What comes next ?

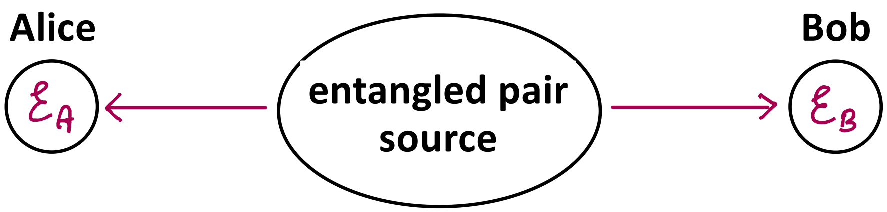
10-3-2024

Congratulations
You Survived Boot Camp

2 Spins, EPR States (Preskill ch. 2.5)

Basic Paradigm:

Shared pair of spin-1/2 particles



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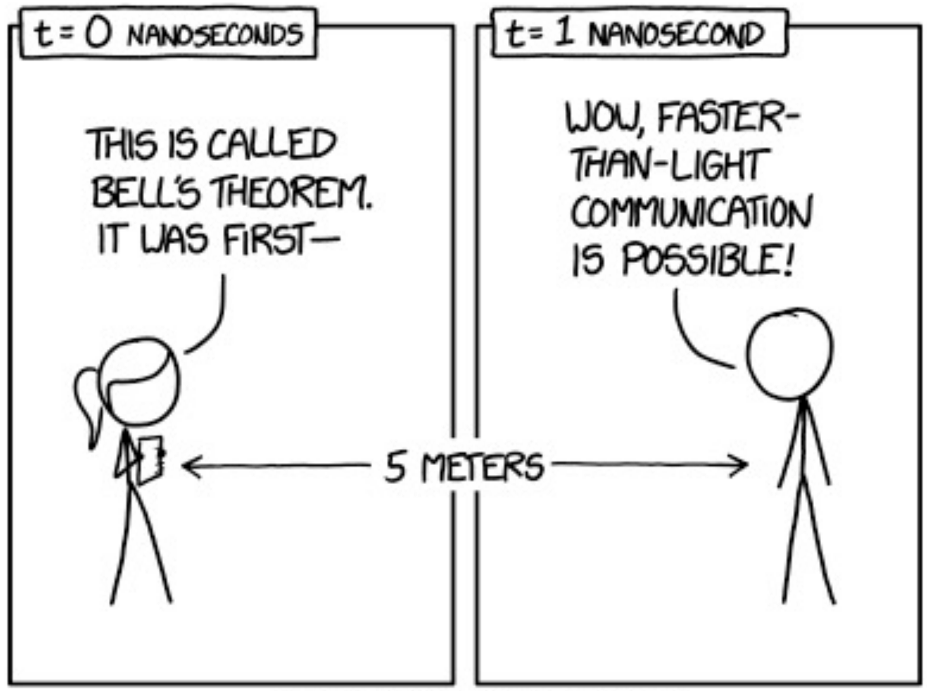

BELL'S SECOND THEOREM:
MISUNDERSTANDINGS OF BELL'S THEOREM
HAPPEN SO FAST THAT THEY VIOLATE LOCALITY.

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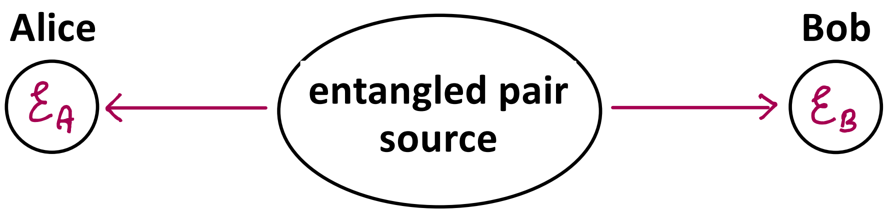

2 – spin state space: $\mathcal{E} = \mathcal{E}_A \otimes \mathcal{E}_B$

Product state Basis: $|\uparrow_{\hat n}\uparrow_{\hat n'}\rangle, |\uparrow_{\hat n}\downarrow_{\hat n'}\rangle, |\downarrow_{\hat n}\uparrow_{\hat n'}\rangle, |\downarrow_{\hat n}\downarrow_{\hat n'}\rangle$

Example of entangled state : $|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\uparrow_z\rangle + |\downarrow_z\downarrow_z\rangle\right)$

Measurement on spin A ⇨ **Need reduced Density Operator**

$$\rho_A = Tr_B[\rho_{AB}] = \sum_{i=\uparrow,\downarrow} {}_B\langle i|\tfrac{1}{2}\left(|\uparrow_z\uparrow_z\rangle + |\downarrow_z\downarrow_z\rangle\right)\left(\langle\uparrow_z\uparrow_z| + \langle\downarrow_z\downarrow_z|\right)|i\rangle_B$$

$$= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$ ← **maximally mixed**

Note: $\boxed{\rho_A \text{ contains no information !}}$

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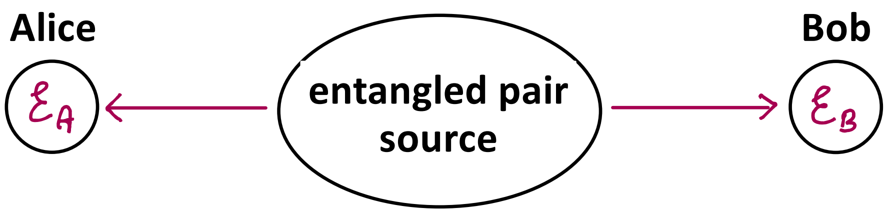

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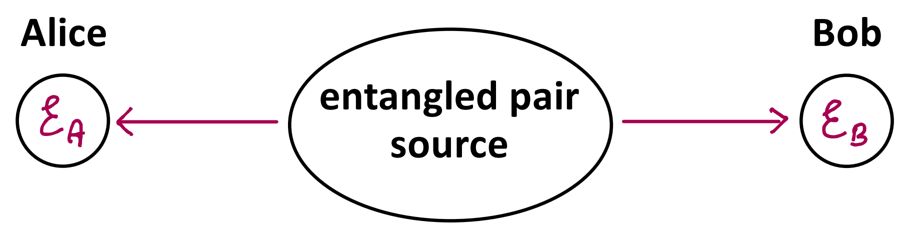
$$P(a) = Tr[P_a \varrho_A] = Tr\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}}_{\text{basis } |a\rangle, |a'\rangle} = Tr\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}$$

observable A
outcomes a, a'
eigenbasis $|a\rangle, |a'\rangle$

for any observable, any outcome

Local Measurements, Correlations ?



Alice and Bob each receive a steady stream of spins with buildt-in correlations according to

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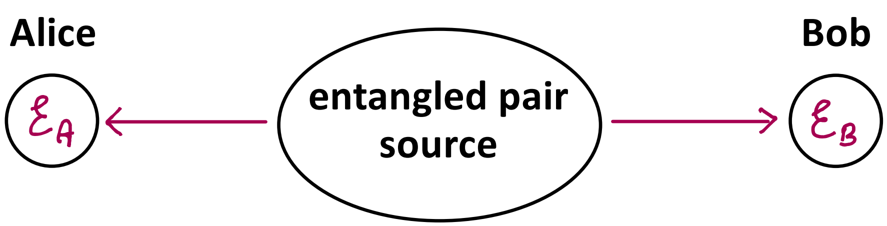

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<u>Note</u>: This holds for any max entangled state and any measurement Bob can make.

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But something is different in 1 vs 2:

Ensemble decomposition, Correlations

Correlations:

1. Bob measures S_z on many pairs ➡ $\uparrow\downarrow\uparrow\downarrow\downarrow\ldots$

   Alice measures S_z on many pairs ➡ $\uparrow\downarrow\uparrow\downarrow\downarrow\ldots$

   ➡ Compare records ➡ perfect correlation

2. Bob measures S_x on many pairs ➡ $\uparrow\downarrow\uparrow\downarrow\downarrow\ldots$

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➡ Alice can tell of Bob measured S_x or S_z if they compare measurement records

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<u>Pure State Distillation</u>:

1. Bob tells Alice he measured S_x , keeps measurement record ↑↓↑↓↓... to himself

   Alice keeps spins w/out measuring

   ➡ $\rho_A = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle\langle\uparrow_x| + |\downarrow_x\rangle\langle\downarrow_x|\right)$

2. Bob shares measurement record with Alice, who then knows which spins are up and which are down. She flips the latter.

   Alice can "distill" a pure state from the ensemble

<u>Conclusion</u>:

$$\boxed{\rho_A \neq \rho_A + \text{information}}$$

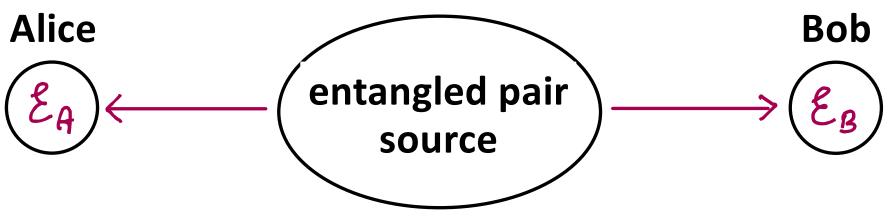
- Information is physical -

The above scenarios and variants thereof are central to Quantum Communication !

EPR and Bell Inequalities (Preskill ch. 4.1)

10-3-2024

We return to our basic scenario:



Einstein: If A & B are separated in space then measurements on A & B can be spacelike separated events ➡ Alice and Bob cannot exchange light speed signals so one of them will know the result of the others measurement before performing their own

⬇

In a complete description of physical reality a measurement performed on A must not modify the description of B.

Seems reasonable, given what we know about **Special Relativity** and **Causality**

How to think about this in a rigorous, testable way?

Thought experiment: Scheduling a date

1. Alice and Bob share qubits in the entangled state

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z \uparrow_z\rangle + |\downarrow_z \downarrow_z\rangle\right)}$$

They agree that if Bob measures $\uparrow_z$ at a specified later date then he will visit Alice.

2. Alice travels to a galaxy far, far away

3. At the agreed-upon time Alice and Bob measure their qubits

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Alice instantly knows about Bob's travel plans. Moreover, **this information did not exist prior to their measurements**

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Let us be clear:

(1) Alice and Bob have no control over the outcome of their measurements – it is equally likely that they both get $\uparrow_z$ or they both get $\downarrow_z$. Thus they cannot signal each other to say "I am bored, come visit"

(2) Alice and Bob would be in the same situation if they shared a mixed state,

$$\rho = \frac{1}{2}\left(|\uparrow_z \uparrow_z\rangle\langle \uparrow_z \uparrow_z| + |\downarrow_z \downarrow_z\rangle\langle \downarrow_z \downarrow_z|\right)$$

Therefore entanglement is not involved !

(3) Alice and Bob would be in the same situation if a machine prepared two boxes with either a green ball in each or a red ball in each, chosen by some fundamentally random process.

Therefore Quantum Mechanics is not involved !

EPR and Bell Inequalities (Preskill ch. 4.1)

10-3-2024

Did we go too far ?

Classical balls ➡ Info was there all along

Entangled spins ➡ Info came into existence when the measurement was made

Note: This distinction is meaningless to Alice and Bob

Nevertheless , the quantum and classical physics appears fundamentally different

Bells inequalities:

Guidelines to set up experiments with entangled spins so quantum mechanics and “reasonable classical models” (Hidden Variable Theories) make different and testable predictions

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Local Hidden Variable (LHV) Theories

Measurement is fundamentally deterministic. It appears probabilistic only because the state of a system is described by the quantum state plus a set of hidden variables whose values are not known and cannot be controlled

QM: Preparation ➡ spin in state $|\psi_s\rangle$

LHV: Preparation ➡ spin state $(|\psi_s\rangle, \{\lambda\})$

LHVs

Example: $|\psi_s\rangle$, one HV $0 \le \lambda \le 1$, uniformly distributed

Measure: σ_z ➡ $\begin{cases} |\uparrow_z\rangle & \text{for } 0 \le \lambda \le \cos^2 \theta/2 \\ |\downarrow_z\rangle & \text{for } \cos^2 \theta/2 \le \lambda \le 1 \end{cases}$

Deterministic if we know λ, probabilistic otherwise

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Take this seriously? Definitely!

EPR and Bell Inequalities (Preskill ch. 4.1)

Einstein: There exist a LHV description of a spin-1/2. Thus, once prepared, the outcome of measuring $\sigma_{\hat{n}} = \vec{\sigma}\cdot\hat{n}$ is completely determined by the LHV state $(|\psi_s\rangle, \{\lambda\})$, and we could Predict the outcome deterministically if only we knew $|\psi_s\rangle$ and the values of all the HV's in the set $\{\lambda\}$.

QM says: Measure $\sigma_{\vec{n}}$ ⇨ wipe out info predicting outcomes of later measurements $\sigma_{\vec{m}}$, where $\vec{n}\cdot\vec{m} = 0$.

HV Theory: This happens because measuring $\sigma_{\vec{n}}$ disturbs the values of the HV's in ways we cannot control and cannot know.

Nevertheless, the original HV state $(|\psi_s\rangle, \{\lambda\})$ contains all the info needed to predict the outcome of any pair of measurements $\sigma_{\vec{n}}$ or $\sigma_{\vec{m}}$.

Physical Reality of HV's?

If QM is always correct and HV theories make no measurably different predictions, then we conclude the $\{\lambda\}$ do not represent any element of physical reality (Occams Razor)

EPR experiment with spins:

(1) Prepare 2 spins in state $|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Note: Total spin $\vec{J} = \vec{S}_1 + \vec{S}_2$, $|\psi_-\rangle = |j=0, m=0\rangle$

(Singlet state, rotationally invariant)

(2) Separate and measure $\sigma_{\vec{n}}(A)$ and $\sigma_{\vec{m}}(B)$ as spacelike separated events ⇨

$$\text{Local descriptions} \begin{cases} \rho_A = \mathrm{Tr}_B\left(|\psi_-\rangle\langle\psi_-|\right) \\ \rho_B = \mathrm{Tr}_A\left(|\psi_-\rangle\langle\psi_-|\right) \end{cases}$$

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EPR and Bell Inequalities (Preskill ch. 4.1)

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(3) Is a LHV description possible?

To test, assign a LHV state $(\rho_i, \{\lambda\})_i$, $i = A, B$ where complete knowledge of the HVs allows deterministic predictions regarding measurements of $\sigma_{\vec{n}}(A), \sigma_{\vec{m}}(B)$ and their correlations.

* Experiments ⇨ we know measurements of $\sigma_{\vec{n}}(A), \sigma_{-\vec{n}}(B)$ are always perfectly correlated
  ⇨ source must build in correlations between $\{\lambda\}_A, \{\lambda\}_B$ to make this happen
* Even so, we still cannot predict if outcomes will be $\uparrow_{\vec{n}}, \downarrow_{-\vec{n}}$ or $\downarrow_{\vec{n}} \uparrow_{-\vec{n}}$

(4) Spacelike interval ⇨ Bobs measurement cannot alter the LHV state $(\rho_A, \{\lambda\}_A)$

* Bob measures $\sigma_{\vec{m}}(B)$ ⇨ we know result if Alice were to measure $\sigma_{\vec{m}}(A)$
* Instead Alice measures $\sigma_{\vec{n}}(A)$ ⇨ we have effectively measured complementary observables $\sigma_{\vec{n}}(A)$, $\sigma_{\vec{m}}(A)$, $\vec{n} \neq \vec{m}$

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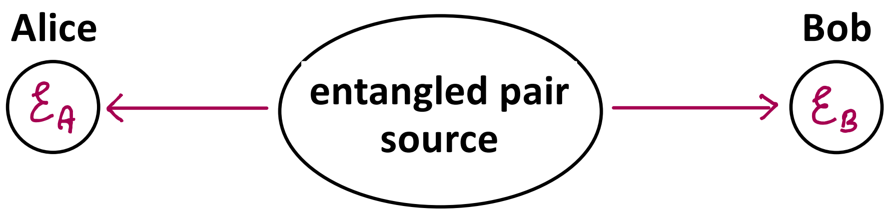
(5) In a LHV description complete info about the measurement outcomes and correlations must reside in the HV state $(\varphi_A, \{\lambda\}_A)$

John Bell: The LHV description above forces us to make certain predictions about the outcomes of joint measurements that are In conflict with those of Quantum Mechanics

EPR and Bell Inequalities (Preskill ch. 4.1)



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* Instead, before a light speed signal can reach her, Alice decides at random to instead measure $\sigma_{\vec{n}}(A)$

⇨ Between them, Alice and Bob have managed to measure the complementary observables

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Baked in at time of pair creation

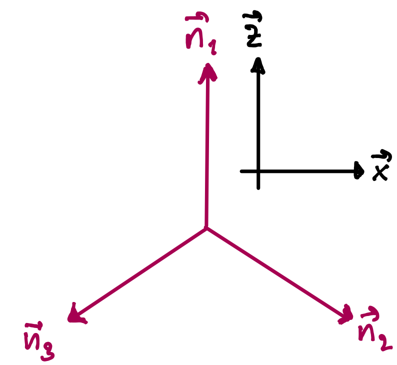
Setup: Alice and Bob choose at random between measurement axes

Alice chooses among

$\vec{n}_1 = (0, 0, 1)$

$\vec{n}_2 = \left(\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$

$\vec{n}_3 = \left(-\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$



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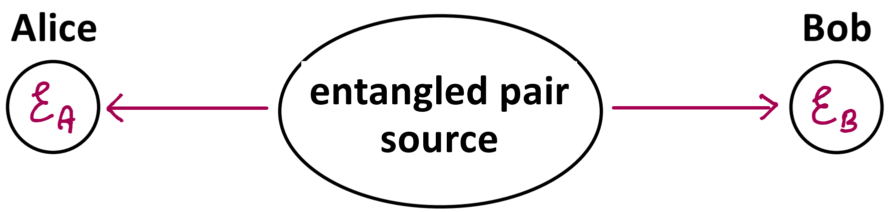
EPR and Bell Inequalities (Preskill ch. 4.1)



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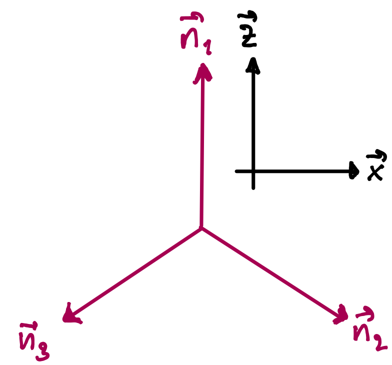

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EPR and Bell Inequalities (Preskill ch. 4.1)

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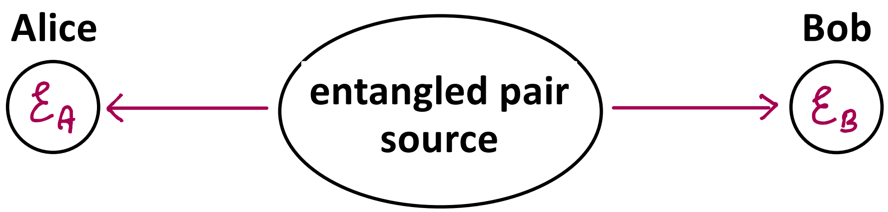

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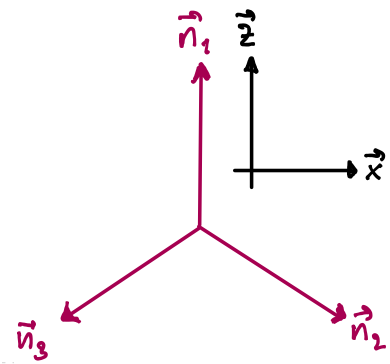

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(1) Repeat many times, keep those where $\vec{m} \neq -\vec{n}$, compare notes.

(2) Outcomes will not be perfectly correlated, but they can estimate probabilities $P_{same}(i,j)$ that the outcomes are $\uparrow_{\vec{n}_i} \downarrow_{-\vec{n}_j}$ or $\downarrow_{\vec{n}_i} \uparrow_{-\vec{n}_j}$ for any pair (i,j), $i \neq j$.

(3) Accept LHV description ➡ must have info about 3 combinations $(i,j) = (1,2), (1,3), (2,3)$ simultaneously, all encoded in Alice's LHV state

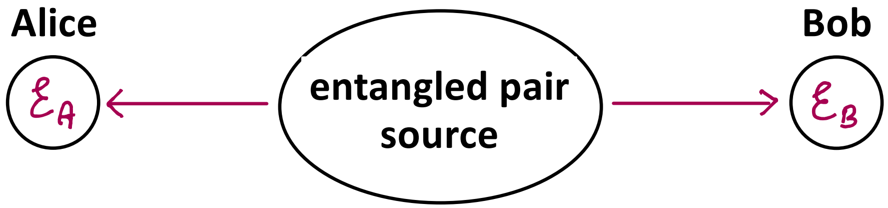
Equivalent Scenario:

Flip 3 coins repeatedly, and each time pick 2 at random and look at those only. The coin flip process builds in correlations between the coins (HV values) that we can observe in measurements. This allows us to estimate $P_{same}(i,j) \; \forall (i,j)$

Note: If we flip 3 coins (heads or tails) then at least 2 of them must have the same value. This gives us the following Bell's inequality:

$$\boxed{P_{same}(1,2) + P_{same}(1,3) + P_{same}(2,3) \geq 1}$$

EPR and Bell Inequalities (Preskill ch. 4.1)



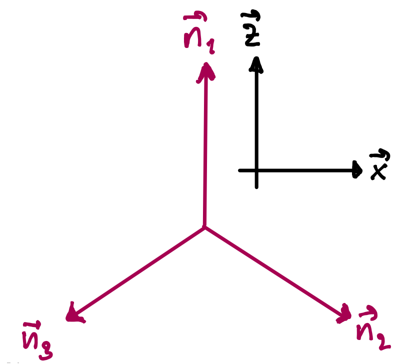
Setup: Alice and Bob choose at random between measurement axes

Alice chooses among

$\vec{n}_1 = (0, 0, 1)$

$\vec{n}_2 = (\frac{\sqrt{3}}{2}, 0, -\frac{1}{2})$

$\vec{n}_3 = (-\frac{\sqrt{3}}{2}, 0, -\frac{1}{2})$



Bob chooses among

$\vec{m}_1 = -\vec{n}_1$

$\vec{m}_2 = -\vec{n}_2$

$\vec{m}_3 = -\vec{n}_3$

(1) Repeat many times, keep those where $\vec{m} \neq -\vec{n}$, compare notes.

(2) Outcomes will not be perfectly correlated, but they can estimate probabilities $P_{same}(i,j)$ that the outcomes are $\uparrow_{\vec{n}_i} \downarrow_{-\vec{n}_j}$ or $\downarrow_{\vec{n}_i} \uparrow_{-\vec{n}_j}$ for any pair (i,j), $i \neq j$.

(3) Accept LHV description ➡ must have info about 3 combinations $(i,j) = (1,2), (1,3), (2,3)$ simultaneously, all encoded in Alice's LHV state

Equivalent Scenario:

Flip 3 coins repeatedly, and each time pick 2 at random and look at those only. The coin flip process builds in correlations between the coins (HV values) that we can observe in measurements. This allows us to estimate $P_{same}(i,j) \,\forall (i,j)$

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EPR and Bell Inequalities (Preskill ch. 4.1)

10-8-2024

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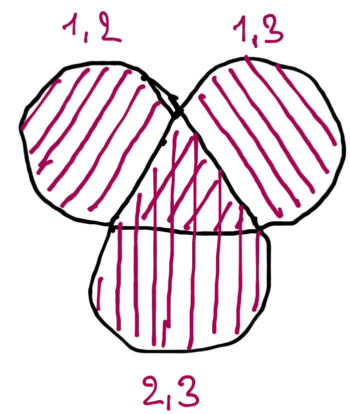
We can show this graphically by considering the overlap of the 3 sets $P(1,2 \vee 1,3 \vee 2,3) = 1$



(It does not matter if the coins are fair)

Note: This conclusion rests on one thing only – that the head-ness or tail-ness is a settled property of the coins before we look at them.

This is exactly what a LVH description says about the spin measurements in an EPR experiment. But It turns out to be in conflict with the predictions of Quantum Mechanics !!

EPR and Bell Inequalities (Preskill ch. 4.1)

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Quantum Mechanics: (From Preskill)

The prob. of an outcome is the expectation value of the corresponding projector. We have

$$E(\vec{n},+) = |\uparrow_{\vec{n}}\rangle\langle\uparrow_{\vec{n}}| = \frac{1}{2}(\mathbb{1}+\vec{n}\cdot\vec{\sigma}) = \frac{1}{2}(1+\sigma_{\vec{n}})$$

$$E(\vec{n},-) = |\downarrow_{\vec{n}}\rangle\langle\downarrow_{\vec{n}}| = \frac{1}{2}(\mathbb{1}-\vec{n}\cdot\vec{\sigma}) = \frac{1}{2}(1-\sigma_{\vec{n}})$$

projectors for outcomes up/down along $\vec{n}$

Probability of identical outcomes

$$\mathcal{P}(\pm,\pm) = \langle\psi_-|E^{(A)}(\vec{n},\pm)\,E^{(B)}(m,\pm)|\psi_-\rangle \;^{1)}$$

$$= \langle\psi_-|\tfrac{1}{2}(\mathbb{1}+\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)} \pm \sigma_{\vec{n}}^{(A)} \pm \sigma_{\vec{m}}^{(B)}|\psi_-\rangle$$

$$= \frac{1}{4}(1+\langle\psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)}|\psi_-\rangle \;^{2)}$$

Next we use

$$(\vec{\sigma}^{(A)}+\vec{\sigma}^{(B)})|\psi_-\rangle = 0 \Rightarrow \vec{\sigma}^{(B)}|\psi_-\rangle = -\vec{\sigma}^{(A)}|\psi_-\rangle \Rightarrow$$

$$\langle\psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)}|\psi_-\rangle = -\langle\psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(A)}|\psi_-\rangle$$

$$= \mathrm{Tr}[\rho_A\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(A)}] = \frac{1}{2}\mathrm{Tr}[(\vec{n}\cdot\vec{\sigma})(\vec{m}\cdot\vec{\sigma})] \qquad (\text{using } \rho_A = \tfrac{1}{2}\mathbb{1})$$

$$= -\frac{1}{2}\sum_{ij} n_i m_j \mathrm{Tr}[\sigma_i^{(A)}\sigma_j^{(A)}] = -\frac{1}{2}\sum_{i,j} n_i m_j \delta_{ij} \qquad (\text{using } \mathrm{Tr}[\sigma_i\sigma_j] = \delta_{ij})$$

$$= -\vec{n}\cdot\vec{m} = -\cos\theta, \quad \theta = \text{angle between } \vec{n}, \vec{m}$$

---

1) $|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\vec{n}}^{(A)}\rangle|\downarrow_{\vec{n}}^{(B)}\rangle - |\downarrow_{\vec{n}}^{(A)}\rangle|\uparrow_{\vec{n}}^{(B)}\rangle\right)$, 2) $\langle\sigma_{\vec{n}}^{(A)}\rangle = \langle\sigma_{\vec{m}}^{(B)}\rangle = 0$

EPR and Bell Inequalities (Preskill ch. 4.1)

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Note: σ_i are the Pauli operators $i=1,2,3$, and $\sigma_{\vec{n}}$ is the component of the Pauli vector along $\vec{n}$

This gives us

$$\boxed{\begin{aligned}\mathcal{P}(\pm,\pm) &= \tfrac{1}{4}(1-\cos\theta)\\ \mathcal{P}(\pm,\mp) &= \tfrac{1}{4}(1+\cos\theta)\end{aligned}}$$

In John Bell's version of the EPR experiment the angles between the $\vec{n}_i$ and the $\vec{m}_j$ are all 60^0, $\cos 60^0 = 1/2$.

QM: $\mathcal{P}_{same}(1,2)+\mathcal{P}_{same}(1,3)+\mathcal{P}_{same}(2,3) = 3/4$

Whereas

LVH: $\mathcal{P}_{same}(1,2)+\mathcal{P}_{same}(1,3)+\mathcal{P}_{same}(2,3) \geq 1$

EPR and Bell Inequalities (Preskill ch. 4.1)

10-8-2024

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Whereas

LVH: $P_{same}(1,2) + P_{same}(1,3) + P_{same}(2,3) \geq 1$

This is in conflict with the LHV model of the experiment !! Actual experiments agree with QM, rules out LHV's by many standard deviations.

David Mermin: Is reality really real ? (Physics Today)

Possible Resolutions

* **New physics** beyond QM – no sign so far

* **Complementarity** – Alice did not measure $\sigma^{(A)}_{\vec{n}_i}$, $\sigma^{(A)}_{\vec{n}_j}$ and it is meaningless to assign probs to measurements that were not done.

* **Nonlocality** – Bobs choice affects outcomes of Alice's measurements

* Alternative: Take QM at face value.

  Nature does not allow us to assign LHV descriptions to Alice and Bob's qubits if they are entangled. Only the Global State Has objective Physical Reality

  This is not hard to accept if we embrace the viewpoint of Quantum Information Science

EPR and Bell Inequalities (Preskill ch. 4.1)

10-8-2024

Quantum States are States of Knowledge. Thus, in the EPR experiment a global observer is permitted by Nature to have only as much information as can be encoded in the global state vector. This allows to predict correlations and nothing else. Nature does not allow local observers with access to only one spin to have any information about it.

Real EPR Experiments:

- Tend to use photons with entangled polarization states
- Earliest experiments used photons produced in atomic cascades (Aspect); modern experiments use photon pairs from spontaneous parametric downconversion.
- Photon experiments use polarization states $|x\rangle$, $|y\rangle$ corresponding to linear polarizations forming a 90° angle

Relevant formulae contains angles that are half of those for spins

Loopholes – some examples

- Locality (space-like separated measurements)
- Fair Sampling (detection efficiency)
- Freedom of choice (truly random meas. settings)
- Coincidence – time (locally defined detection windows)
- Memory (trials not identical and independent)

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- First good experiments Aspect et al. (3 papers)
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From Preskill's notes, chapter 4.1

Bell's logic seemed compelling but something went wrong, so we are forced to reconsider his tacit assumptions. First, Bell assumed that there is a joint probability distribution that governs the possible outcomes of all measurements that Alice and Bob might perform. This is the hidden-variable hypothesis. He imagines that if the values of the hidden variables are exactly known, then the outcome of any measurement can be predicted with certainty — measurement outcomes are described probabilistically because the values of the hidden variables are drawn from an ensemble of possible values. Second, Bell assumed that Bob's decision about what to measure in Chicago has no effect on the hidden variables that govern Alice's measurement in Pasadena. This is the assumption that the hidden variables are local. If we accept these two assumptions, there is no escaping Bell's conclusion. We have found that the correlations predicted by quantum theory are incompatible with theses assumptions.

What are the implications? Perhaps the moral of the story is that it can be dangerous to reason about what might have happened, but didn't actually happen — what are sometimes called *counterfactuals*. Of course, we do this all the time in our everyday lives, and we usually get away with it; reasoning about counterfactuals seems to be acceptable in the classical world, but sometimes it gets us into trouble in the quantum world. We claimed that Alice knew what would happen when she measured along $\hat{a}_1$, because Bob measured along $-\hat{a}_1$, and every time we have ever checked, their measurement outcomes are always perfectly correlated. But Alice did *not* measure along $\hat{a}_1$; she measured along $\hat{a}_2$ instead. We got into trouble by trying to assign probabilities to the outcomes of measurements along $\hat{a}_1, \hat{a}_2$, and $\hat{a}_3$, even though Alice can perform just one of those measurements. In quantum theory, assuming that there is a probability distribution that governs the outcomes of all three measurements that Alice might have made, even though she was able to carry out only one of these measurements, leads to mathematical inconsistencies, so we had better not do it. We have affirmed Bohr's principle of *complementary* — we are forbidden to consider simultaneously the possible outcomes of two mutually exclusive experiments.

EPR and Bell Inequalities (Preskill ch. 4.1)

10-8-2024

Clauser-Horne-Shimony-Holt (C.H.S.H.) Inequality

(Different version of Bell's inequality)

Alice: 2 settings ⇨ measure $\begin{cases} a \text{ w/outcome } \pm 1 \\ a' \text{ w/outcome } \pm 1 \end{cases}$

Bob: 2 settings ⇨ measure $\begin{cases} b \text{ w/outcome } \pm 1 \\ b' \text{ w/outcome } \pm 1 \end{cases}$

Note: $a, a' = \pm 1$ ⇨ $\begin{cases} a + a' = 0 \ \& \ a - a' = \pm 2 \\ a - a' = 0 \ \& \ a + a' = \pm 2 \end{cases}$

Combine w/ $b, b' = \pm 1$ ⇨

$$C = (a + a')b + (a - a')b' = \pm 2$$

EPR and Bell Inequalities (Preskill ch. 4.1)

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It follows that $|\langle c \rangle| \leq \langle |c| \rangle = 2$ ➡

$$\boxed{|\langle ab \rangle + \langle a'b \rangle + \langle ab' \rangle - \langle a'b' \rangle| \leq 2}$$

(C.H.S.H. inequality)

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For $\theta = 45°$
(max violation)

$$\boxed{|\langle ab \rangle + \langle a'b \rangle + \langle ab' \rangle - \langle a'b' \rangle| = \frac{4}{\sqrt{2}} = 2\sqrt{2} > 2}$$

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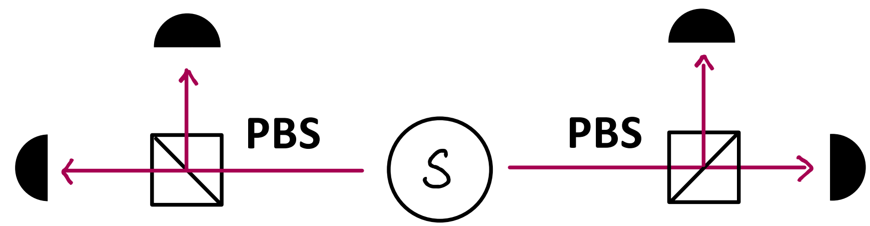
- violates C.H.S.H. inequality

Laboratory Experiment (Aspect, many others)



polarizer settings