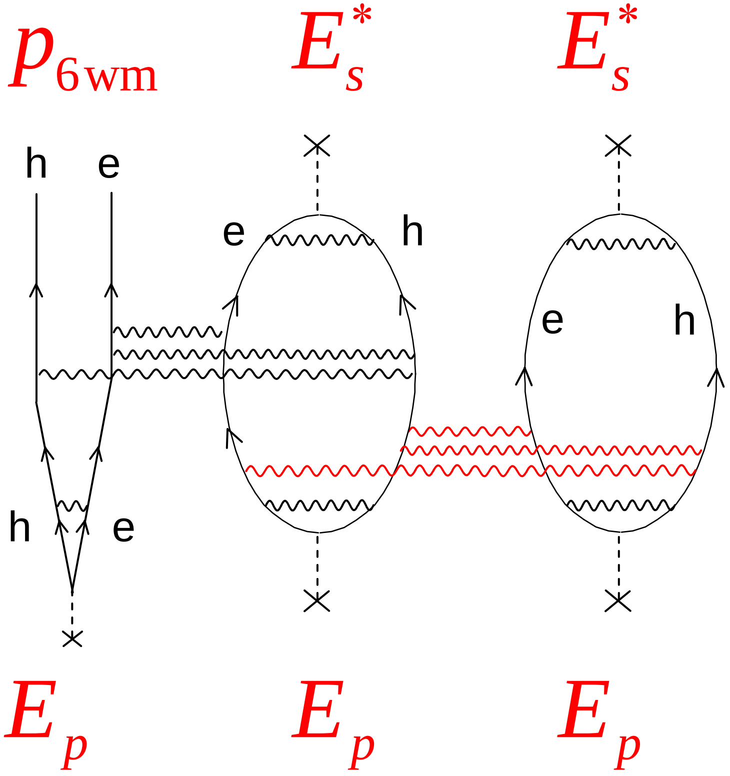
Correlations beyond $\chi^{(3)}$

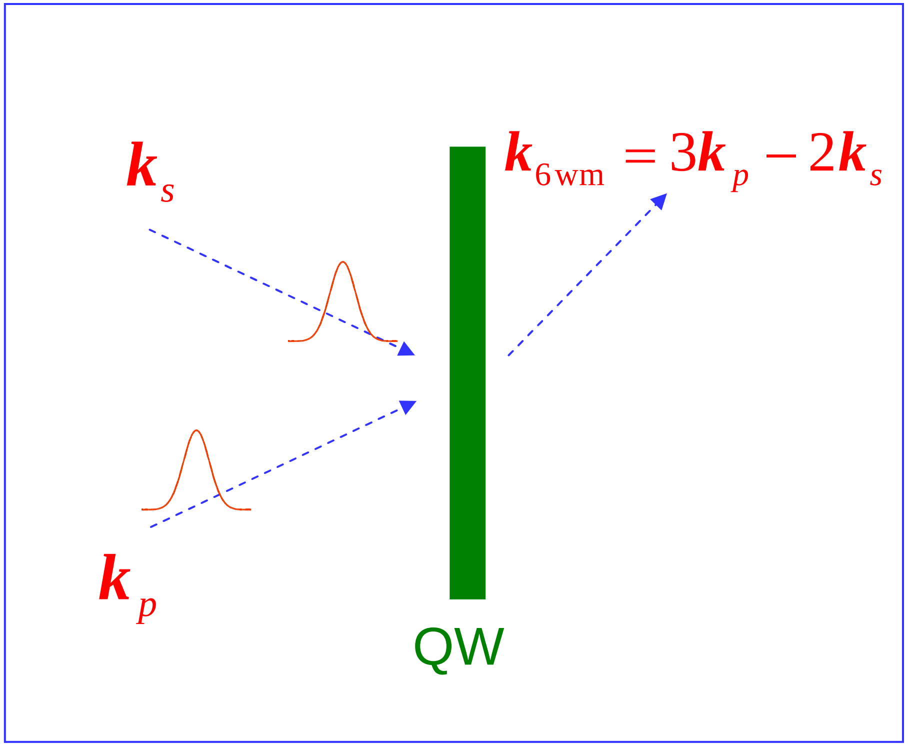
$i\dot{p} =$		
$V\,p^*\,p\,p$	$V\left\langle a^\dagger a^\dagger\right\rangle\left\langle a\,a\right\rangle\left\langle a\,a\right\rangle$	HF
$p^*\left[G\,p\,p\right]$	$V\left\langle a^\dagger a^\dagger\right\rangle\left\langle a\,a\,a\,a\right\rangle_{corr}$	biexciton
$V\,n\,p$	$V\left\langle a^\dagger a^\dagger a\,a\right\rangle_{corr}\left\langle a\,a\right\rangle$	"incoh. density"
$\left[G\,n\,p\right]$	$V\left\langle a^\dagger a^\dagger a\,a\,a\,a\right\rangle_{np}$	"incoh. dens. assist. trans."

plus:

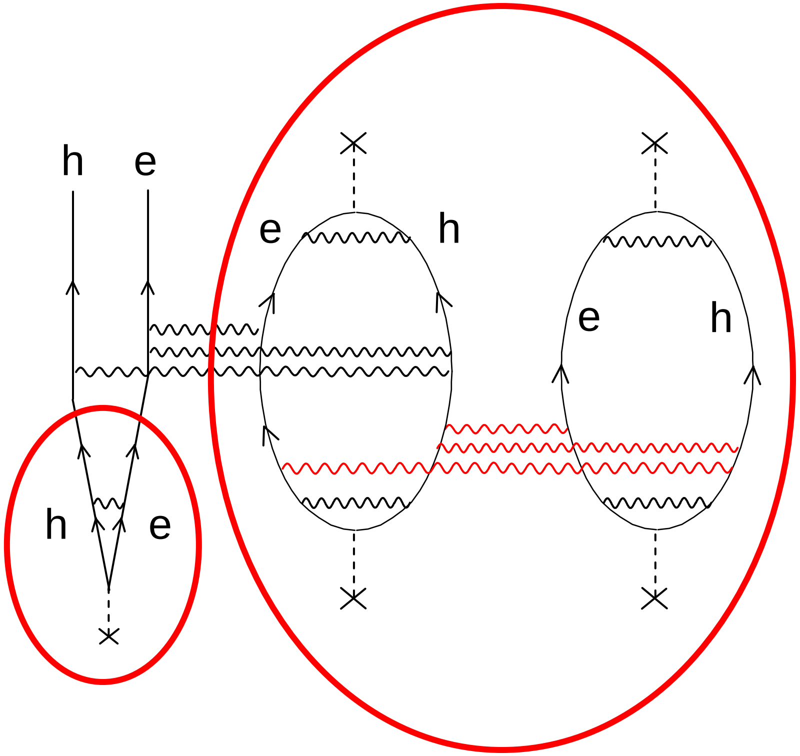
- renormalization of V or G
- triexciton

Six Wave Mixing

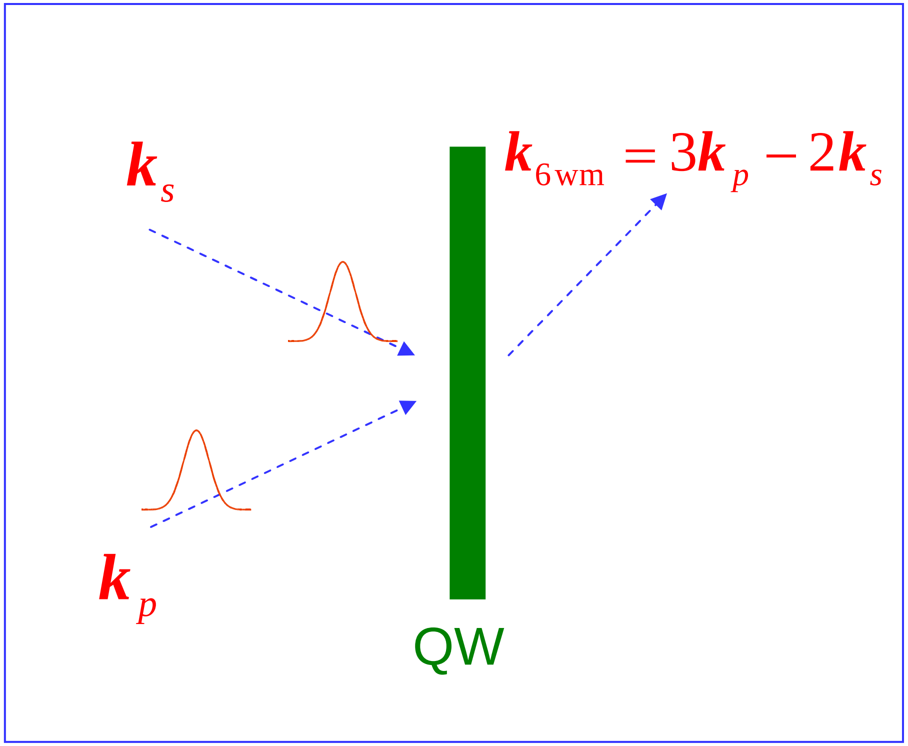




Six Wave Mixing



$$\left[G\,n\,p\right]$$



VOLUME 85, NUMBER 9 PHYSICAL REVIEW LETTERS 28 AUGUST 2000

Demonstration of Sixth-Order Coulomb Correlations in a Semiconductor Single Quantum Well

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(Received 9 March 2000)

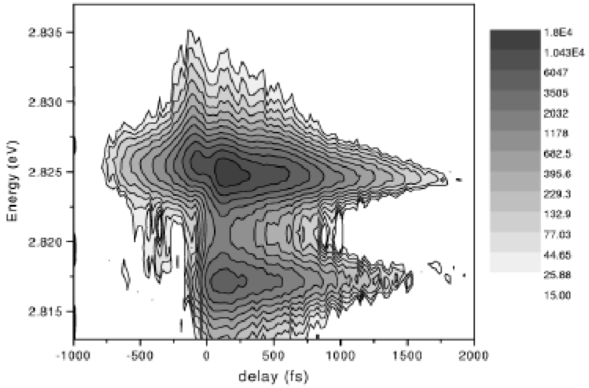

FIG. 1. Six-wave mixing emission measured in the $\{X, Y, Y\}$ configuration, shown over 3 orders of magnitude in emission intensity.

Experiment

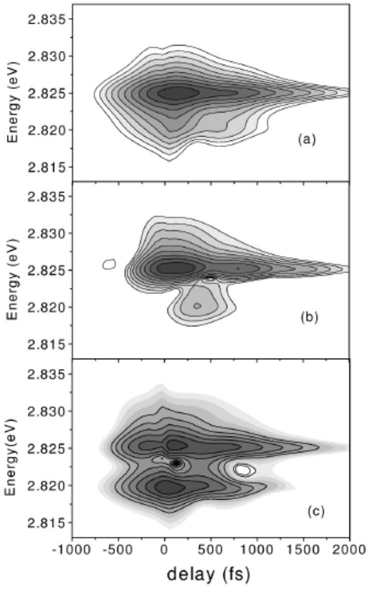

FIG. 2. Theoretical calculations of SWM emission in the $\{X, Y, Y\}$ configuration, shown over 3 orders of magnitude in emission intensity: (a) calculation in the coherent limit CL including Y and $\bar{B}$; (b) calculation including contributions of Y, $\bar{B}$, and $\bar{N}$, i.e., the ID level; (c) calculation on the IDAT level including the four- and six-point contributions Y, $\bar{B}$, $\bar{N}$, and $\bar{Z}$.

Theory

"coherent limit"

"incoherent densities"

"incoherent-density assisted transitions"

PHYSICAL REVIEW B **66**, 155301 (2002)

Biexcitonic effects in the coherent control of the excitonic polarization detected in six-wave-mixing signals

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(Received 5 July 2002; published 1 October 2002)

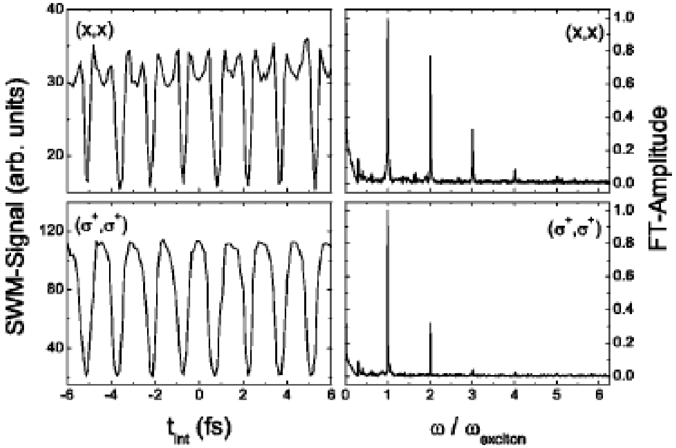
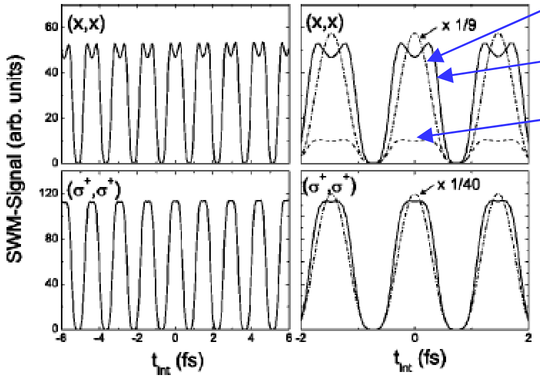


FIG. 2. Interferometric SWM signal measured for (x,x) colinearly (top, with biexcitonic contributions) and (σ^+,σ^+) cocircularly (bottom, without biexcitonic contributions) polarized incident beams at $t_{del}=0.6$ ps. The corresponding FT spectra (right) are each normalized to the value of ω_{exc}.

Experiment



mean field theory

full theory

without biexciton

FIG. 3. Calculated interferometric SWM signal for (x,x) colinearly (top) and (σ^+,σ^+) cocircularly (bottom) polarized incident beams. Left panels: full theory; right panels: (solid) full theory, (dashed) full theory without biexciton contributions, and (dot-dashed) mean-field theory scaled by the indicated factors. Parameters used in the calculations: electron mass $m_e=0.14m_0$, hole mass $m_h=0.67m_0$, m_0=free-electron mass, static dielectric constant $\epsilon_s=8.7$, interband dephasing time $T_2=3.3$ ps, and pulse area $A=0.07$ for all three pulses.

Theory

Eur. Phys. J. B **42**, 175–180 (2004)
DOI: 10.1140/epjb/e2004-00369-4

THE EUROPEAN
PHYSICAL JOURNAL B

Light-polarization and intensity dependence of higher-order nonlinearities in excitonic FWM signals

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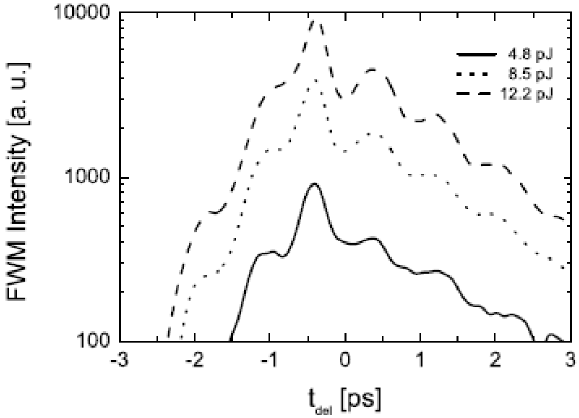


Fig. 2. Experiment: FWM signal vs. delay time for equal intensities of both pulses which are linearly polarized enclosing an angle of $\phi_{\mathrm{pol}} = 75^\circ$. The pulse energies are varied from 4.8 to 12.2 pJ. Signal detection is at the spectral position of the excitonic resonance.

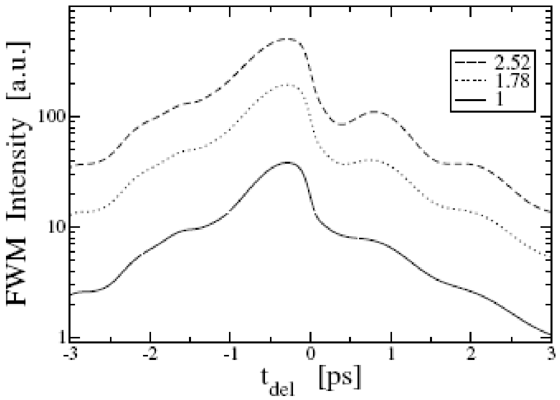
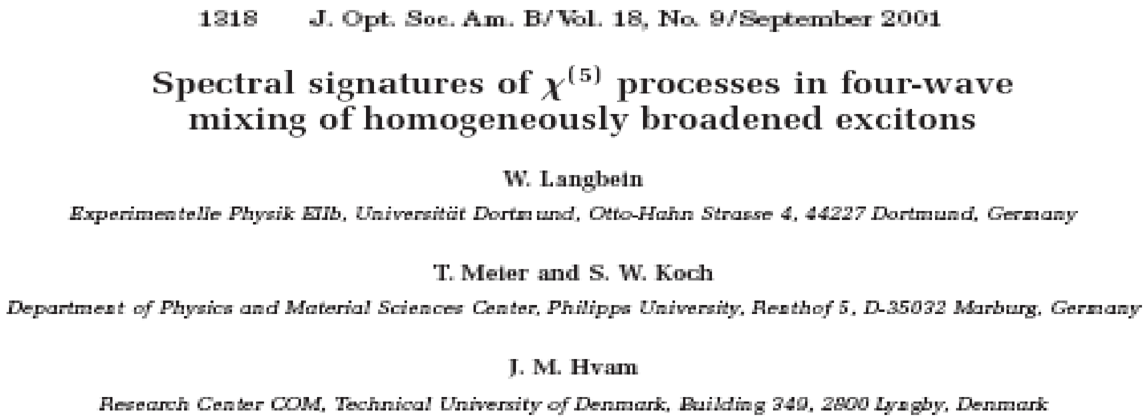


Fig. 5. Theoretical result for the same excitation and detection conditions as in Figure 2. The lowest intensity corresponds to a Rabi energy of $dE/E_B = 6.3 \times 10^{-3}$ in units of the $3d$ excitonic Rydberg energy E_B.

Experiment

Theory



1318 J. Opt. Soc. Am. B/Vol. 18, No. 9/September 2001

Spectral signatures of $\chi^{(5)}$ processes in four-wave mixing of homogeneously broadened excitons

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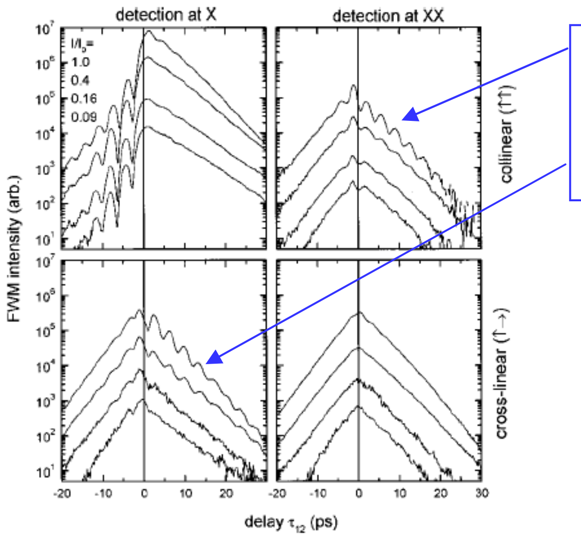

Fig. 4. FWM traces for various excitation intensities, as labeled (I_0 = 300 nJ/cm^2). All traces are scaled by the same factor. Top, collinearly polarized excitation (↑↑); bottom, cross-linearly polarized excitation (↑→); left, detection energy at the exciton resonance (X); right, detection energy at the exciton–biexciton resonance (X–XX).

beats with inverse biexciton binding energy period

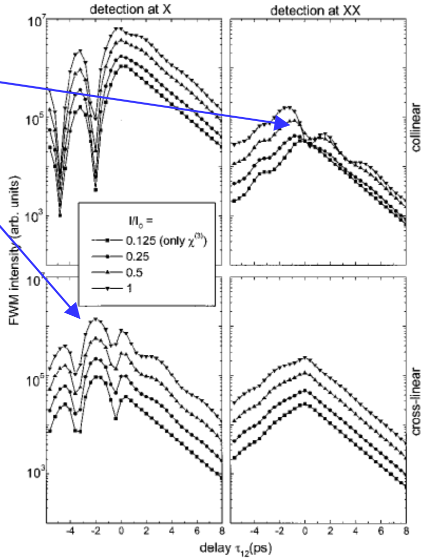

Fig. 6. Calculated FWM traces for various excitation intensities, as labeled. Top, collinearly polarized excitation (↑↑); bottom, cross-linearly polarized excitation (↑→); left, detection energy at the exciton resonance (X); right, detection energy at the exciton–biexciton resonance (X–XX). The symbols indicate the calculated time delays; the lines are guides for the eye.

Experiment

Theory

See also: Meier, Koch, Phys. Rev. B 59, 13202 (1999)

1318 J. Opt. Soc. Am. B/Vol. 18, No. 9/September 2001

Spectral signatures of $\chi^{(5)}$ processes in four-wave mixing of homogeneously broadened excitons

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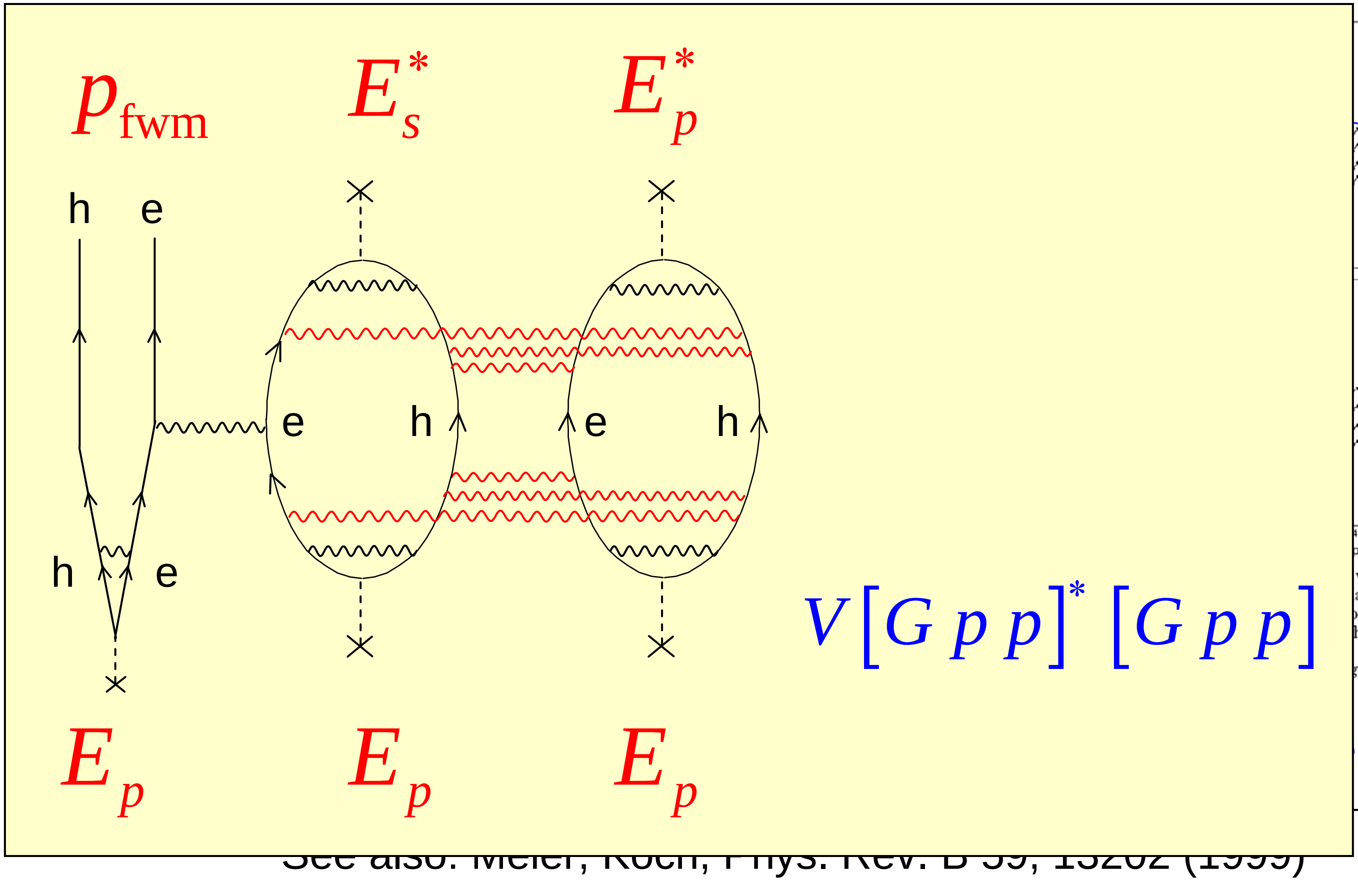
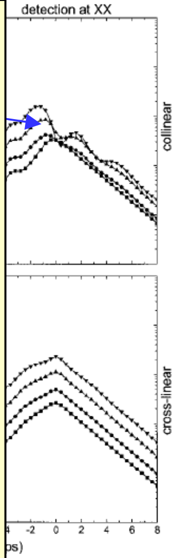


$$V\,[G\,p\,p]^*\,[G\,p\,p]$$

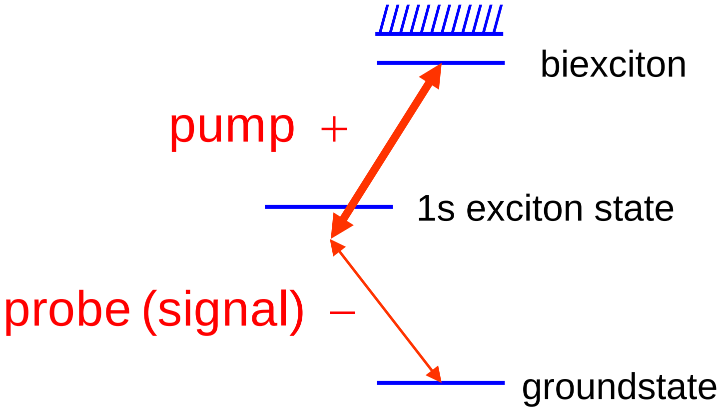


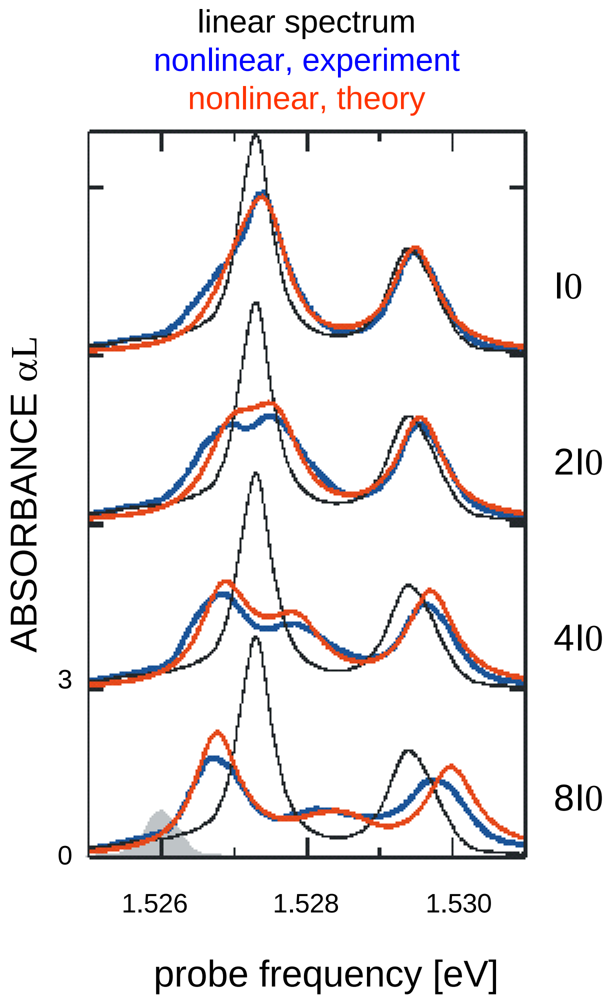
various excitation intensi-
arized excitation (↑↑); bot-
n (↑→); left, detection en-
ht, detection energy at the
The symbols indicate the
uides for the eye.

ory

See also: Meier, Koch, Phys. Rev. B 59, 13202 (1999)

Electromagnetically-induced transparency (EIT)





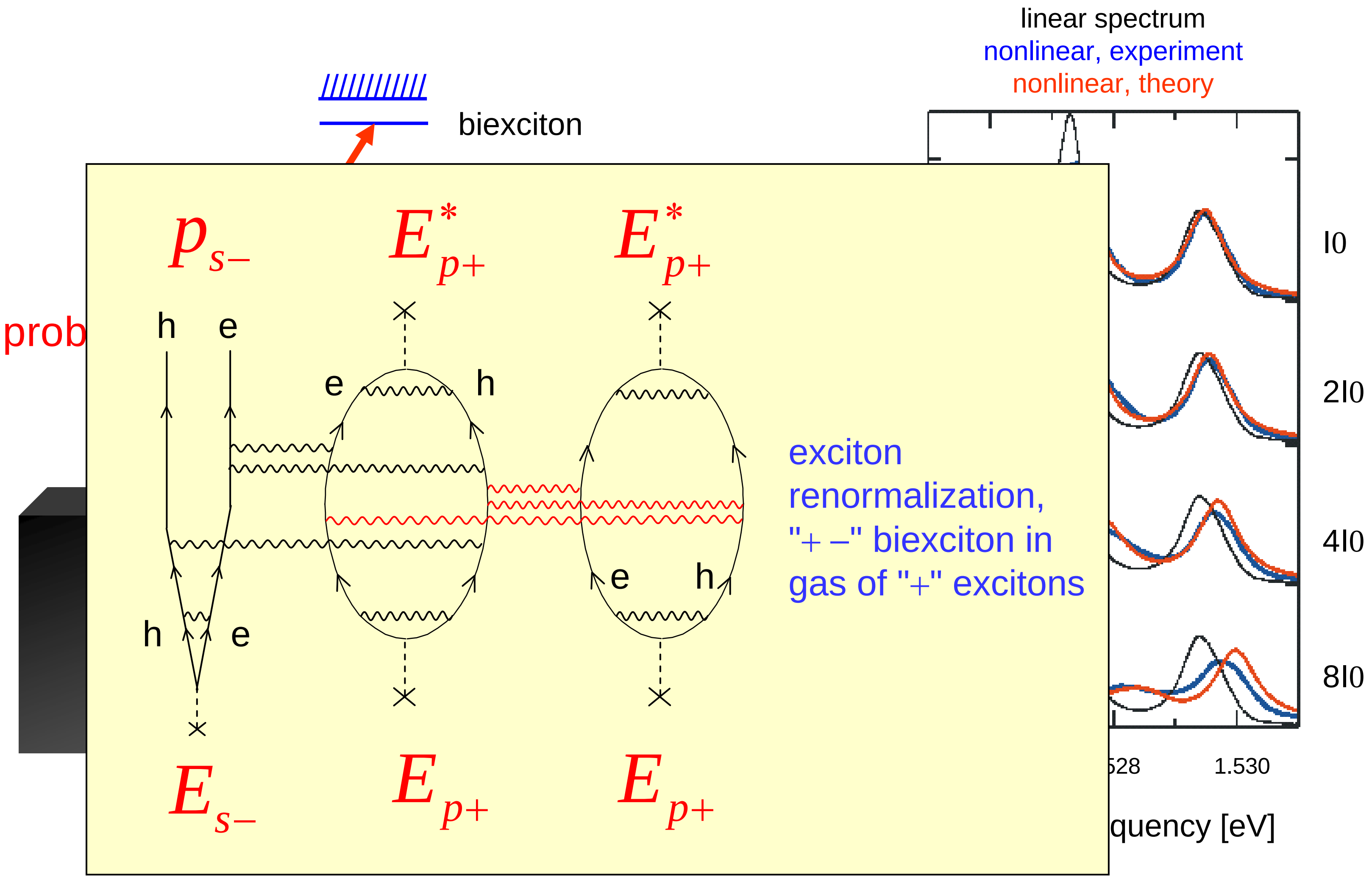
EIT dip at

$$\hbar\omega_{pump} + \hbar\omega_{probe} \approx \varepsilon_{biexciton}$$

shifts with increasing pump intensity

Phillips, Wang, Rumyantsev, Kwong, Takayama, Binder, Phys. Rev. Lett. 91, 183602 (2003)

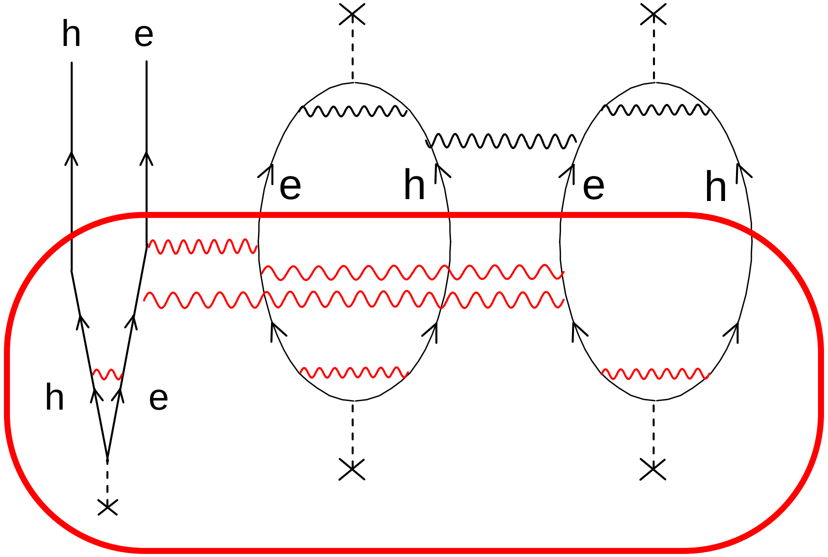
Electromagnetically-induced transparency (EIT)



exciton renormalization, "+ –" biexciton in gas of "+" excitons

Phillips, Wang, Rumyantsev, Kwong, Takayama, Binder, Phys. Rev. Lett. 91, 183602 (2003)

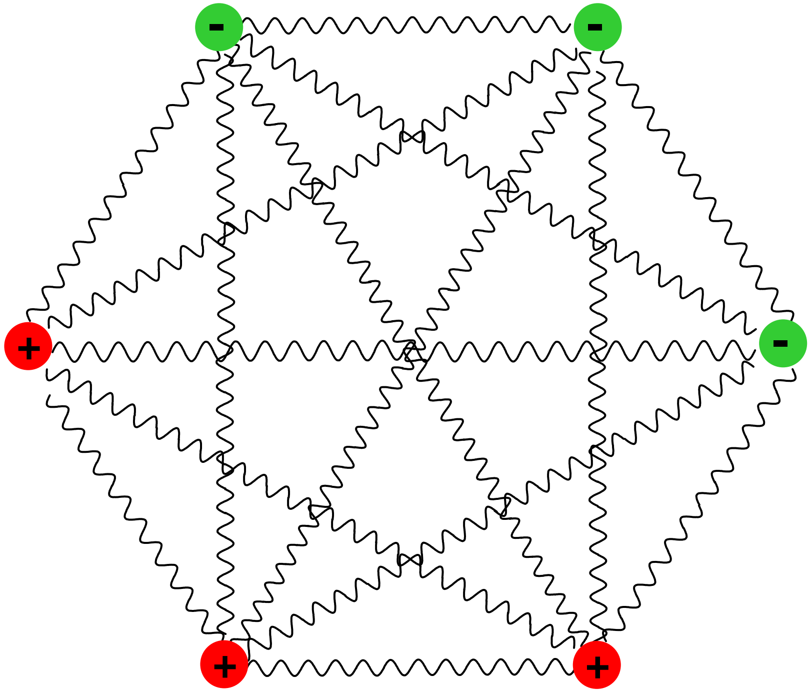
Triexciton states?



$$i\frac{d}{dt}\langle a\,a\rangle\Big|_{corr}$$

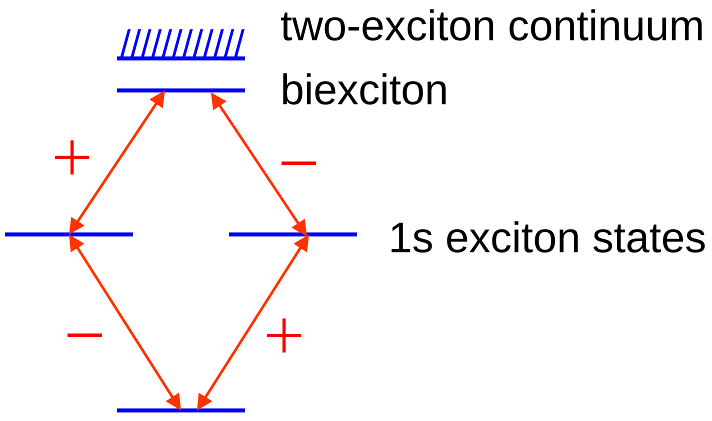
$$= V\langle a^{\dagger}\,a^{\dagger}\,a^{\dagger}\,a^{\dagger}\rangle\langle a\;a\;a\;a\;a\;a\rangle_{corr}$$

triexciton



Next:

Few-level systems



Few level models:

- are useful for conceptual analysis of optical nonlinearities
- can be used in "double-sided Feynman diagrams"

(e.g. in Li, Zhang, Borca, Cundiff, Phys. Rev. Lett. 96, 057406 (2006))

Question: can they be strictly related to many-particle theory?

Answer: yes, at least in $\chi^{(3)}$ regime!

PRL **96,** 057406 (2006) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2006

Many-Body Interactions in Semiconductors Probed by Optical Two-Dimensional Fourier Transform Spectroscopy

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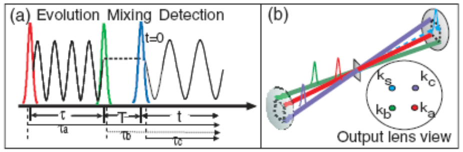


FIG. 1 (color). (a) Schematic of the excitation sequence showing the relevant time intervals. Note the dashed line indicating that the phase during time period t is related to the phase evolution during time τ. (b) Diagram showing geometry of excitation pulses and generated signal.

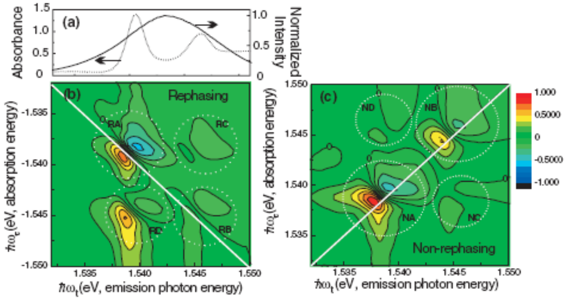


FIG. 3 (color). Linear absorption, excitation pulse spectrum (a) and experimental real spectra for the rephasing (b) and nonrephasing (c) pulse sequences. Both spectra in (c) and (c) are normalized to the most intense peak.

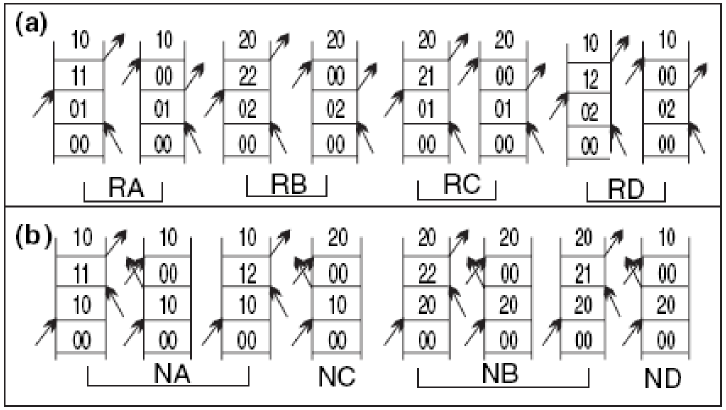
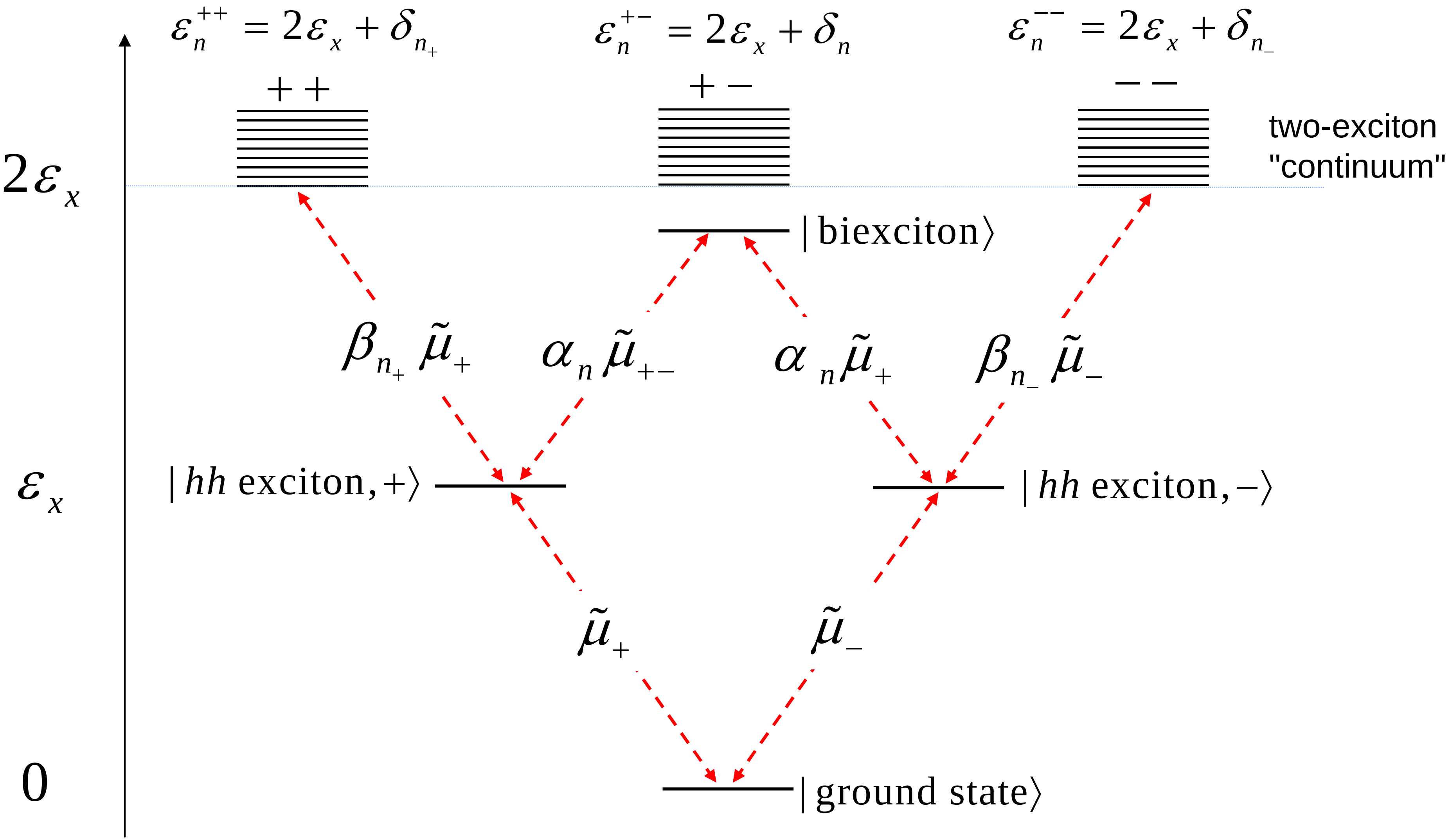


FIG. 2. Feynman diagrams for all possible quantum mechanical pathways for (a) rephasing and (b) nonrephasing measurements in a V system, where 0, 1, 2 corresponds to crystal ground state, heavy-hole, and light-hole excitons, respectively.

see Robert W. Boyd, *Nonlinear Optics*, (Academic Press, London, 1992)



$\varepsilon_n^{++} = 2\varepsilon_x + \delta_{n_+}$ $\varepsilon_n^{+-} = 2\varepsilon_x + \delta_n$ $\varepsilon_n^{--} = 2\varepsilon_x + \delta_{n_-}$

$++$ $+-$ $--$

two-exciton "continuum"

$2\varepsilon_x$

$|\,\text{biexciton}\rangle$

$\beta_{n_+}\tilde{\mu}_+$ $\alpha_n\tilde{\mu}_{+-}$ $\alpha_n\tilde{\mu}_+$ $\beta_{n_-}\tilde{\mu}_-$

ε_x

$|\,hh\ \text{exciton},+\rangle$ $|\,hh\ \text{exciton},-\rangle$

$\tilde{\mu}_+$ $\tilde{\mu}_-$

0

$|\,\text{ground state}\rangle$

$$i\hbar \frac{d}{dt}\rho_{ij} = \sum_k \left(H_{ik}\rho_{kj} - \rho_{ik}H_{kj} \right)$$

Initial condition:
system in ground state

$$P_+(t) = N_a \, \tilde{\mu}_+^* \, p_+(t)$$

↑

density of few-level 'atoms'

$$p_+(t) = \rho_{\mathrm{exc}+,g}(t) + \sum \alpha_n^* \rho_{n,\mathrm{exc}+}(t) + \sum \beta_n^* \rho_{n_+,\mathrm{exc}+}(t)$$

$$i\hbar \dot{p}^+ = (\varepsilon_x - i\gamma\ )\, p^+ - [1 - 2v\left|p^+\right|^2]\, \tilde{E}^+$$

$$+ V_{+-}^{phen} \left|p^-\right|^2 p^+$$

$$+ V_{++}^{phen} \left|p^+\right|^2 p^+$$

$$+ 2\ p^{+*} \int_{-\infty}^{\infty} d\,t'\, G_{++}^{phen}(t-t')\ p^+(t')\, p^+(t')$$

$$+ p^{-*} \int_{-\infty}^{\infty} d\,t'\, G_{+-}^{phen}(t-t')\ p^+(t')\, p^-(t')$$

$+$ terms proportional to γ's

$$i\hbar \dot{p}^+ = (\varepsilon_x - i\gamma)\, p^+ - [\phi_{1s}^*(0) - 2A^{\mathrm{PSF}}\, |\, p^+\, |^2\,]\, E^+$$

Phase-space filling

$$+V^{\mathrm{HF}}\, |\, p^+\, |^2\ p^+$$

Hartree-Fock
Coulomb interaction

$$\left.\begin{aligned} &+2\ p^{+*} \int_{-\infty}^{\infty} d\,t'\ \ G^{++}(t-t')\ p^+(t')\ p^+(t') \\ &+\ p^{-*} \int_{-\infty}^{\infty} d\,t'\, G^{+-}(t-t')\ p^+(t')\ p^-(t') \end{aligned}\right\}$$

Time-retarded
two-exciton
correlations
(incl. biexciton)

We have set $\sum \left|\alpha_n\right|^2 = 1$

$$v = 1 - \frac{1}{2} \sum \left|\beta_n\right|^2$$

$$V_{++}^{phen} = \frac{1}{2} \sum \left|\beta_n\right|^2 \delta_{n_+}$$

$$V_{+-}^{phen} = \sum \left|\alpha_n\right|^2 \delta_n$$

$$G_{++}^{phen}\left(t - t'\right) = \left(-\frac{i}{\hbar}\right) \theta\left(t - t'\right) \sum \frac{\beta_n^* \delta_{n_+}}{2} e^{-\frac{i}{\hbar}\left(2\varepsilon_x + \delta_{n_+} - i\gamma_b\right)\left(t - t'\right)} \frac{\beta_n \delta_{n_+}}{2}$$

$$G_{+-}^{phen}\left(t - t'\right) = \left(-\frac{i}{\hbar}\right) \theta\left(t - t'\right) \sum \alpha_n^* \delta_n e^{-\frac{i}{\hbar}\left(2\varepsilon_x + \delta_n - i\gamma_b\right)\left(t - t'\right)} \alpha_n \delta_n$$

Identification of few-level parameters

$$N_a \, |\tilde{\mu}|^2 \leftrightarrow |\phi_{1s}(0)|^2 \; |\mu|^2$$

$$\frac{1}{N_a} v \leftrightarrow A^{PSF} / |\phi_{1s}(0)|$$

$$\frac{1}{N_a} V_{++}^{phen} \leftrightarrow V^{HF}$$

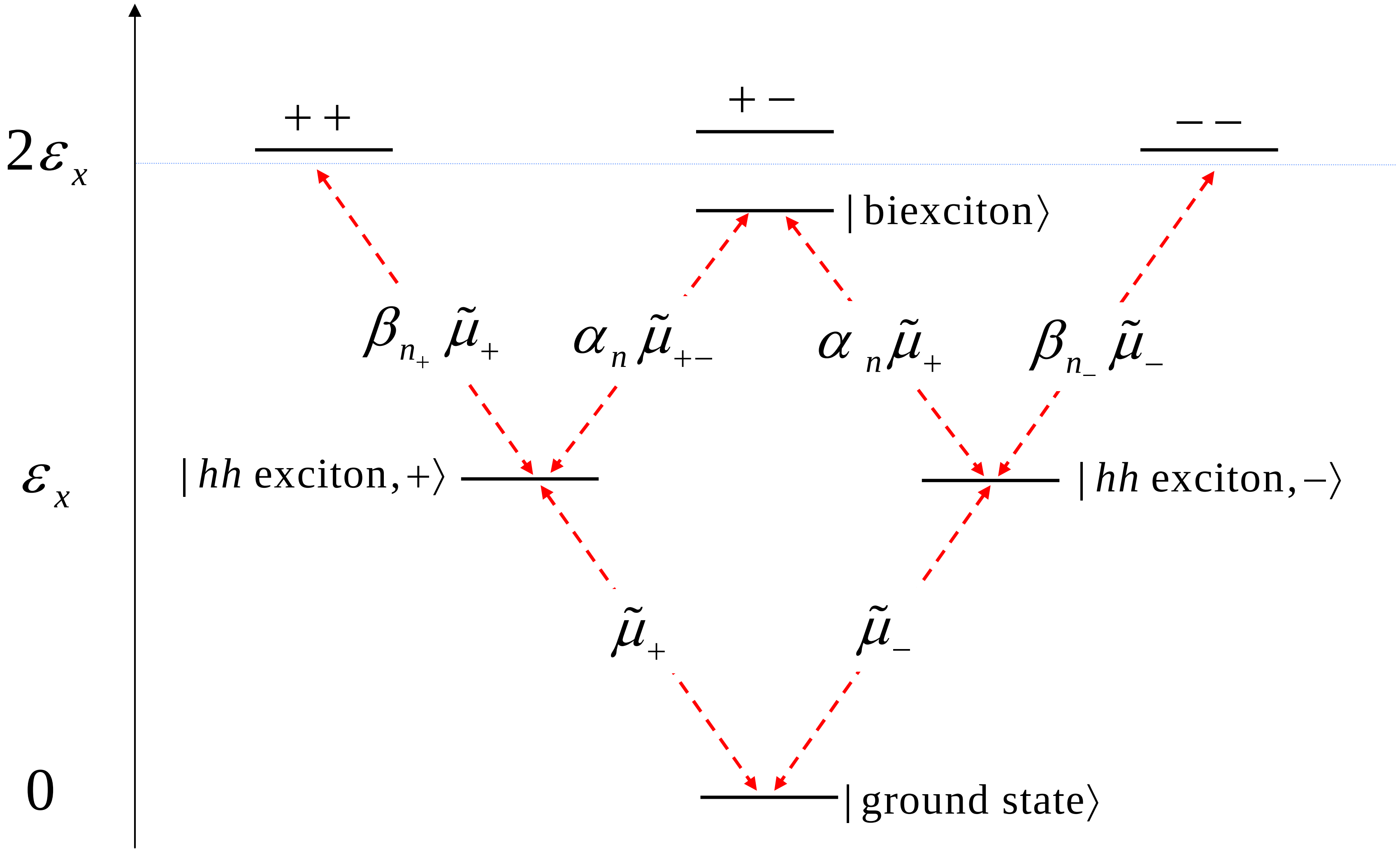
$$\frac{1}{N_a} V_{+-}^{phen} \leftrightarrow 0$$

$$\left.\begin{array}{l} \dfrac{1}{N_a} G_{++}^{phen}\left(t-t'\right) \leftrightarrow G_{++}\left(t-t'\right) \\ \dfrac{1}{N_a} G_{+-}^{phen}\left(t-t'\right) \leftrightarrow G_{+-}\left(t-t'\right) \end{array}\right\}$$

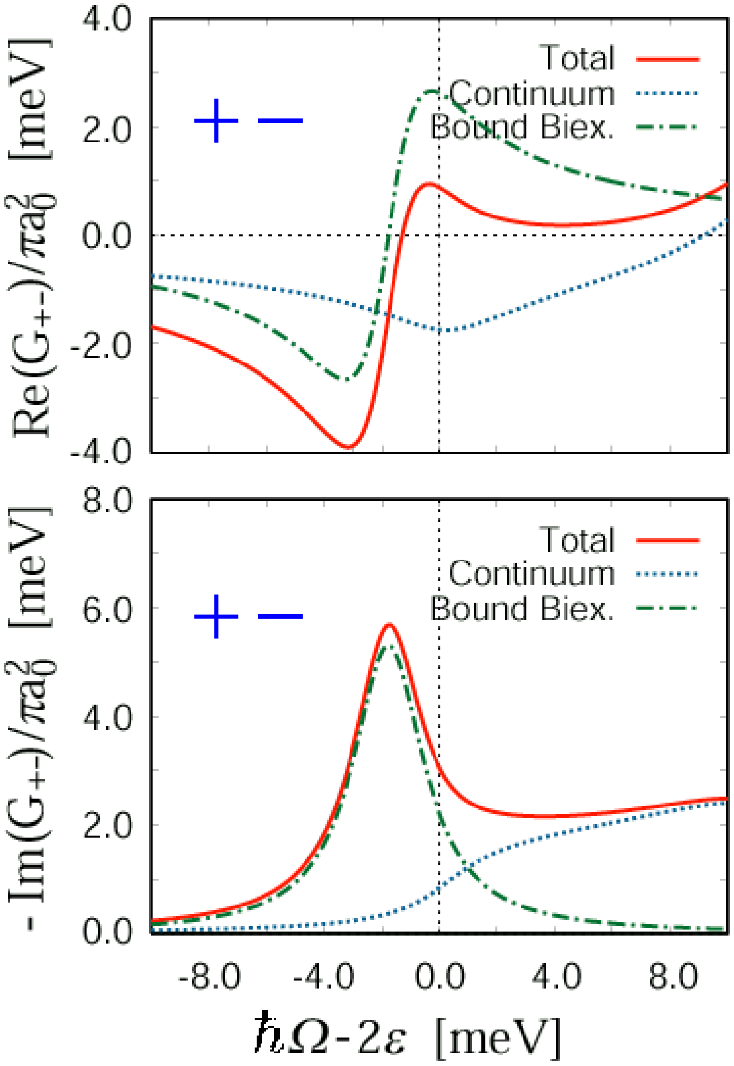
← approximate
(more 'atomic' levels
yield better agreement)

Kwong, Rumyantsev, Binder, Smirl, Phys. Rev. B 72, 235312 (2005)

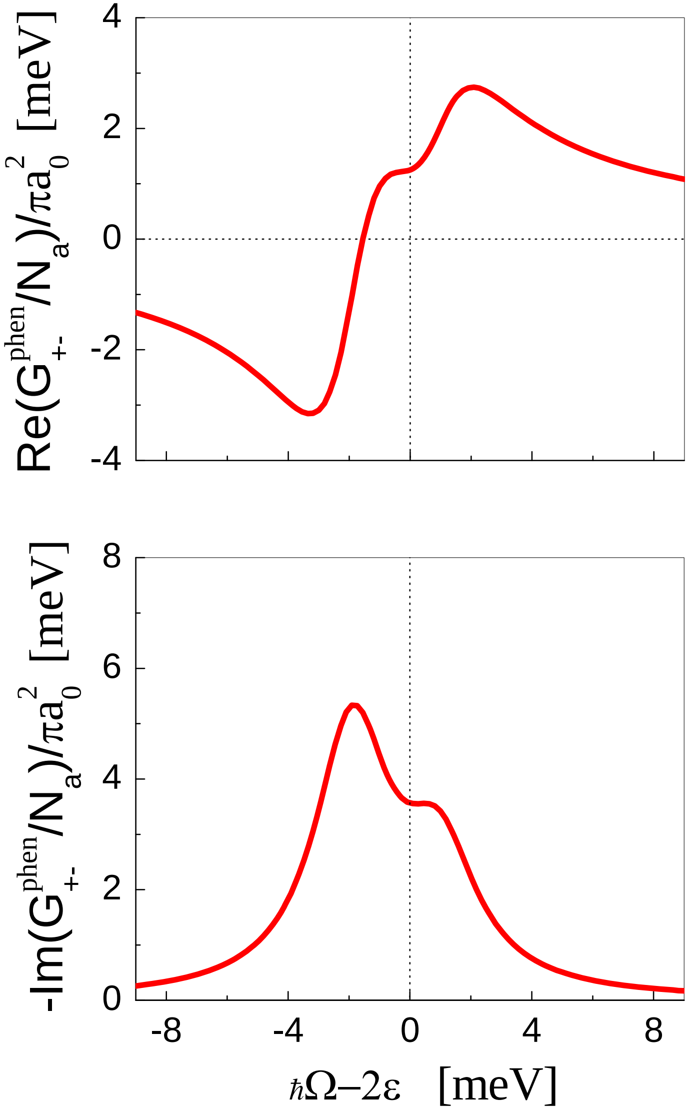
Example: 7-level system



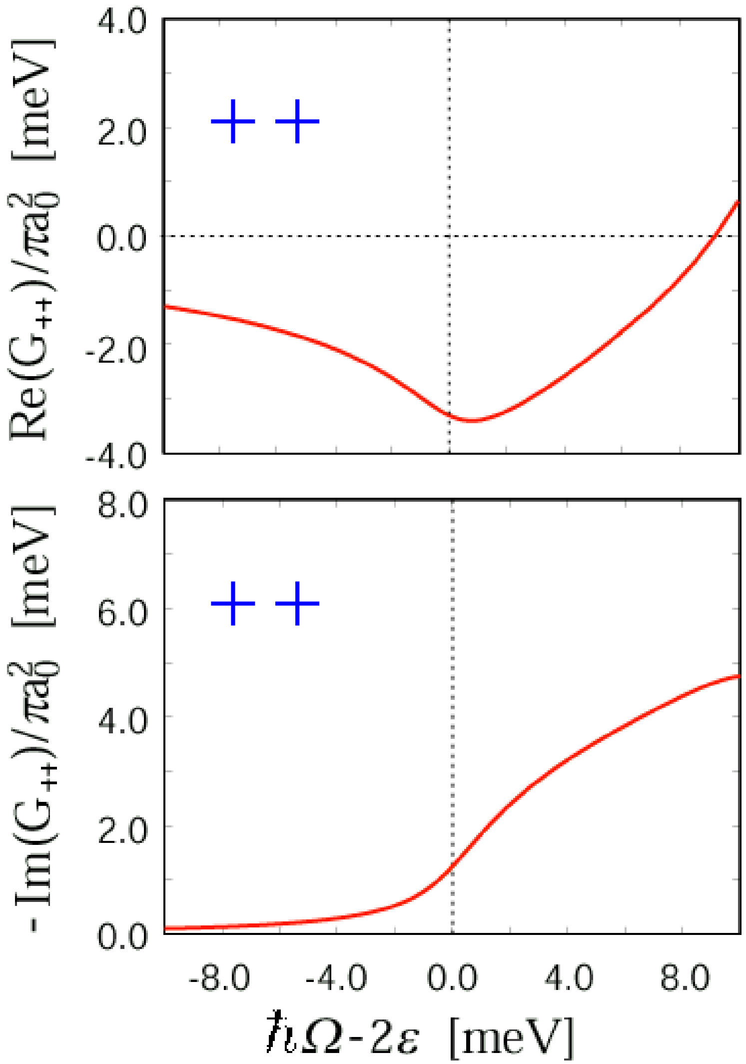
Microscopic theory



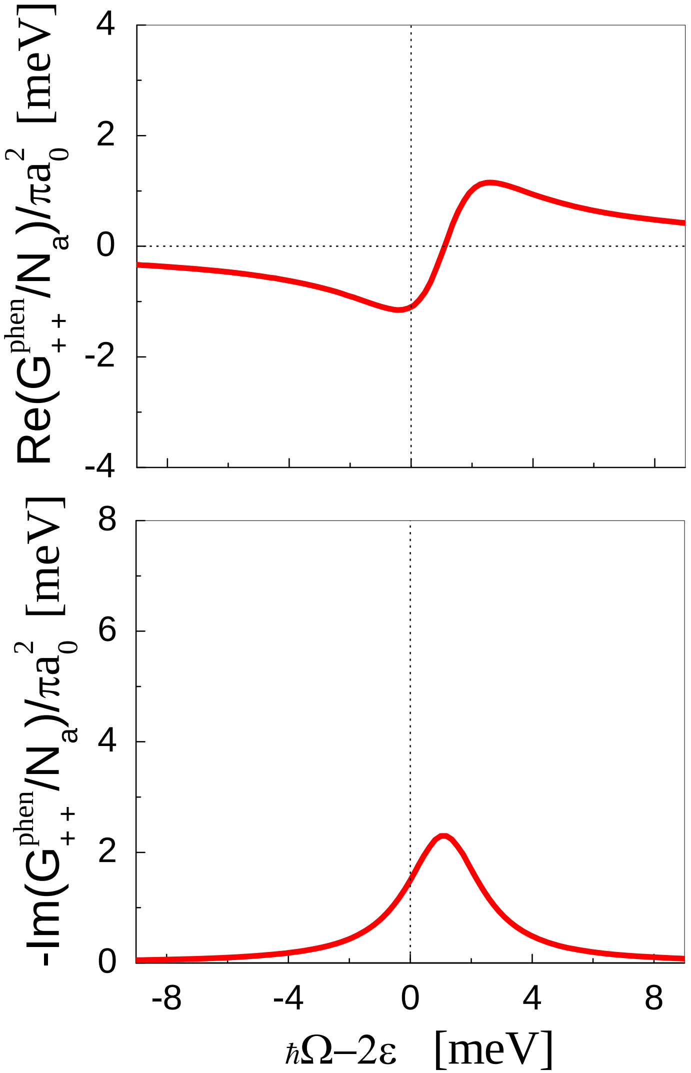
7-level system



Microscopic theory

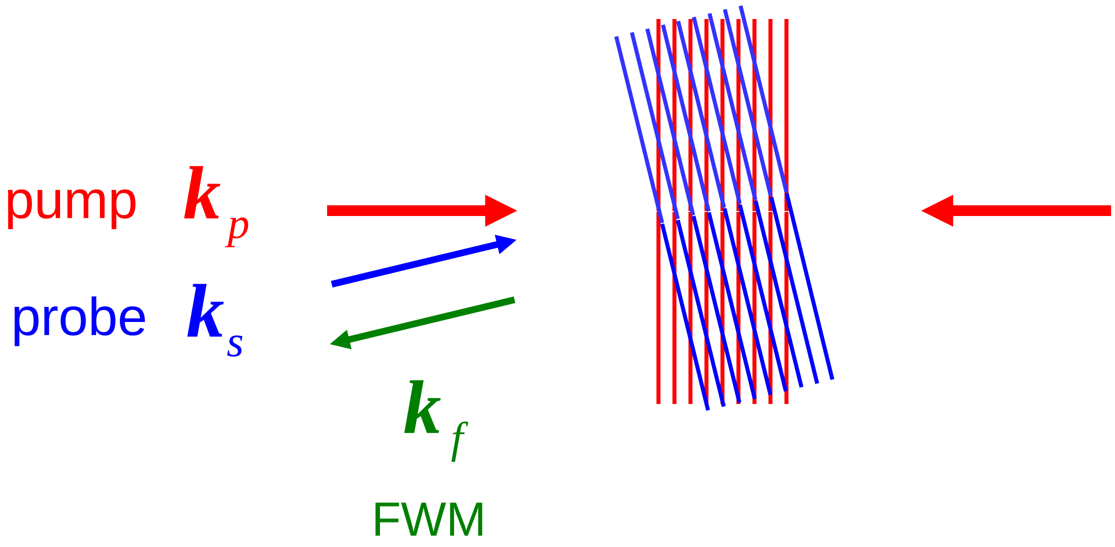


7-level system



Next:

FWM instabilities

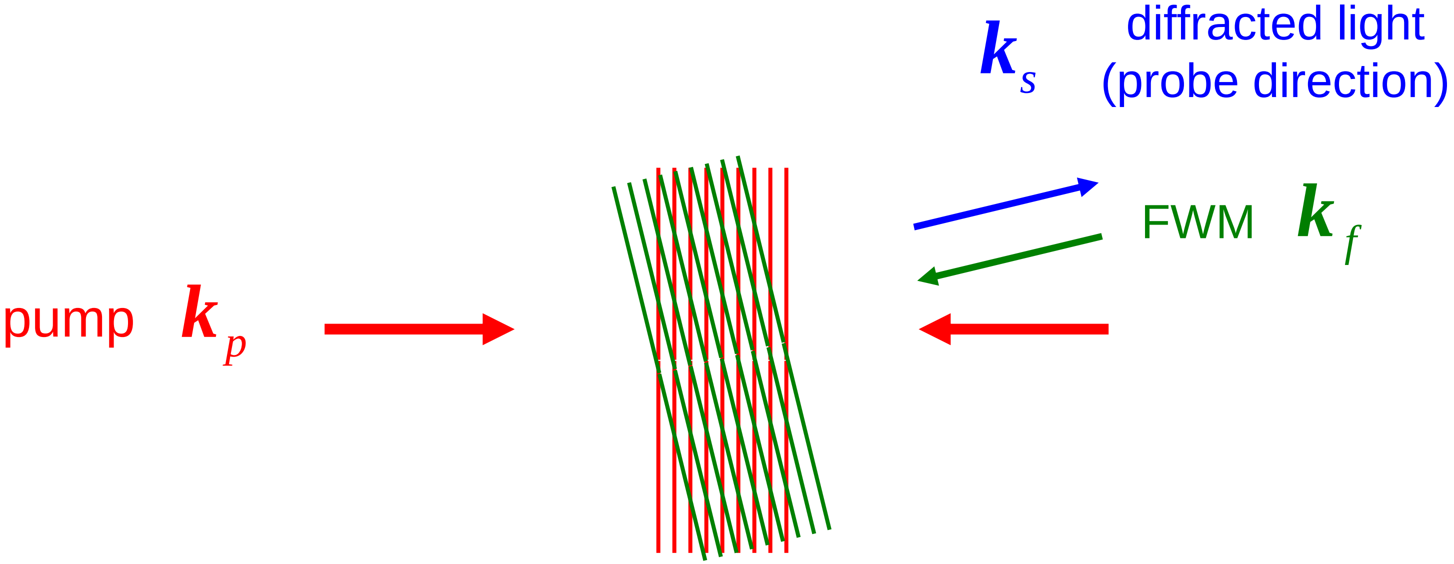


pump $\boldsymbol{k}_p$

probe $\boldsymbol{k}_s$

$\boldsymbol{k}_f$

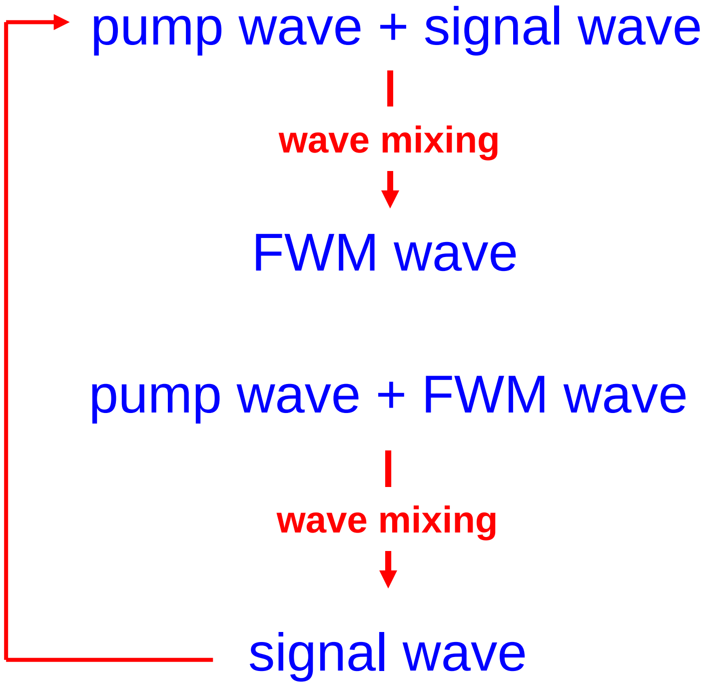
FWM



$\boldsymbol{k}_s$ diffracted light
(probe direction)

FWM $\boldsymbol{k}_f$

pump $\boldsymbol{k}_p$



pump wave + signal wave

wave mixing

FWM wave

pump wave + FWM wave

wave mixing

signal wave

feedback and possible dynamic instability

$$i\hbar\,\dot{p}_f = V^{HF}\,p_p\,p_p\,p_s^*$$
$$i\hbar\,\dot{p}_s = V^{HF}\,p_p\,p_p\,p_f^*$$

self consistency (positive feedback)

assume $p_p(t) = \tilde{p}_p\,e^{-i\omega_p t}$ and $p_s(t) = \tilde{p}_s(t)\,e^{-i\omega_p t}$

$p_f(t) = \tilde{p}_f(t)\,e^{-i\omega_p t}$

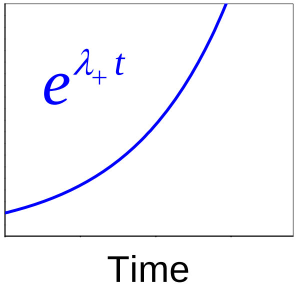
$$\dot{A} = M\ A$$

with $\boldsymbol{A} = \begin{pmatrix} \tilde{p}_s \\ \tilde{p}_f^* \end{pmatrix}$ $\quad M = \begin{pmatrix} 0 & -i\kappa \\ i\kappa & 0 \end{pmatrix}$ $\quad \kappa = V^{HF}\,\tilde{p}_p^2$

Linear stability analysis:

$$\boldsymbol{A} = \boldsymbol{A}_0\,e^{\lambda t}$$

$$\det(M - \lambda) = 0$$



$$\lambda_\pm = \pm|\kappa|$$

Stimulated polariton scattering in semiconductor microcavities

- Savvidis, Baumberg, Stevenson, Skolnick, Whittaker, and Roberts, Phys. Rev. Lett. 84, 1547 (2000)
- Huang, Tassone, Yamamoto, Phys. Rev. B 61, R7854 (2000)
- Ciuti, Schwendimann, Deveaud, Quattropani, Phys. Rev. B 62, R4825 (2000)
- Stevenson, Astratov, Skolnik, Whittaker, Emam-Ismail, Tartakovskii, Savvidis, Baumberg, Roberts, Phys. Rev. Lett. 85, 3680 (2000)
- Savasta, DiStefano, Girlanda, Phys. Rev. Lett. 90, 096403 (2003)
- Savasta, DiStefano, Savona, Langbein, Phys. Rev. Lett. 94,246401 (2005)
- Klopotowski, Martin, Amo, Vina, Shlykh, Glazo, Malpuech, Kavokin, Andre, Solid State Commun. 139, 511 (2006)
- Kasprzak, Richard, Kundermann, Baas, Jeambrun, Keeling, Marchetti, Szymanska, Andre, Staehli, Savona, Littlewood, Deveaud, LeSiDang, Nature 443, 409 (2006)

⋮

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- Stevenson, Astratov, Skolnick, Wh
  Baumberg, Roberts, Phys. Rev.
- Savasta, DiStefano, Girlanda, Ph
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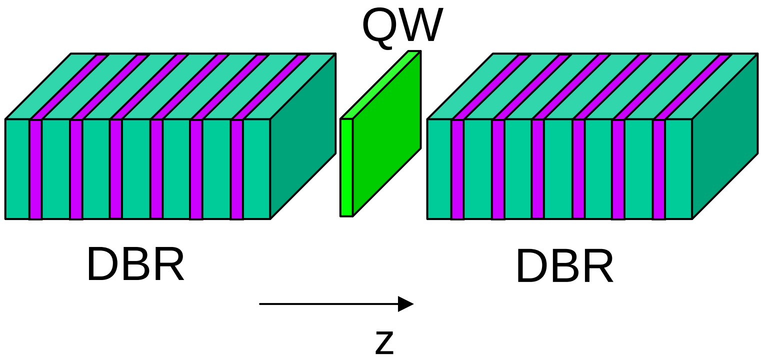
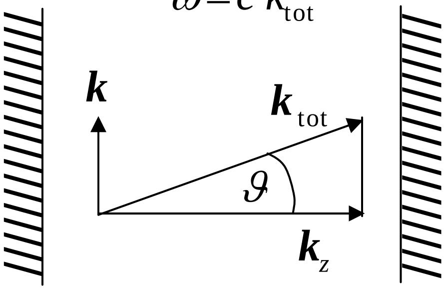
⋮

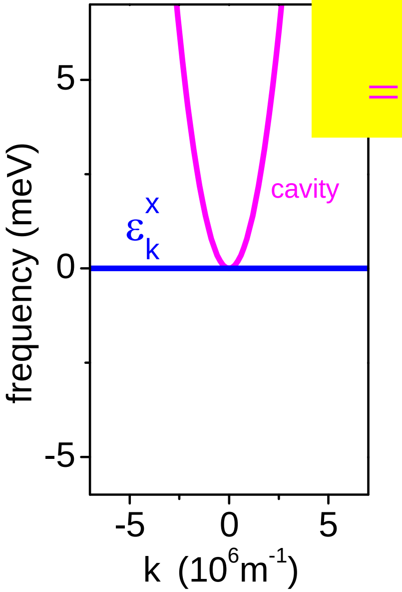
The publications in 2000 spurred major activities in bosonic aspects of excitons. In this tutorial, only FWM aspects are covered, not the bosonic aspects.



$$\omega = c\,k_{\text{tot}}$$

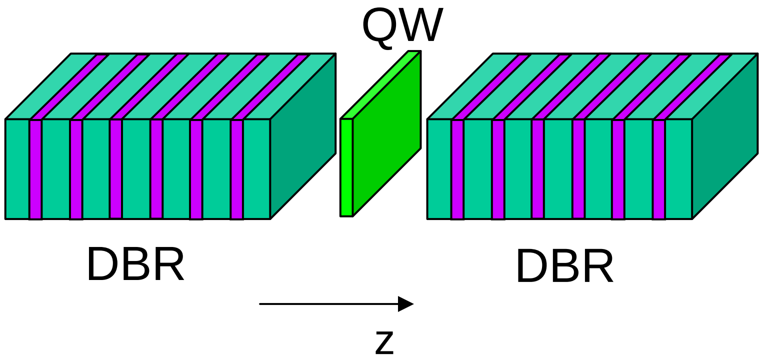


$$ck_{\text{z}} = \omega_{\text{cav}}$$

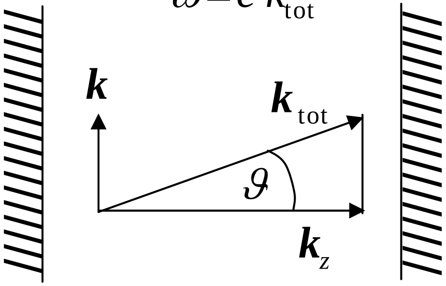


$$\omega = c\sqrt{k^2 + k_{\text{z}}^2}$$
$$= c\sqrt{k^2 + \omega_{\text{cav}}^2 / c^2}$$

$$\varepsilon_k^x = \varepsilon_0^x + \frac{\hbar^2 k^2}{2M}$$

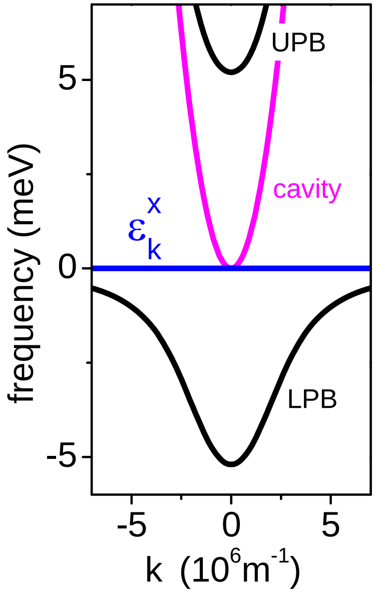


$\omega = c\, k_{tot}$



$c k_z = \omega_{cav}$

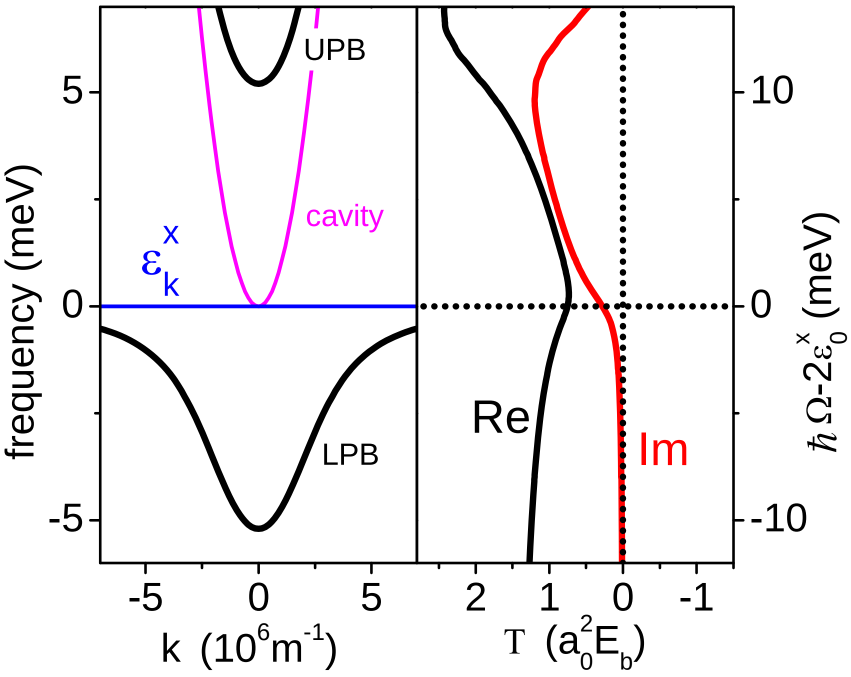
Coupled modes:



upper polariton branch

lower polariton branch

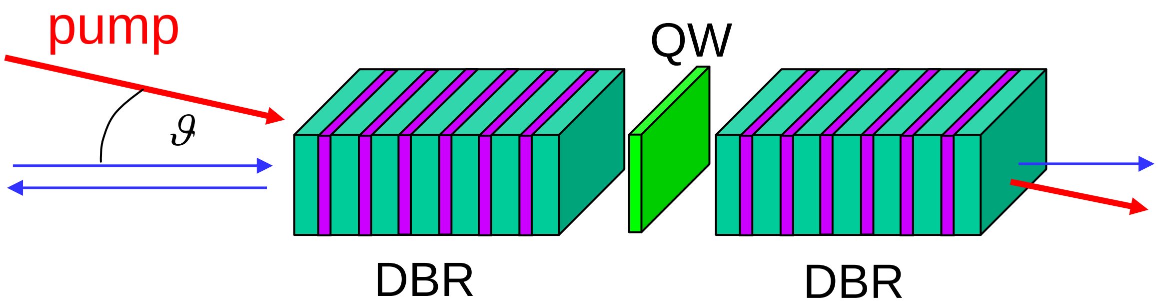
$$V^{HF} + G^{++}$$

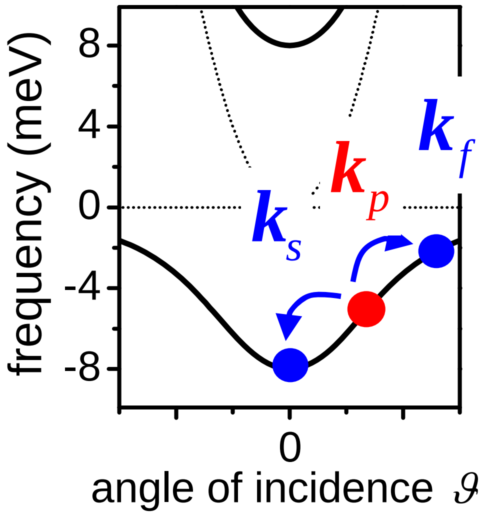


neglecting polariton effects in exciton-exciton scattering

- On LPB, two-exciton correlation dominated by Hartree-Fock
- Small excitation-induced dephasing at LPB facilitates instability (in contrast to single quantum well$^{(*)}$)

- Savasta, DiStefano, Girlanda, Phys. Rev. Lett. 90, 096403 (2003)
- Schumacher, Kwong, Binder, Phys. Rev. B 76, 245324 (2007)
- $^{(*)}$Schumacher, Kwong, Binder, Europhys. Lett. 81, 27003 (2008)
- $^{(*)}$Schumacher, Kwong, Binder, Smirl, Phys. Stat. Sol. (b) 246, 307 (2009)



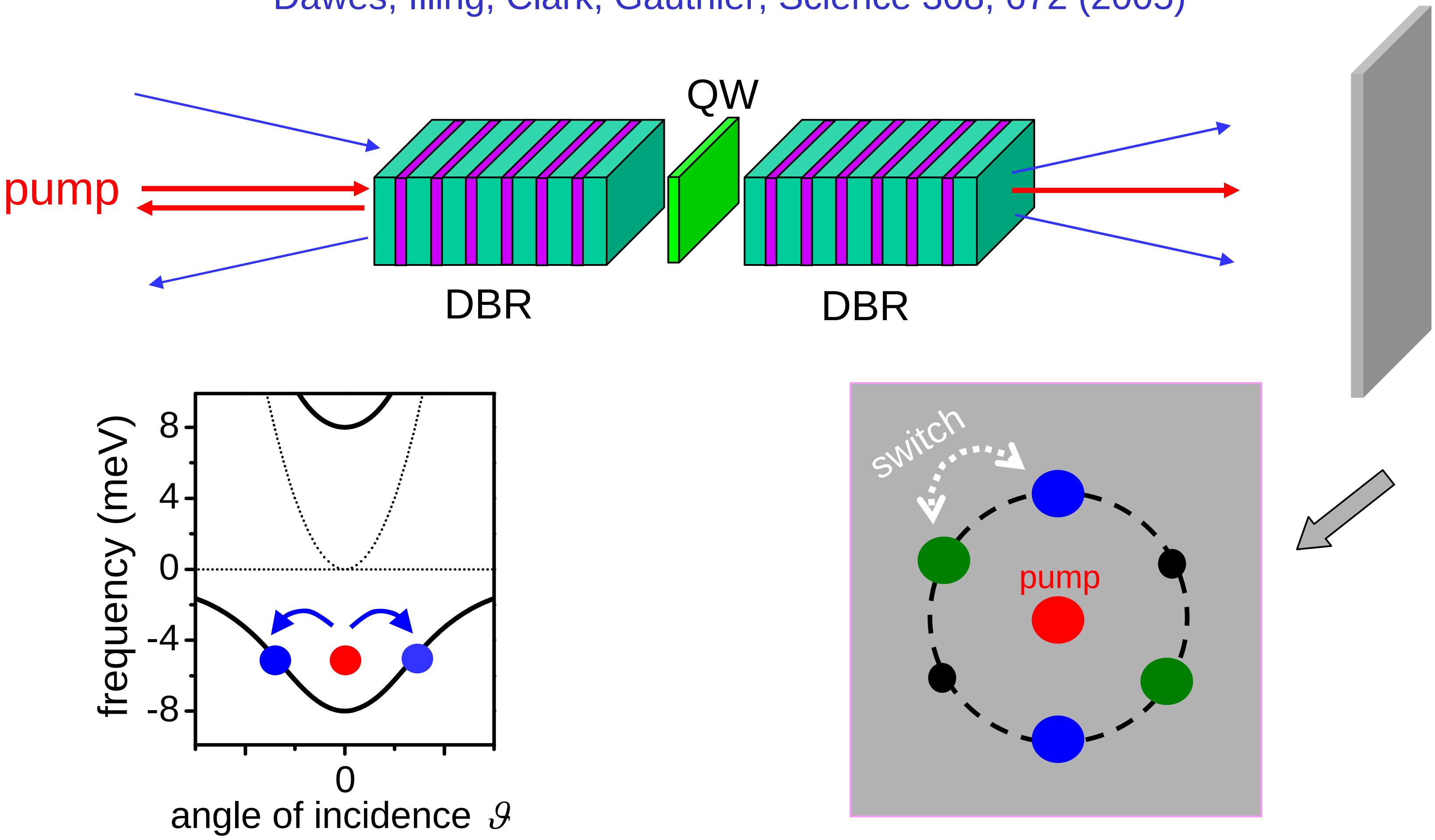


instability threshold low at "magic angle"

- Savvidis, Baumberg, Stevenson, Skolnick, Whittaker, and Roberts, Phys. Rev. Lett. 84, 1547 (2000)
- Huang, Tassone, Yamamoto, Phys. Rev. B 61, R7854 (2000)
- Ciuti, Schwendimann, Deveaud, Quattropani, Phys. Rev. B 62, R4825 (2000)

Low-intensity directional manipulation

semiconductor proposal for low-intensity switch demonstrated in
Dawes, Illing, Clark, Gauthier, Science 308, 672 (2005)



- Schumacher, Kwong, Binder, Smirl, Phys. Stat. Sol. RRL 3, 10 (2009)
- Dawes, Gauthier, Schumacher, Kwong, Binder, Smirl, Laser & Photon. Rev. (2009)

Last:

Conclusion

During the last 20 years, FWM techniques developed by D.S. Chemla and others have given us deep insight into many-particle processes in optically excited semiconductors, including the observation of excitonic correlation effects.

The physical processes underlying these effects can be visualized (and analyzed) with the help of Feynman diagrams.

This tutorial talk is available at

www.optics.arizona.edu/binder