Introduction and Overview (Preskills Notes)

9-16-2024

Quantum Computing in the NISQ era and beyond

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Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

Review of Quantum Mechanics Part 1

9-16-2024

Cohen-Tannoudji Ch. II & III, Preskill 2.1 & 2.3

Review of Quantum Mechanics Part 1

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Note: Everyone is assumed to be familiar with grad level QM

Review of 2-level systems, Tensor Products of States, Operators, and Hilbert Spaces. Density Matrix formalism

State vectors

("Rays" in Preskill)

Unique quantum state ↔ unique state vector

$|\psi\rangle \in \mathcal{E}$ ← State Space

Scalar product

$\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$

complex number

($\mathcal{E}$ is a Hilbert Space)

Linear Operators

$\forall\, |\psi\rangle \in \mathcal{E}: \quad A|\psi\rangle = |\psi'\rangle \in \mathcal{E}$

Projectors

$P_\psi = |\psi\rangle\langle\psi|$ ← Projector on $|\psi\rangle$

$P_{\mathcal{E}_q} = \sum_{i=1}^{q} |\varphi_q^i\rangle\langle\varphi_q^i|$ ← projector on subspace $\mathcal{E}_q$

Basis in q dimensional $\mathcal{E}_q$

Hermitian Operators

$A^+ = A$

Adjoint $|\psi'\rangle = A|\psi\rangle \leftrightarrow \langle\psi'| = \langle\psi|A^+$

Physical (measurable) quantities!

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Eigenvalue Equation

$A|\psi\rangle = \lambda|\psi\rangle$

A Hermitian

- Eigenvalues of A are real-valued
- Eigenvectors $A|\psi\rangle = \lambda|\psi\rangle$, $A|\varphi\rangle = \mu|\varphi\rangle$ are orthogonal if $\lambda \neq \mu$
- Eigenvectors of A form orthonormal basis in $\mathcal{E}$

Commuting Observables

$[A,B] \equiv AB - BA = 0$ ⇒

$\exists$ orthonormal basis in $\mathcal{E}$ of common eigenvectors of A, B

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C.S.C.O (Complete set of commuting observables)

Set $A, B, C\ldots$ such that basis $\exists$ in $\mathcal{E}$ of eigenvectors $|a_m, b_m, c_m \ldots\rangle$ uniquely labeled by the set of eigenvalues a_m, b_m, c_m

Example H, L^2, L_z for the Hydrogen atom

Unitary Operators

U is unitary ⇨ $U^{-1} = U^{+} \leftrightarrow U^{+}U = UU^{+} = \mathbb{1}$

Scalar product invariant: $\langle\psi|\varphi\rangle = \langle\psi|U^{+}U|\varphi\rangle$

⇨ U is a change of basis in $\mathcal{E}$

$U|v\rangle = \lambda|v\rangle$ ⇨ $\lambda = e^{i\theta}$

eigenvecs for $\lambda \neq \lambda'$ are orthogonal

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Representation and bases

The set $\{|u_i\rangle\}$ forms a basis in $\mathcal{E}$ if the expansion

$$|\psi\rangle = \sum_i \langle u_i|\psi\rangle |u_i\rangle$$

is unique and exists $\forall |\psi\rangle \in \mathcal{E}$

States $|\psi\rangle \Leftrightarrow \begin{bmatrix} \vdots \\ \langle u_i|\psi\rangle \\ \vdots \end{bmatrix}$

Operators $A \Leftrightarrow \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}$

Review of Quantum Mechanics Part 1

Postulates of Quantum Mechanics

(1) At a fixed time t the state of a physical system is defined by specifying a ket $|\psi(t)\rangle$ belonging to the state space $\mathcal{E}$.

(2) Every measurable physical quantity $\mathcal{A}$ is described by an operator A acting in $\mathcal{E}$; this operator is an observable.

(3) The only possible result of a measurement of A physical quantity $\mathcal{A}$ is one of the eigenvalues of the corresponding observable A.

(4) (Discrete non-degenerate spectrum) When the physical quantity $\mathcal{A}$ is measured on A system in the normalized state $|\psi\rangle$, the probability $\mathcal{P}(a_n)$ of obtaining the non-degenerate eigenvalue a_n of the observable A is:

$$\mathcal{P}(a_n) = |\langle a_n|\psi\rangle|^2 = \langle\psi|P_n|\psi\rangle$$

where $|a_n\rangle$ is the normalized eigenvector of A associated with the eigenvalue a_n, and $P = |a_n\rangle\langle a_n|$ is the projector onto $|a_n\rangle$.

Postulates of Quantum Mechanics

(5) If the measurement of the physical quantity $\mathcal{A}$ on the system in state $|\psi\rangle$ gives the result a_n, then the state immediately after the measurement is the normalized projection of $|\psi\rangle$ onto $|a_n\rangle$:

$$|\psi_{after}\rangle = \frac{P_n|\psi\rangle}{\langle\psi|P_n|\psi\rangle}$$

Degenerate case: use projector onto the Subspace associated with a_n.

(6) The time evolution of the state vector $|\psi(t)\rangle$ Is governed by the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

where $H(t)$ is the observable associated with the total energy of the system.

See also Note on the Bayesian Update Rule for “classical” probability distributions

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9-16-2024

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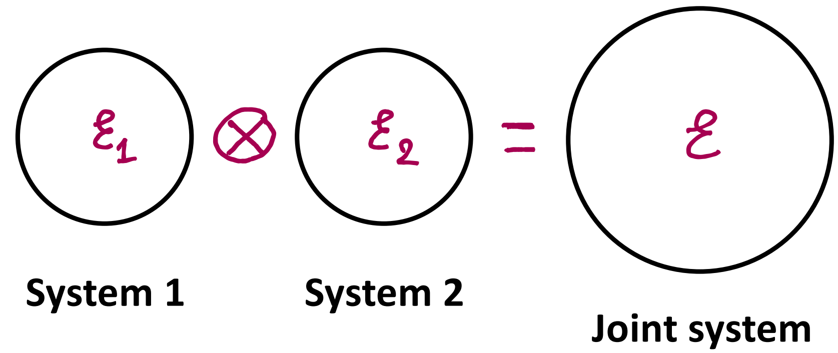
Quantum Mechanics of systems that consist of multiple parts



Def: Let $\mathcal{E}_1$, $\mathcal{E}_2$ be vector spaces of dimension N_1, N_2

The vector space $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$ is called the

Tensor Product of $\mathcal{E}_1$ and $\mathcal{E}_2$ iff

$\forall$ pairs $|\varphi(1)\rangle \in \mathcal{E}_1, |\chi(2)\rangle \in \mathcal{E}_2, \exists$ vector $\in \mathcal{E}$

such that

1. The association is linear with respect to multiplication with complex numbers

$$\lambda|\varphi(1)\rangle \otimes \mu|\chi(2)\rangle = \lambda\mu\left[|\varphi(1)\rangle \otimes |\chi(2)\rangle\right]$$

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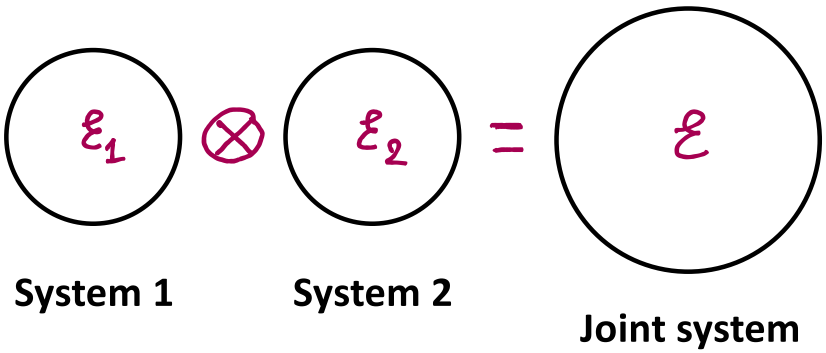

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2. Distributive $|\varphi(1)\rangle \otimes \left[a|\chi_1(2)\rangle + b|\chi_2(2)\rangle \right]$

$$= a|\varphi(1)\rangle \otimes |\chi_1(2)\rangle + b|\varphi(1)\rangle \otimes |\chi_2(2)\rangle$$

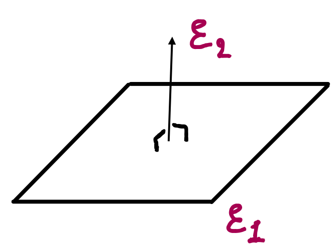
3. Bases $\{|u_i(1)\rangle\}$ in $\mathcal{E}_1$, $\{|v_\ell(2)\rangle\}$ in $\mathcal{E}_2$

$\Rightarrow$ $\{|u_i(1)\rangle \otimes |v_\ell(2)\rangle\}$ is a basis in $\mathcal{E}$

Iff N_1, N_2 are finite, then $\mathrm{Dim}(\mathcal{E}) = N_1 \times N_2$

These properties $\Rightarrow$ The usual linear algebra works in $\mathcal{E}$

Analogy: Tensor product of 1D & 2D geometrical space



Note: $\mathcal{E}_1 \otimes \mathcal{E}_2 \neq 3D$ geom. space

SP of vectors in $\mathcal{E}_1$ w/vectors in $\mathcal{E}_2$

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9-16-2024

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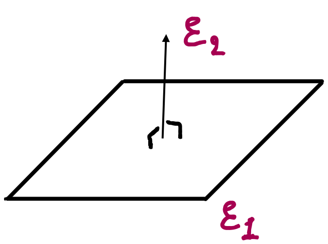

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SP of vectors in $\mathcal{E}_1$ w/vectors in $\mathcal{E}_2$ not defined

Vectors in $\mathcal{E}$

Let $|\varphi(1)\rangle = \sum a_i |u_i(1)\rangle$
$|\chi(2)\rangle = \sum b_\ell |v_\ell(2)\rangle$

Then $|\varphi(1)\rangle \otimes |\chi(2)\rangle = \sum_{i,\ell} a_i b_\ell |u_i(1)\rangle \otimes |v_\ell(2)\rangle$

Hugely important:

There are vectors in $\mathcal{E}$ that are not tensor products of vectors from $\mathcal{E}_1, \mathcal{E}_2$

General vector $\in \mathcal{E}$ can be written as

$$|\psi\rangle = \sum_{i,\ell} c_{i\ell} |u_i(1)\rangle \otimes |v_\ell(2)\rangle$$

How to see? There are $N_1 \times N_2$ prob. ampl's $c_{i\ell}$

These cannot all be written as $a_i \times b_\ell$ where the sets $\{a_i\}, \{b_\ell\}$ are valid probability amplitudes.

Review of Quantum Mechanics Part 1

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Example: $\mathcal{E}_1, \mathcal{E}_2$ are qubits, $N_1 = N_2 = 2$

$|\varphi(1)\rangle = a_1 |u_1(1)\rangle + a_2 |u_2(1)\rangle$

$|\chi(2)\rangle = b_1 |v_1(2)\rangle + b_2 |v_2(2)\rangle$ ➡ 2 real-valued variables each

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In basis $\{|u_i(1)\rangle \otimes |v_\ell(2)\rangle\}$

Product state $\begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix}$ ↑ 4 real-valued variables

General state $\begin{bmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \end{bmatrix}$ ↑ 6 real-valued variables

---

N qubits ➡ $\begin{cases} \text{product state} \rightarrow 2N \text{ real variables} \\ \text{general state} \rightarrow 2^{N+1} - 2 \text{ real var's} \end{cases}$

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$|\chi(2)\rangle = b_1 |v_1(2)\rangle + b_2 |v_2(2)\rangle$

$\Rightarrow$ 2 real-valued variables each

In basis $\{ |u_i(1)\rangle \otimes |v_j(2)\rangle \}$

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N qubits $\Rightarrow$ $\begin{cases} \text{product state} \to 2N \text{ real variables} \\ \text{general state} \to 2^{N+1} - 2 \text{ real var's} \end{cases}$

Note: States $\in \mathcal{E}$ that are not product states are known as

Entangled States or Correlated States