Nonlinear atom optics

Brian P. Anderson and Pierre Meystre
Optical Sciences Center, The University of Arizona
Tucson, Arizona 85721

I. INTRODUCTION

The idea that light carries momentum and can modify the trajectories of massive objects can be traced back to Kepler, who offered it as an explanation for the direction of the tail of comets being always directed away from the Sun. More rigorously, the force exerted by light on atoms is implicit in Maxwell's equations. For example, it is readily derived from the classical Lorentz model of atom and radiation interaction, where the force of light on atoms is found to be

$$\mathbf{F}(\mathbf{r},t) = -\nabla_r V(\mathbf{x},\mathbf{r},t),$$

where $\mathbf{r}$ is the center-of-mass coordinate of the atom, $\mathbf{x}$ the position of the electron relative to the nucleus, and

$$V(\mathbf{x},\mathbf{r},t) = -q\mathbf{x}\cdot\mathbf{E}(\mathbf{r},t),$$

is the dipole potential due to the light. $\mathbf{E}(\mathbf{r},t)$ is the electric field at the center-of-mass location of the atom and $q=-e$ is the electron charge. The force $\mathbf{F}(\mathbf{r},t)$ is often called the dipole force, or the gradient force. It indicates that it is possible to use light to manipulate atomic trajectories, even when considered at the classical level.

An additional key element in understanding the interaction between atoms and light derives from basic quantum mechanics: since the work of Louis de Broglie [1] in 1923, we know that any massive particle of mass M possesses wave-like properties, characterized by a de Broglie wavelength

$$\lambda_{\rm dB} = \frac{h}{Mv}$$

where h is Planck's constant and v the particle velocity. Likewise, we know from the work of Planck and Einstein that it is oftentimes useful to think of light as consisting of particles that are now called photons. This wave-particle duality of both light and atoms is the cornerstone of quantum mechanics, yet ironically is still one of its most unsettling aspects.

Combining de Broglie's matter wave hypothesis with the idea that light can exert a force on atoms, it is easy to see that in addition to conventional optics, where the trajectory of light is modified by material elements such as lenses, prisms, mirrors, and diffraction gratings, it is possible to manipulate matter waves with light, resulting in *atom optics.* Indeed, atom optics [2] often proceeds by reversing the roles of light and matter, so that light serves as the "optical" elements for matter waves. There are however notable exceptions, such as the use of material gratings in atomic and molecular beam diffraction and interference experiments [3], and the use of magnetic fields for trapping of cold atoms and the realization of Bose-Einstein condensates.

Very much like conventional optics can be organized into ray, wave, nonlinear, and quantum optics, matter-wave optics has recently witnessed parallel developments. Ray atom optics is concerned with those aspects of atom optics where the wave nature of the atoms doesn't play a central role, and the atoms can be treated as point particles. Wave atom optics deals with topics such as matter-wave diffraction and interferences. Nonlinear atom optics considers the mixing of matter-wave fields, such as in atomic four-wave mixing, and the photoassociation of ultracold atoms — a matter-wave analog of second-harmonic generation. In such cases, the nonlinear medium *appears* to be the atoms themselves, but in a proper treatment it turns out to be the electromagnetic vacuum, as we discuss in some detail later on. Finally, quantum atom optics deals with topics where atom statistics are of central interest, such as the generation of entangled and squeezed matter waves. Note that in contrast to photons, which obey bosonic quantum statistics, atoms are either composite bosons or fermions. Hence, in addition to the atom optics of bosonic matter waves, which finds much inspiration in its electromagnetic counterpart, the atom optics of fermionic matter waves is starting to be actively studied by a number of groups. This emerging line of investigations is likely to lead to novel developments completely absent from bosonic atom optics.

The experimental confirmation of the wave nature of atoms followed soon after de Broglie's revolutionary conjecture, when Otto Stern [4] demonstrated the reflection and diffraction of atoms at metal surfaces. These experiments can be considered as marking the birth of atom optics. But at that time, a major stumbling block toward the further development of atom optics was the dependence of the thermal de Broglie wavelength on the temperature T,

$$\Lambda_{\rm dB} = \frac{h}{\sqrt{2\pi M k_B T}}$$

where k_B is Boltzman's constant. At room temperature, $\Lambda_{\rm dB}$ is of the order of a few hundredths of nanometers for atoms, which makes the fabrication of atom-optical elements as well as the detection of wave characteristics of atoms exceedingly difficult.

The invention of the laser changed this state of affairs dramatically, leading to a considerable improvement of our understanding of the way light interacts with atoms, and to the invention of a number of exquisite ways to manipulate atomic trajectories. These include the use of evanescent waves acting as atomic mirrors, standing waves of light serving as diffraction gratings, internal-atomic-state transitions enabling novel atomic beam splitters, and configurable optical fields for the creation of atomic traps and resonators. Most important among the developments centering about atom-laser interactions was the invention of laser cooling techniques, which permit the relatively straightforward realization of atomic samples with temperatures in the microkelvin range or colder. At such temperatures, the atomic thermal de Broglie wavelength is of the order of microns, comparable to visible optical wavelengths. There is then a perfect match between the length scales of light and atomic matter waves that can be exploited not just in the preparation and manipulation of matter waves, but also in their detection.

By decreasing the temperature of an atomic sample even further, typically to the nanokelvin range, with techniques such as evaporative cooling, one reaches an even more remarkable situation where the quantum statistics of the atomic sample play a fundamental role in the behavior of the atoms. Samples of identical bosonic atoms at even modest densities can undergo a phase transition to a state of matter called a Bose-Einstein condensate (BEC), while fermions are predicted to witness the creation of Cooper pairs and a Bardeen-Cooper-Schrieffer (BCS) phase transition from a normal fluid to a superfluid. One remarkable property of BECs is that they are characterized by coherence properties similar to those of laser light, leading to the realization of "atom lasers." Thus a BEC is one of the key ingredients for experiments on nonlinear and quantum atom optics, much like the laser is central to the fields of nonlinear and quantum optics.

II. OVERVIEW OF BOSE-EINSTEIN CONDENSATION

To produce a BEC, an isolated sample of identical bosonic atoms is cooled to extremely low temperatures [5]. At some point during the cooling process, a macroscopic fraction of the atoms fall into a state that is essentially the ground state of the confining potential (usually a magnetic trap). This point in the cooling cycle, characterized by a sudden macroscopic occupation of a single quantum state, is the transition to Bose-Einstein condensation. Atomic Bose-Einstein condensates have now been achieved for many alkali atom isotopes [6] (^{7}Li, ^{23}Na, ^{41}K, ^{85}Rb, ^{87}Rb, ^{133}Cs) as well as for Hydrogen and for metastable Helium, providing fascinating systems whose quantum-mechanical behavior is observable on a macroscopic scale [7]. The experimental configurations needed to produce a BEC are widely variable, and the complexity of an apparatus depends upon the type of atom to be used, the number of atoms desired in the BEC, the BEC production time needed, and of course the specific experimental goals.

An atomic BEC can be compared with other fields of physics in which a macroscopic number of particles occupy a single quantum state. Thus a BEC can be thought of as a dilute superfluid, or alternatively as the matter-wave equivalent to laser light: the entire sample is described to an excellent degree of approximation by the simple macroscopic wave function of a monoenergetic and phase-coherent atomic ensemble. To be more specific, we recall that the state of a manybody bosonic system is of the general form

$$\begin{aligned} |\phi_N\rangle &= \frac{1}{\sqrt{N!}}\int d\mathbf{r}_1\dots d\mathbf{r}_N \phi_N(\mathbf{r}_1\dots\mathbf{r}_N,t) \\ &\times\ \hat\Psi^\dagger(\mathbf{r}_N)\dots\hat\Psi^\dagger(\mathbf{r}_1)|0\rangle, \end{aligned}$$

where the Schrödinger field creation operator $\hat\Psi^\dagger(\mathbf{r})$, the matter-wave analog of the electric field operator $\hat E^-(\mathbf{r})$ in optics, satisfies the bosonic commutation relations

$$[\hat\Psi(\mathbf{r}),\hat\Psi^\dagger(\mathbf{r}')] = \delta(\mathbf{r}-\mathbf{r}')$$

and

$$[\hat\Psi(\mathbf{r}),\hat\Psi(\mathbf{r}')] = 0.$$

The manybody wave function $\phi_N(\mathbf{r}_1\dots\mathbf{r}_N,t)$ describes the state of a system of N particles created from the vacuum $|0\rangle$ by the successive application of N creation operators $\hat\Psi^\dagger(\mathbf{r})$.

While the formalism of field quantization involved here may not be completely familiar, one can obtain intuition for its physical meaning by drawing an analogy between the field creation operator $\hat\Psi^\dagger(\mathbf{r})$ and the electric field operator $\hat E^-(\mathbf{r})$. Specifically, just as it is oftentimes more useful in optics to think in terms of electric fields rather than individual photons, we find that when quantum statistics becomes important, it is more convenient to think of the atomic Schrödinger field rather than individual atoms. One reason why this is so is that the use of the creation and annihilation operators $\hat\Psi^\dagger(\mathbf{r})$ and $\hat\Psi(\mathbf{r})$ and the commutation relations appropriate for bosons (or fermions) automatically lead to the proper symmetrization of the many-particle wave-function, a task that is otherwise cumbersome. But this is not all: very much like it is difficult to explain interference phenomena in terms of single photons, a task that is trivial in terms of the electric field, it is also difficult (although possible) to explain matter-wave interference in terms of single atoms, but it is almost trivial to do so in terms of a matter-wave field. The reader not completely familiar with the powerful tools of field quantization may find it useful to draw analogies with the optical case to eliminate confusion. This usually (but not always) works, at least for bosons.

A perfect Bose-Einstein condensate at absolute zero temperature $T = 0$ is characterized by all atoms of the condensate occupying precisely the same quantum state, so that the manybody wave function factorizes as

$$\phi_N(\mathbf{r}_1 \ldots \mathbf{r}_N, t) = \varphi(\mathbf{r}_1) \ldots \varphi(\mathbf{r}_N).$$

Alternatively (and ignoring some subtleties beyond the scope of this paper), the quantum state of a condensate can be approximated as an eigenstate of the Schrödinger annihilation operator $\hat{\Psi}(\mathbf{r})$, that is, the matter-wave version of a Glauber coherent state. We recall that Glauber coherent states, or more briefly coherent states, can be thought of as the "most classical" states of the electromagnetic field, and play a central role in quantum optics. In particular, it can be shown that a single-mode laser operating far above threshold is described to a good approximation as such a state. For the case of matter waves, the quantum state of a condensate can be characterized by an amplitude and a phase, in analogy with the coherent states of the electromagnetic field.

The description of Bose-Einstein condensates in terms of coherent states makes their analogy with a laser particularly compelling. The atom-optical analog of a laser cavity is the atomic trap, and an output coupler extracts atoms from the trapped condensate, very much like partially reflecting mirrors extract light from the optical resonator in conventional lasers. Several atom output couplers have been demonstrated, including simply switching off the trap, resulting in a rather primitive "pulsed" atom laser, coupling of a fraction of the trapped atoms to an unbound electronic level [8–10], and atom tunneling from a periodic potential, leading to the atom-laser analog of mode locking [11]. By slowly and continuously extracting atoms out of a stabilized magnetic trap for a duration of about 0.1 seconds, researchers at the Max-Planck Institute for Quantum Optics (MPQ) [9] measured a coherence length of several millimeters for their atom-laser configuration. This should be compared to the dimensions of the atom cavities, which are effectively on the order of tens of microns.

The first-order coherence properties of atom lasers have now been investigated quantitatively by several groups. Particularly noteworthy in this context is the recent work in Orsay, France, in which the ABCD formalism of paraxial optics was adapted to the description of the matter waves emitted by an atom laser [12]. In another recent experiment at MPQ, the interference between an incident and a retro-reflected atom laser beam was studied as a function of the time delay between the beams [13]. The linewidth and coherence length of the beam was determined from the contrast of the interference pattern.

III. COLLISIONS AS A NONLINEAR MEDIUM

Just as laser light enabled the development of nonlinear optics, in which a light field in a medium effectively interacts with itself through mediating forces arising from the atoms in the medium, Bose-Einstein condensation has instigated the analogous field of nonlinear atom optics in which the atomic field effectively interacts with itself.

It is well known that photons do not interact with each other in free space. In order to achieve the wave-mixing phenomena that are the hallmark of nonlinear optics, it is necessary that light propagates within a medium, be it an atomic vapor, a crystal, a plasma, etc. Under many circumstances, it is possible to eliminate the material dynamics, leading to effective nonlinear equations for the optical fields. This is for instance the case in the traditional formulation of perturbative nonlinear optics [14], where the material properties are described in terms of a nonlinear susceptibility χ_{NL}. This leads to familiar effects such as four-wave mixing, second-harmonic generation, sum-frequency generation, and soliton propagation.

At first sight, the situation appears to be fundamentally different for atoms: for high enough atomic densities, atoms undergo collisions. That is, the presence of other atoms modifies the evolution of a given atom, a signature of nonlinear dynamics. As such, it would appear that atom optics is intrinsically nonlinear. However, we should keep in mind that atomic collisions are *not* fundamental processes: in the range of energies characteristic of atom optics, the only fundamental interaction of relevance is the electromagnetic interaction. All atomic interactions result from the exchange of photons (real or virtual) between atoms, and the concept of collisions is merely a convenient way to describe them. That collisions are represented by *effective* interatomic potentials, with no Maxwell fields present, results from a relatively simple mathematical step, the elimination of (part of) the electromagnetic field from the system dynamics.

From this point of view, then, there is a simple reversal of roles between the situations in optics and in atom optics: in optics, the light field appears to undergo nonlinear dynamics when the material evolution is eliminated, and in atom optics, the atoms appear to undergo nonlinear dynamics when (part of) the electromagnetic field is eliminated. It is, however, not very common to think of collisions in this way. What, then, is different with ultracold atoms that makes this point-of-view particularly useful? And how are we to understand collisions in Bose-Einstein condensates, since as we recall their defining characteristic is that at zero temperature, all atoms are precisely in the same quantum state. The familiar picture of two atoms approaching each other, interacting while they are sufficiently close, and then parting ways, is clearly no longer appropriate.

One way to proceed is to consider the manybody Hamiltonian of a bosonic Schrödinger field subject to two-body collisions,

$$\begin{aligned} H &= \int d\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) H_0 \hat{\Psi}(\mathbf{r}) \\ &+ \int d\mathbf{r}_1 d\mathbf{r}_2 \hat{\Psi}^\dagger(\mathbf{r}_1) \hat{\Psi}^\dagger(\mathbf{r}_2) V(\mathbf{r}_1 - \mathbf{r}_2) \hat{\Psi}(\mathbf{r}_2) \hat{\Psi}(\mathbf{r}_1), \end{aligned}$$

where H_0 is the single-particle Hamiltonian (the sum of the kinetic energy of the atoms and a possible external potential), and $V(\mathbf{r}_1-\mathbf{r}_2)$ the two-body Hamiltonian describing two-body collisions.

The description of the N-particle system in terms of the many-body Hamiltonian H can be shown to be completely equivalent to its description in terms of the N individual Hamiltonians for the N particles involved, plus $N(N-1)/2$ two-body Hamiltonians describing two-body collisions between those particles. Clearly, though, it presents the distinct advantage of being much more compact. The Hamiltonian H has a simple physical interpretation if we recall that the field operator $\hat{\Psi}(\mathbf{r})$ annihilates a particle at location $\mathbf{r}$. Hence, one can understand the way the first term of H operates on the many-particle system by noting that it picks a particle at $\mathbf{r}$, acts on it with the Hamiltonian H_0, and then puts it back into place. The integral guarantees that all particles in the ensemble are subjected to the same treatment. Similarly, the second term in H picks two particles, acts on them via the two-body Hamiltonian $V(\mathbf{r}_1-\mathbf{r}_2)$, and puts them back into place in the right order — the order in which things are done is of course important for particles obeying quantum statistics, as evidenced by their commutation (or anticommutation) relations.

The two-body Hamiltonian $V(\mathbf{r}_1-\mathbf{r}_2)$ can take a very complicated form in general, but the situation is drastically simplified for collisions between ultracold atoms, in which case it can be approximated in the s-wave scattering limit (appropriate for bosons) by the local Hamiltonian

$$V(\mathbf{r}_1-\mathbf{r}_2)=\frac{4\pi\hbar^2 a}{M}\delta(\mathbf{r}_1-\mathbf{r}_2),$$

where a, the so-called s-wave scattering length, completely characterizes the two-body collisions between condensate atoms of mass M. Remarkably, it is experimentally possible to change the magnitude, and even the sign, of the scattering length, giving considerable flexibility in studying a number of aspects of nonlinear and quantum atom optics, using for example Feshbach resonances. These resonances occur in situations where the total energy of two colliding atoms equals the energy of a bound molecular state. Recalling that atomic internal energies are affected by external magnetic fields through Zeeman shifts, it can be seen that the application of magnetic fields of particular strengths can allow the collisional energy to be tuned to match the molecular bound state energy. In this way, it is possible to experimentally tune two-body collisions, and hence the sign and strength of the scattering length, through such resonances, leading to a dependence of a such as illustrated in Fig. 1.

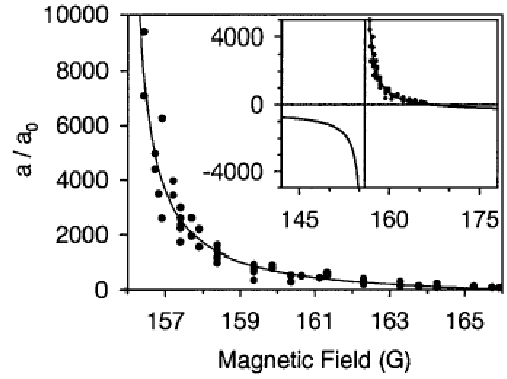

FIG. 1: Scattering length a for ^{85}Rb in units of the Bohr radius a_0 as a function of the magnetic field. The data are derived from measurements of ^{85}Rb condensate widths. The solid line illustrates the expected shape of the Feshbach resonance, i.e. the dependence of scattering length on magnetic field strength. For reference, the shape of the full resonance has been included in the inset. From reference [15], courtesy of C. Wieman.

In the s-wave scattering limit, the Heisenberg equation of motion for the Schrödinger field,

$$\frac{d\hat{\Psi}(\mathbf{r},t)}{dt}=\frac{i}{\hbar}[H,\hat{\Psi}(\mathbf{r},t)],$$

gives simply

$$i\hbar\frac{d\hat{\Psi}(\mathbf{r},t)}{dt}=H_0\hat{\Psi}(\mathbf{r},t)+\frac{4\pi\hbar^2 a}{M}\hat{\Psi}^\dagger(\mathbf{r},t)\hat{\Psi}(\mathbf{r},t)\hat{\Psi}(\mathbf{r},t).$$

Taking the expectation value of this operator equation and approximating the quantum state of the condensate by an eigenstate $\Phi(\mathbf{r},t)$ of the field annihilation operator $\hat{\Psi}(\mathbf{r},t)$, the so-called condensate wave function, finally gives the Gross-Pitaevskii equation (GPE) [16]

$$i\hbar\frac{\partial\Phi(\mathbf{r},t)}{\partial t}=H_0\Phi+\frac{4\pi\hbar^2 a}{M}|\Phi(\mathbf{r},t)|^2\Phi(\mathbf{r},t), \qquad (1)$$

showing that for ultracold bosonic atoms collisions simply result in a nonlinear phase shift of the condensate wave function. This equation has proven immensely useful in describing the most prominent properties of low-density atomic BECs.

It is important to realize that at this stage, the multi-particle description of the atoms has been replaced by a single-particle-like condensate wave function $\Phi(\mathbf{r})$. This is similar to what happens to the description of the individual photons in the classical description of light. The condensate wave function $\Phi(\mathbf{r},t)$ can be thought of as the "semiclassical" version of the field operator $\hat{\Psi}(\mathbf{r},t)$. As such it is somewhat analogous to the classical electric field used in classical optics. In atom optics, just like in optics, it is frequently useful to think in terms of such fields rather than their individual constituents, be they photons or atoms. Keep in mind, however, that just like in optics, the use of a "semiclassical" approximation is not necessarily justified.

The GPE (1) is reminiscent of the nonlinear wave equation describing the paraxial propagation of light in a nonlinear medium characterized by an instantaneous cubic nonlinearity $\chi^{(3)}$,

$$2ik\frac{dA(\mathbf{r},t)}{dz} + \nabla_T^2 A(\mathbf{r},t) \propto \chi^{(3)}|A(\mathbf{r},t)|^2 A(\mathbf{r},t),$$

where we have introduced the slowly varying envelope $A(z,t)$ of the electric field in z and t via $E(\mathbf{r},t) = A(\mathbf{r})\exp[i(kz-\omega t)] + c.c.$ and we recognize that the squared transverse Laplacian ∇_T^2 is mathematically analogous to the atomic kinetic energy that appears in H_0 — recall that $p^2/2M \to -(\hbar^2/2M)\nabla^2$ in the coordinate representation. (The difference between having a time derivative in the GPE and a spatial derivative in the paraxial wave equation, and a Laplacian rather than a transverse Laplacian, are of no further significance for the present discussion.) This latter equation is a cornerstone of nonlinear optics, and has led to the prediction and demonstration of numerous effects, including e.g. optical four-wave mixing and phase conjugation (the property that a nonlinear medium can act as a "mirror" that time-reverses the phase of an optical field, and can thus be used to compensate optical aberrations), optical solitons and vortices, and more. In view of the similarity between the inherently nonlinear GPE and paraxial wave equation, it is thus not surprising that many of the nonlinear effects first predicted and demonstrated in optics can now be observed with matter waves.

The prediction of nonlinear atom optics actually predates the realization of atomic Bose-Einstein condensation. Two groups [17] independently pointed out that the electric dipole-dipole interaction between atoms leads to an effective cubic nonlinearity in the atomic dynamics, which is then governed by a nonlinear Schrödinger equation similar to the GPE. Matter-wave four-wave mixing, phase conjugation, and atomic solitons were predicted at that time. Until the mid-1990s, however, atom optics was in a situation similar to that of optics before the invention of the laser: monoenergetic (or, equivalently, monochromatic) atomic beams obtained by spectral filtering contained only a small fraction of an atom per elementary phase space cell. Very much like the laser changed the situation for electromagnetic waves by producing vast amounts of photons per mode, the experimental realization of atomic Bose-Einstein condensation led to the availability of quantum-degenerate atomic samples, soon followed by the first nonlinear atom optics experiments.

Matter-wave four-wave mixing [18], dark [19, 20] and bright matter-wave solitons [21, 22], and matter-wave vortices [23, 24] have already been demonstrated. Modulational instabilities have been shown to exist as a soliton decay mechanisms, leading to the formation of vortex rings in condensates [25]. Processes through which two atoms can be induced to combine into a diatomic molecule are also objects of intensive studies, and are formally equivalent to second-harmonic generation in optics. In addition, the nonlinear mixing of optical and matter waves has led to the demonstration of matter-wave superradiance [26], coherent matter-wave amplification [27], and the joint parametric amplification of optical and matter waves. Finally, we remark that BECs are also characterized by the existence of "quasi-particles," whose total number is not conserved and that correspond to phonons is the limit of long wavelengths. These quasi-particles can also undergo a number of nonlinear mixing phenomena, but are not discussed further in this article.

IV. SELF-FOCUSING AND SELF-DEFOCUSING OF MATTER WAVES

As noted above, the principal parameter that governs the nonlinear dynamics of a BEC is the scattering length a. The scattering length can be interpreted phenomenologically in the following way: when the scattering length is positive, $a > 0$, the effective interatomic interactions are repulsive and the equilibrium size of the condensate is larger than for a noninteracting ($a = 0$) case for the same number of atoms. In contrast, when $a < 0$, the interactions are attractive, and the BEC contracts to minimize its overall energy. The nonlinear term in the GPE thus indicates a condensate behavior similar to optical "self-defocusing" for condensates with repulsive interactions, and to "self-focusing" when the interactions are attractive. This implies that large condensates are unstable for the case where $a < 0$ and will collapse onto themselves. While this prediction is correct in free space, the situation is actually more complicated in traps, where the contraction competes with the "quantum pressure" from the kinetic energy, or the zero-point energy due to the trap. As a result, it is possible to maintain small $a < 0$ condensates, containing typically a few hundred to a few thousand atoms. But for strong enough attractive interactions or high enough atomic densities, the quantum pressure is insufficient to stabilize the BEC. The GPE then has no steady-state solution, and the BEC implodes. There is a close analogy between this implosion and beam collapse in nonlinear optics. In that case, the competition is between diffraction, which tends to expand the beam, and a self-focusing nonlinearity. If the nonlinearity is strong enough, the beam will focus as it propagates until the intensity exceeds the damage threshold of the medium and the beam self-destructs.

Most of the experimental work with BECs to date, including the first experimental observations of condensation, utilize condensates that have repulsive interactions ($a > 0$) in low magnetic fields. However, an increasing number of significant and exciting experiments are being done with condensates with a negative scattering length, or with a scattering length that can be adjusted using magnetic fields to be negative. The first condensates with attractive interactions were produced in Randy Hulet's group at Rice University [28]. In these experiments, a gas of lithium atoms was cooled, and the maximum number of atoms that could be condensed was observed to be around 1250 atoms. When more than this critical number of atoms were put into the condensate, the attractive energies overwhelmed the stabilizing energy of the trap, and the BEC suddenly collapsed and lost atoms. In more recent BEC experiments, it has become possible to adjust the strength of the scattering length a over a wide range of values by applying an adjustable magnetic field that exploits the existence of so-called Feshbach resonances (more on those resonances in Section VIII). This ability to tune a was first observed for a BEC in Wolfgang Ketterle's group at MIT with sodium atoms [29], but it was of limited usefulness for studying tunable interactions due to strongly enhanced inelastic losses around the resonance. The scattering length for ^{85}Rb atoms can also be controlled in this fashion, without the three-body collisions problem. For most magnetic field values, the scattering length is so strongly negative that the condensate number is limited to about 70 atoms. Experiments with ^{85}Rb at JILA have exploited the change in sign of the scattering length a as a magnetic field drives the system across a Feshbach resonance to produce condensates with up to 15,000 atoms, and then to manipulate the interatomic interactions of these condensates by applying magnetic-field ramps [15]. More recent experiments with adjustable scattering lengths have enabled the production of large-number ^{7}Li condensates [21, 22] and ^{133}Cs condensates [30], an atom that also normally has a negative scattering length.

In the following sections, we review a few of the main topics of the nonlinear atom optics field that have been explored for both attractive and repulsive interatomic interactions in condensates. This is certainly not meant to be a comprehensive list, but rather an exploration of some of the highlights in the BEC field that have direct comparison to topics in nonlinear optics.

V. ATOMIC FOUR-WAVE MIXING

Following several theoretical proposals, the first experimental verification of matter-wave four-wave mixing was achieved at NIST [18]. The experiment proceeded by first releasing a sodium condensate from its trap potential at time $t = 0$. After a period of free evolution, optical pulses were made to interact with the condensate. The resulting exchange of momenta between atoms and photons produced three moving matter-wave packets of momenta $\mathbf{p}_1 \simeq 0$, $\mathbf{p}_2$, and $\mathbf{p}_3$ such that the momentum differences $\mathbf{p}_i - \mathbf{p}_j$ were much larger than the momentum spread of the initial condensate wave packet. Since the experimental time it takes to create these side modes was very short compared with the time scale over which the wave packets evolve, the three wave packets initially overlapped and were merely momentum-shifted copies of the initial condensate. As they flew apart, they interacted nonlinearly to produce a fourth matter wave with a new momentum $\mathbf{p}_4 = \mathbf{p}_1 - \mathbf{p}_2 + \mathbf{p}_3$. The four matter-wave momentum components, and their relative intensities, were imaged via absorption imaging after the components had spatially separated, as shown in Fig. 2.

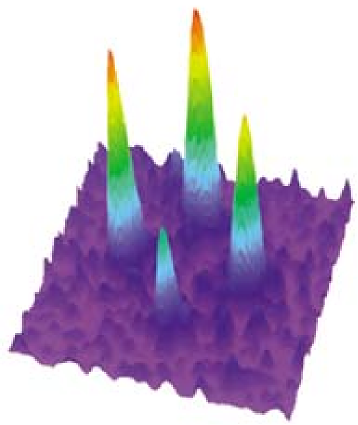

FIG. 2: Experimental results for four-wave mixing of matter waves [18]. The spatial distribution of atoms after a given time of flight is represented by the four different false-color peaks, showing four different momentum components. The smallest peak indicates the momentum component generated during the four-wave mixing process. Figure courtesy of the NIST, Gaithersburg BEC team.

It is known in nonlinear optics that atomic four-wave mixing can be exploited to generate phase-conjugate waves, which can in turn be though of as "real-time holography." Holography is a two-step process where first the information about the object is stored in a hologram. The second step is the reconstruction, which is performed by shining a reading beam similar to the reference beam onto the hologram. The diffraction of the reading beam from the recorded pattern yields a virtual as well as a real optical image of the original object. In atom holography, at least the final reading step is performed with an atomic beam. In this way, an atom-optical image of the object is created which in certain situations can be thought of as some sort of material replica of the original. One way to realize an atom hologram is actually based on linear atom optics: it rests on the diffraction of atoms from a mechanical mask. The first successful realizations of such an approach have recently been reported [31]. Atom holography offers the promise of practical applications from atom lithography to the manufacturing of nanostructures, but one of the prerequisites for a practical implementation is the availability of a reading beam of sufficient monochromaticity and coherence. Given the rapid advances in nonlinear atom optics and especially in the realization of atom lasers, this promise appears realistic.

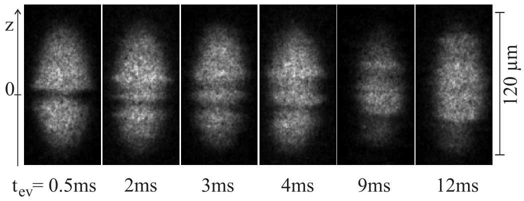

FIG. 3: Absorption images of dark solitons in a BEC with repulsive interatomic interactions. The images are shown for different times after soliton creation in the magnetic trap prior to the release of the BEC from the trap. The solitons are seen to propagate along the long axis of the BEC. From reference [19], courtesy of K. Sengstock.

VI. SOLITONS

The Gross-Pitaevskii equation is a nonlinear Schrödinger equation. It is known from several areas of physics, in particular nonlinear optics, that soliton solutions are generic to this equation. Hence, it is natural to ask whether matter-wave solitons can be launched in Bose-Einstein condensates.

One difficulty is that as mentioned above, the generation of large condensates requires positive scattering lengths a, the analog of self-defocusing media in optics, and it is known that bright solitons are not possible in that case. As a result, much atom optics work has focused on the formation of dark solitons, which correspond to low-density regions surrounded by regions of higher atomic density. A difference in the macroscopic phases between the two high-density regions partially stabilizes the dark soliton; a phase difference of π is ideal, and provides a soliton with no instantaneous velocity. In the first experimental observations of dark solitons in BECs [19, 20], dark solitons of variable velocity were launched via "phase imprinting" of a BEC by a light-shift potential, similarly to the way a phase mask can imprint a soliton or vortex on a light beam. The soliton velocity could be selected by applying a laser pulse to only half of the BEC and choosing the laser intensity and duration to select a desired phase step. Example images of dark solitons propagating in a BEC are shown in Fig. 3.

Later experiments with dark solitons [25] confirmed the predicted onset of dynamical instabilities originating from undulations of the soliton, much like the so-called "snake" instabilities of optical dark solitons. In the first of these experiments, a dark soliton was created in a spherically symmetric BEC in such a way that the soliton nodal region was filled with a different group of condensed atoms, thereby creating a "dark-bright" soliton combination. When the inner soliton-filling component was removed, dynamical instabilities drove the BEC into a more topologically stable configuration in which vortex rings were found to be embedded in the BEC.

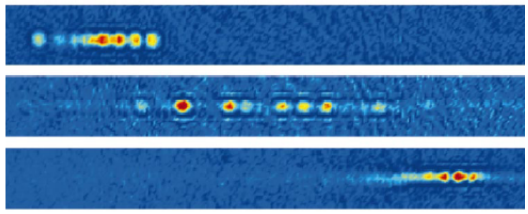

FIG. 4: Repulsive interactions between solitons in a train of bright atomic solitons. The three images show examples of a soliton train near the two turning points of a nearly one-dimensional atom trap (first and third image), and near the center of oscillation (second image). The spacing between solitons is compressed at the turning points, and spread out at the center of the oscillation. From reference [21], courtesy of R. Hulet.

For atom optics applications such as atom interferometry, it is desirable to achieve the dispersionless transport of a spatially localized ensemble of atoms, rather than propagation of a "hole" within a group of atoms. Bright solitons in an attractive ($a < 0$) condensate are one possible way to achieve this goal, and have recently been demonstrated in experiments at Rice University [21] and at ENS in Paris [22]. In these two experiments, large elongated condensates of ^{7}Li were created under conditions where the scattering length was positive. Using a Feshbach resonance, the scattering length was then changed to $a < 0$, at which point condensed atoms were observed to propagate within an effectively one-dimensional trap. In the Rice University experiment, a train of bright solitons was observed, and interactions between the solitons were found to be repulsive, keeping the solitons spatially separated as shown in the data of Fig. 4.

The predicted existence of gap solitons offers an elegant alternative [32] to using $a < 0$ condensates. Their existence is based on the observation that in a periodic potential it is possible to reverse the sign of the effective mass of the particles. For particles with a negative mass, the roles of attractive and repulsive interactions are reversed, resulting in the possibility of launching and propagating bright solitons for both attractive and repulsive two-body interactions. Physically, gap solitons result from the balance between the nonlinearity and the effective linear dispersion of a coupled system, e.g., counter-propagating waves in a grating structure, and appear in the gaps associated with avoided crossings. Again, this is a direct extension to atom optics of a nonlinear optics situation, where gap solitons have previously been predicted and demonstrated [33].

VII. VORTICES

Quantized vortices in BEC provide further common ground between optics and atom optics, as well as research in superfluid helium, superconductivity, and even the rotation of neutron stars. A vortex is a topological singularity created and stabilized by fluid flow around the singularity, or vortex core. The vortex flow pattern is irrotational, that is, it is curl-free, and the local fluid velocity is fastest near the vortex core. In a superfluid such as a BEC, the quantum phase must be single-valued at any point. Thus it can only change by integer multiples of 2π around any closed loop encircling a phase singularity. Because the superfluid velocity is proportional to the gradient of the phase, the quantization of phase change directly leads to the quantization of vortex angular momentum. In a BEC, right at the singularity, the fluid velocity would be undefined. However, fluid is expelled from a vortex core, and the flow pattern thus remains well-defined throughout the fluid.

Single vortices in a BEC were first generated and observed in a ^{87}Rb condensate in Eric Cornell's group at JILA in 1999, using a phase and density engineering technique [23]. Starting from a condensate composed of one internal hyperfine spin state, a second condensate of ^{87}Rb in a different spin state was created, having the precise spatial phase dependence of a vortex. A singly quantized vortex was thus formed in just one of the two superimposed condensates; the non-rotating condensate filled the vortex core of the rotating one. This scenario is reminiscent of nonlinear optics experiments in which the core of an optical vortex is used as a waveguide for another beam of light. In the BEC experiment, the core-filling condensate could be slowly or quickly removed with a properly tuned laser beam incident on the atoms, allowing the study of a continuum of possible configurations between filled and empty vortex cores. Without the filling, the nonlinear interactions of the BEC determine the size of the core, restricting it to dimensions usually less than 1 micron, making vortex cores difficult to observe optically. One benefit to having the filled vortex core is that it made the core large enough to be directly observed. This permitted observations of the precession of an off-centered vortex core around the center of the BEC, an inherent part of vortex dynamics in a confined fluid.

Soon after the creation of these single vortices, Jean Dalibard's group at ENS in Paris [24], followed by Wolfgang Ketterle's group at MIT [34], created multiple vortices in their condensates by stirring a BEC with a laser beam that repelled the atoms via the AC Stark shift. The BECs were then released from their traps and allowed to expand before imaging. Such work is similar to lattices of optical vortices in beams of light.

More recent work at JILA has shown that by inducing rotation in a thermal cloud of atoms above the BEC critical temperature, and then cooling the cloud through the BEC transition, large vortex lattices can be generated. This method of vortex creation is most similar to the method of inducing vortices in a liquid of superfluid helium. The number, positions, and lifetimes of vortex cores, and the overall structures of the vortex lattices generated, depend on numerous experimental conditions. A few examples of vortex lattices in a BEC are shown in Fig. 5. Further work at JILA using this vortex creation technique has shown that upon excitation of surface waves on the condensates, the dynamics of the atomic vortex lattices have been examined. Fundamental changes to the vortex lattice structure away from the equilibrium hexagonal pattern have also been observed [35].

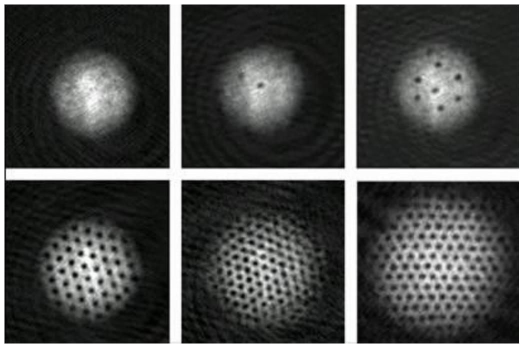

FIG. 5: Images of vortex lattices in Bose-Einstein condensates at JILA. The dark holes within the light areas are vortex cores. Each image was acquired after releasing a BEC from its magnetic trap, allowing it and the vortex cores to spatially expand enough to be optically resolved with a probe laser beam. Images provided by E. Cornell's TOP trap team at JILA [36].

VIII. SECOND-HARMONIC GENERATION

Second-harmonic generation was the first nonlinear optics effect to be observed. It results when light at frequency ω propagates through a crystal that exhibits a second-order nonlinearity. In terms of photons, it can be thought of as the process where two photons at frequency ω are annihilated and a new photon at frequency 2ω is created. At first sight, it might appear that such a process is impossible in atom optics, due to the conservation of the number of atoms. However, this is too simplified a picture: Since atoms can be combined to form molecules, a matter-wave optics analog of second-harmonic generation would correspond to the annihilation of two atoms and the creation of a diatomic molecule. Alternatively, second-order nonlinear effects such as sum and difference frequency generation and second-harmonic generation can be produced with quasi-particles, since their number is not conserved.

We concentrate here on the case of molecular condensates. So far, it has not been possible to reach condensation by directly cooling molecules to their BEC transition temperature. An alternative method that has proved much more successful consists in first creating an atomic condensate, and then combining the atoms into molecules. This can be achieved in principle by using either photoassociation or Feshbach resonances. Photoassociation proceeds by using laser light to capture atoms and combine them into a diatomic molecule during a collision, while Feshbach resonances operate by using a magnetic field to tune the energies of colliding atom pairs so that they can combine into a molecule. Resonances occur when the total energy of two colliding atoms equals the energy of a bound molecular state. In the vicinity of the resonance, it is essential to properly account for the molecular dynamics, and a proper description of the system requires that one accounts for the coherent coupling between the atomic and molecular field. The remarkable agreement between theory based on such approaches and experiments provides almost certain proof that in their recent experiment [37], Wieman and coworkers at JILA observed coherent oscillations between Rb atoms and Rb_2 molecules in an atomic condensate.

IX. MIXING OF OPTICAL AND MATTER WAVES

We have discussed in Section III how under appropriate circumstances, one can formally eliminate the material dynamics in the description of light-matter interactions, resulting in effective nonlinear interactions between light waves. Under a different set of conditions one can eliminate the electromagnetic field dynamics, resulting in effective atom-atom interactions —collisions— and nonlinear atom optics. Outside of these two regimes neither field is readily eliminated. This leads to new possibilities, including the nonlinear mixing of optical and matter waves and coherent matter-wave amplification.

Consider for example a Bose-Einstein condensate interacting with both a strong laser and a weak probe optical field. Assuming that the probe field begins in or near the vacuum state and the atomic field consists initially of a trapped BEC, the initial dynamics of the coupled system is dominated by a stimulated scattering process: the transfer of an atom to a momentum side-mode of the original condensate is accompanied by the transfer of a photon from the pump to the probe. This process may be thought of as the joint parametric amplification of an optical and a matter-wave field. It can be understood intuitively in essentially classical terms.

In matter-wave superradiance, both the weak optical field and the matter-wave field side-mode are spontaneously generated from, respectively, vacuum fluctuations and density fluctuations, the atomic version of vacuum fluctuations. As the condensate side-mode becomes populated, it interferes with the original condensate to create a spatial matter-wave density grating [38]. While the generated photons rapidly escape from the condensate region, the slow recoil velocity of the atoms results in the matter-wave grating remaining stored for a long time inside the BEC. This grating provides a feedback mechanism that leads under appropriate conditions to the sequential amplification of a series of momentum side modes of the condensate, i.e., the diffraction of atoms off of the matter-wave grating. This effect was demonstrated in a series of elegant experiments at MIT. Furthermore, sufficient feedback is provided by the matter-wave grating inside the condensate for the phase of the state being amplified to be preserved. Hence the matter-wave amplification is phase coherent with the amplified state [27].

X. FERMIONS

While there are many analogies between optical and atomic waves, perhaps the most fundamental difference is that while photons are bosons, atoms can be either bosons or fermions. Fermion behavior is strongly constrained by Pauli's exclusion principle. This, then, begs the question as to whether nonlinear atom optics is also possible with quantum-degenerate fermionic atoms.

Consider for example matter-wave four-wave mixing. This phenomenon may be interpreted in terms of atom scattering from a density grating of period $\Lambda_{\text{grating}} = 2\pi/K_{\text{grating}}$ generated inside the condensate. Alternatively, it is possible to understand this process in terms of a stimulated scattering process among different matter-wave momentum states, in which case amplification is attributed to Bose enhancement. This naturally leads one to ask whether matter-wave four-wave mixing is also possible in a quantum-degenerate Fermi gas [39]. This question was addressed in two recent papers [40] that demonstrate that this is indeed possible under appropriate conditions. In the case of a degenerate Bose gas, stimulated four-wave mixing can indeed be attributed to Bose enhancement, whereas in the fermionic case it can be shown to originate from quantum interferences between "paths" that lead to indistinguishable final states. A central tenet of quantum mechanics is that if it is impossible, even in principle, to distinguish between the paths that lead from the same initial state to the same final state, then the transition amplitudes for these processes must be added, rather than their probabilities. In that case, four-wave mixing is not interpreted as a result of quantum statistics, but rather of "cooperation." [41] Both mechanisms lead to practically the same enhancement proportional to the square N^2 of the number of atoms involved in forming the matter-wave grating, provided that the fermionic grating is properly prepared in a momentum-space Dicke state, that is, a highly entangled quantum state with all atoms in a coherent superposition of states with center-of-mass momenta 0 and K_{grating}. Hence, effects that can be interpreted as Bragg scattering from atomic-matter wave gratings, such as atomic four-wave mixing, BEC superradiance, and matter-wave amplification, can in principle work as efficiently in both degenerate Bose and Fermi systems.

XI. OUTLOOK

In the few short years since the first experimental realization of atomic Bose-Einstein condensates, the progress in atom optics has been astounding, rapidly moving the field into nonlinear and, more recently, quantum atom optics. It is becoming increasingly apparent that these developments will lead not just to a more profound understanding of the dynamics of ultracold atoms, quantum degenerate systems, and their interaction with light, but that intriguing technological developments will soon be realized.

Most obvious perhaps is the application of atom optics to the design of novel devices such as gravimeters, gravity gradiometers and inertial sensors. When compared with optical rotation sensors, atom interferometers present the advantage of producing a phase shift proportional to the atomic mass. It follows that, *everything else being equal*, a matter-wave rotation sensor is more sensitive than an optical gyroscope by a staggering factor of the order of 10^{11}. Of course, not everything is equal, and there are also major benefits to using optical interferometers as sensors. Nonetheless, it is daunting to observe that laboratory-based atom rotation sensors and gravity gradiometers [42] already compare favorably to their best optical counterparts. Other applications of atom optics will likely include nanofabrication, atom holography, and nanolithography. Quantum atom optics and the generation of nonclassical atomic fields might find applications in quantum information processing; the quantum entanglement between the quantum state of atoms, which can easily be stored, and light, which can easily propagate, might be of particular interest in this context. The nonlinear atom optics of fermionic matter, and the mixing between bosonic and fermionic fields, also open up new directions for investigations out of reach of conventional optics. As a last example, we mention the realization of condensates on optical lattices, which provides us with toy model systems for fundamental studies of a number of phase-transition-like phenomena such as ferromagnetism [43] and ferrimagnetism, but also opens the way to potential applications in quantum information technology [44].

Practical matter-wave devices are likely to be built on chips, very much like electronic sensors. This leads to the need to develop integrated atom optics, a goal actively pursued in a number of laboratories. It has recently become possible to generate Bose-Einstein condensates directly on a chip [45] and also to coherently couple atomic condensates into atomic waveguides. These spectacular and rapid developments bode well for the future of atom optics.

Acknowledgments

This work is supported in part by the U.S. Office of Naval Research under Contract No. 14-91-J1205, by the National Science Foundation under Grant No. PHY98-01099, by the NASA Microgravity Program Grant NAG8-1775, by the U.S. Army Research Office, and by the Joint Services Optics Program.

[1] L. de Broglie, Comp. Rend. Ac. Sci. **177**, 507 (1923).
[2] For a review of atom optics, see P. Meystre, *Atom Optics*, Springer-Verlag, New York, 2001.
[3] For a discussion of atom interferometry, see for example, R. M. Godun *et al.*, Cont. Phys. **42**, 77 (2001).
[4] F. Knauer and O. Stern, Zeit. Phys. **53**, 779 (1929).
[5] For an extensive review and references, see F. Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463 (1999). See also M. H. Anderson and M. Holland, Opt. Photon. News **7**, 23 (1996).
[6] See the Georgia Southern University BEC web page: http://amo.phy.gasou.edu/bec.html/.
[7] D. S. Hall, preprint: http://xxx.lanl.gov/abs/cond-mat/0302518.
[8] M. O. Mewes *et al.*, Phys. Rev. Lett. **78**, 582 (1997).
[9] I. Bloch, T. W. Hänsch, and T. Esslinger, Phys. Rev. Lett. **82**, 3008 (1999).
[10] E. W. Hagley *et al.*, Science **283**, 1706 (1999).
[11] B. P. Anderson and M. A. Kasevich, Science **282**, 1686 (1998).
[12] Y. Le Coq *et al.*, Phys. Rev. Lett. **87**, 170403 (2001).
[13] M. Kohl, T. W. Hänsch, and T. Esslinger, Phys. Rev. Lett. **87**, 160404 (2001).
[14] See for example, R. W. Boyd, *Nonlinear Optics*, Academic Press, San Diego, 1992.
[15] S. L. Cornish *et al.*, Phys. Rev. Lett. **85**, 1795 (2000).
[16] E. P. Gross, Nuovo Cimento **20**, 454 (1961); L. P. Pitaevskii, Sov. Phys. JETP **13**, 451 (1961).
[17] G. Lenz, P. Meystre, and E. M. Wright, Phys. Rev. Lett. **71**, 3271 (1993); Weiping Zhang and D. F. Walls, Phys. Rev. A **49**, 3799 (1994).
[18] L. Deng *et al.*, Nature **398**, 218 (1999).
[19] S. Burger *et al.*, Phys. Rev. Lett **83**, 5198 (1999).
[20] J. Denschlag *et al.*, Science **287**, 97 (2000).
[21] K. E. Strecker *et al.*, Nature **417**, 150 (2002).
[22] L. Khaykovich *et al.*, Science **296**, 1290 (2002).
[23] M. R. Matthews *et al.*, Phys. Rev. Lett. **83**, 2498 (1999).
[24] K. W. Madison *et al.*, Phys. Rev. Lett. **84**, 806 (2000).
[25] B. P. Anderson *et al.*, Phys. Rev. Lett. **86**, 2926 (2001); Z. Dutton *et al.*, Science **293**, 663 (2001).
[26] S. Inouye *et al.*, Science **285**, 571 (1999).
[27] S. Inouye *et al.*, Nature **402**, 641 (1999); M. Kozuma *et al.*, Science **286**, 2309 (1999).
[28] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997).
[29] S. Inouye *et al.*, Nature **392**, 151 (1998).
[30] T. Weber *et al.*, Science **299**, 232 (2003).
[31] M. Morinaga *et al.*, Phys. Rev. Lett. **77**, 802 (1996).
[32] O. Zobay *et al.*, Phys. Rev. A **5**, 643 (1999).
[33] D. N. Christodoulides and R. I. Joseph, Phys. Rev. Lett. **62**, 1746 (1989); C. M. de Sterke and J. E. Sipe, Gap soli-

tons, in *Progress in Optics Vol. XXXIII*, E. Wolf, editor, p. 203, Elsevier, Amsterdam, 1994.
[34] J. R. Abo-Shaeer *et al.*, Science **292**, 476 (2001).
[35] P. Engels *et al.*, Phys. Rev. Lett. **89**, 100403 (2002).
[36] http://jilawww.colorado.edu/bec/CornellGroup/index.html
[37] E. A. Donley *et al.*, Nature **417**, 529 (2002)
[38] M. R. Andrews *et al.*, Science **275**, 637 (1997).
[39] B. DeMarco and D. S. Jin, Science **285**, 1703 (1999).
[40] M. G. Moore and P. Meystre, Phys. Rev. Lett. **86**, 4199 (2001); W. Ketterle and S. Inouye, Phys. Rev. Lett. **86**, 4203 (2001).
[41] Cooperation is most familiar in optics in the situation where N excited atoms reside within a fraction of an optical wavelength from each other. In that case, spontaneous emission becomes cooperative, and as they decay to their ground state, the atoms emit a pulse of light whose intensity scales as N^2, a dramatic effect called superradiance.
[42] T. L. Gustavson, P. Bouyer, and M. A. Kasevich, Phys. Rev. Lett. **78**, 2046 (1997); M. J. Snadden *et al.*, Phys. Rev. Lett. **81**, 971 (1998).
[43] H. Pu *et al.*, Phys. Rev. Lett. **87**, 140405 (2001).
[44] D. Jaksch *et al.*, Phys. Rev. Lett. **82**, 1975 (1999); C. Orzel *et al.*, Science **291**, 2386 (2001); M. Greiner *et al.*, Nature **415**, 30 (2002).
[45] W. Hänsel *et al.*, Nature **413**, 498 (2001); H. Ott *et al.*, Phys. Rev. Lett. **89**, 230401 (2001).

About the authors: Brian P. Anderson received his Ph.D. in Applied Physics from Stanford University in 1999. After a postdoctoral position at JILA, he joined the Optical Sciences Center at the University of Arizona, where he has been an Assistant Professor since 2001. He is working on Bose-Einstein condensation experiments, and is particularly interested in nonlinear atom optics and superfluidity of Bose-Einstein condensates.

Pierre Meystre obtained his Ph.D. from the Swiss Federal Institute of Technology in Lausanne in 1974. He joined the Max-Planck Institute for Quantum Optics in 1977, following a postdoctoral position with M. O. Scully. Since 1986, he has been at the Optical Sciences Center, University of Arizona, where he is currently a Regents Professor. He is the author or co-author of over 200 papers on topics ranging from laser theory to cavity QED and nonlinear atom optics. He coauthored the textbook "Elements of Quantum Optics," and recently published the monograph "Atom Optics." He is a Fellow of the APS and of the OSA, and has been awarded the Humboldt Research Prize and the R.W. Wood Prize.