

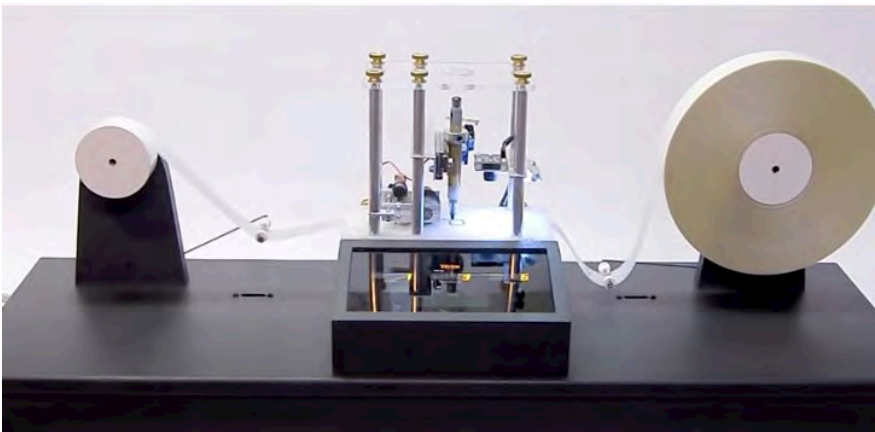
Physics of Information: **Turing
von Neumann**

Notions: **What is a computation ?
What is computable**

**Formulation of Computer Science
that is **Device Independent****



1937 Turing Machine:



<https://www.youtube.com/watch?v=E3keLeMwfHY>

Wikipedia:

A Turing Machine (TM) is a mathematical model of computation describing an abstract machine that manipulates symbols on a strip of paper according to a table of rules.

The TM operates on an infinite tape divided into cells, each of which can hold a symbol drawn from a finite set.

At each step the head reads the symbol in the cell. Then, based on the symbol and the TM's present state, the machine writes a symbol in the cell, and moves the head one step to the left or the right, or halts the computation.

https://en.wikipedia.org/wiki/Turing_machine

Church – Turing Thesis:

Everything that is computable can be computed on a Turing Machine with at most polynomial overhead.

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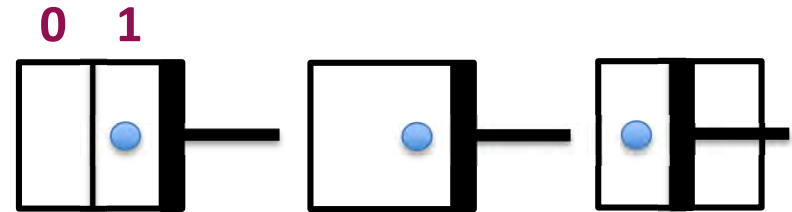
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Landaur: Information is Physical!

Example: Erasure = Dissipation



Entropy: $\Delta S_{\text{gas}} = -k \ln 2$

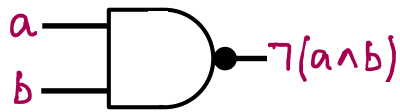
Work: $W = kT \ln 2 = 0.96 \times 10^{-23} \frac{\text{J}}{\text{K}} \cdot 300\text{K}$
 $\sim 3 \times 10^{-21} \text{J} \sim 0.02 \text{eV}$

Is there a way around it ?

Reversible Computation!

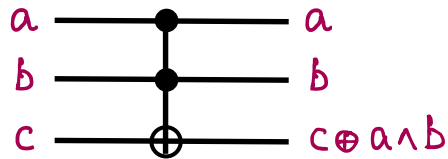
But we need a different gate set !

NAND Gate:



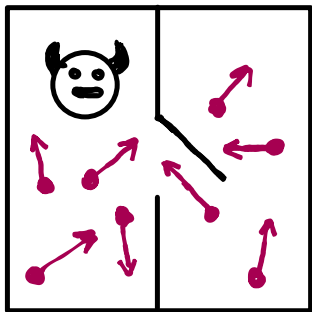
irreversible

Toffoli Gate:



reversible

Maxwells Demon:



Information is Physical!

Quantum Information

Carl Caves: Quantum States are states of knowledge

Physics is Information!

End 8-29-2024

New properties of QM

Measurement:

$$[A, B] \neq 0 \Rightarrow \Delta A \Delta B \geq \frac{\hbar}{2} |\langle [A, B] \rangle|$$

Acquire Info \rightarrow Disturb system

Randomness:

Outcome fundamentally unpredictable

"Collapse" of wavefunction

Cannot determine state of a single quantum if initially unknown

Cannot Copy
No cloning theorem

Entanglement:

Non-local correlations

pure state, entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$\rho = \frac{1}{2} (|00\rangle\langle 00| + |11\rangle\langle 11|)$$

mixed state, not entangled

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Quantum Computing

Does QM impact Computation?

Peter Shor (1994): YES! \rightarrow Quantum Fourier Transform

\downarrow
Factoring !

DFT on N bits	$\mathcal{O}[(2^N)^2]$	steps
FFT on “	$\mathcal{O}[N2^N]$	“
QFT on “	$\mathcal{O}[N \log N]$	“

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Efficient Factoring

Factoring $n = p \times q$ \rightarrow RSA encryption

large integer \rightarrow \leftarrow large prime numbers

- easy to verify solution $\log n$ steps
- very hard to solve $\mathcal{O}[n]$ steps



\rightarrow Exponential in # of digits

QFT \rightarrow Solvable

Preskill Ch. 1, p. 5-6 $T \propto e^{1.9(\log n)^{1/3}} e^{(\log \log n)^{2/3}}$
 Best Classical Algorithm

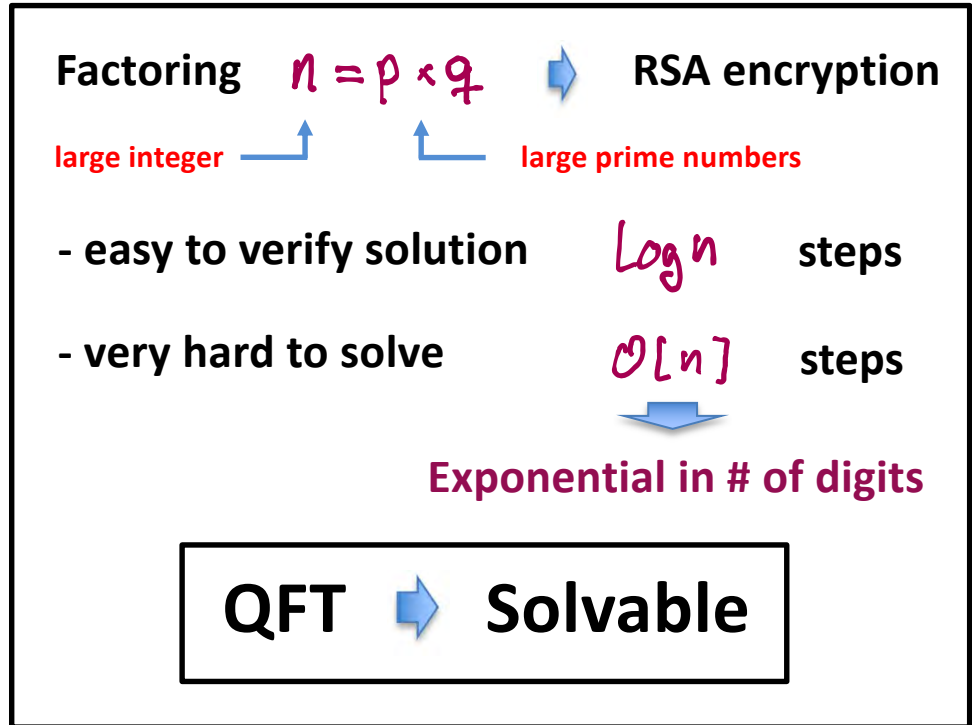
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(1998) 130 digits in 1 month



400 digits in 10^{10} years

(2022) 24 yrs = 16 Moores Law doublings

$2^{16} = 65,536$  400 digits \sim 150kYrs

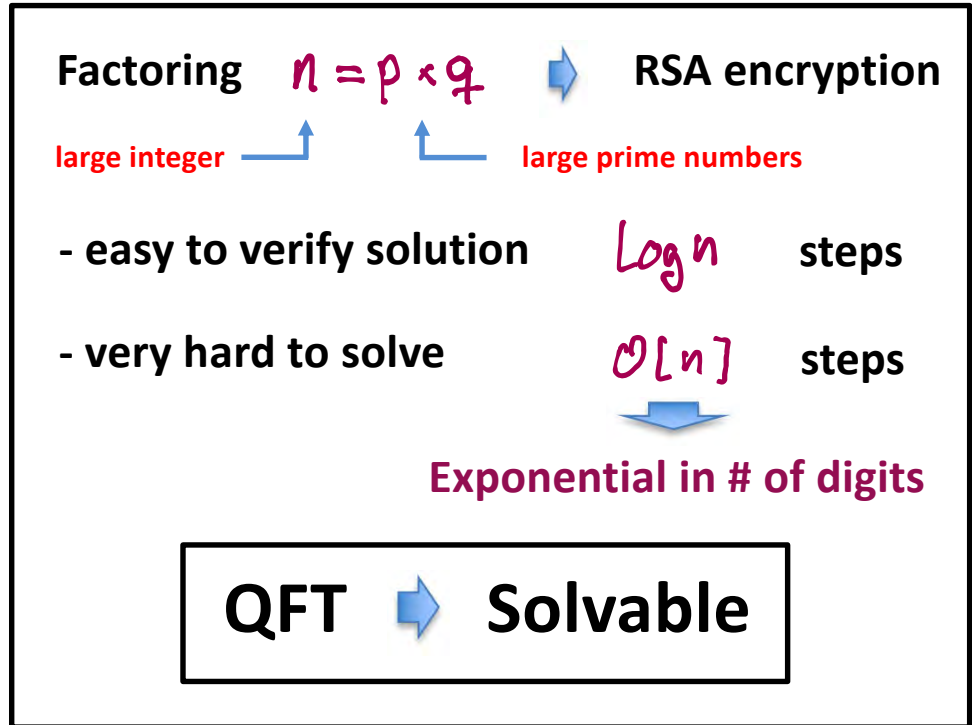
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\rightarrow 400 digits/ 10^{10} years Polynomial in # of digits

Shors algorithm: $\mathcal{O}[(\log n)^3]$ \leftarrow

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Polynomial in # of digits

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Quantum Complexity

Benioff, Feynman \rightarrow Simulating QM is hard

Simulating Physics with computers (1982)

qubit $\begin{matrix} \text{---} & |1\rangle \\ \text{---} & |0\rangle \end{matrix}$ $|2\rangle = a|0\rangle + b|1\rangle$

N qubits $\rightarrow |01101011\dots 010\rangle = |x\rangle \in \mathcal{E}$

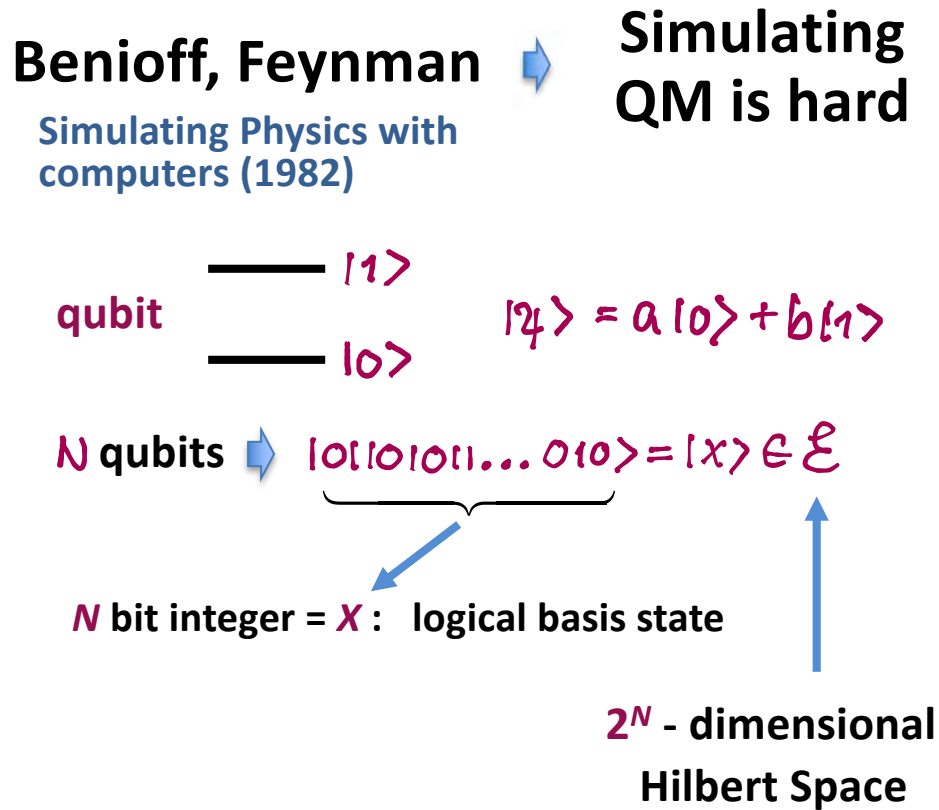
N bit integer = x : logical basis state

2^N - dimensional Hilbert Space

General State:

$$|\psi\rangle = \frac{1}{2^{N/2}} \sum_{x=0}^{2^N-1} a_x |x\rangle$$

Quantum Complexity



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Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

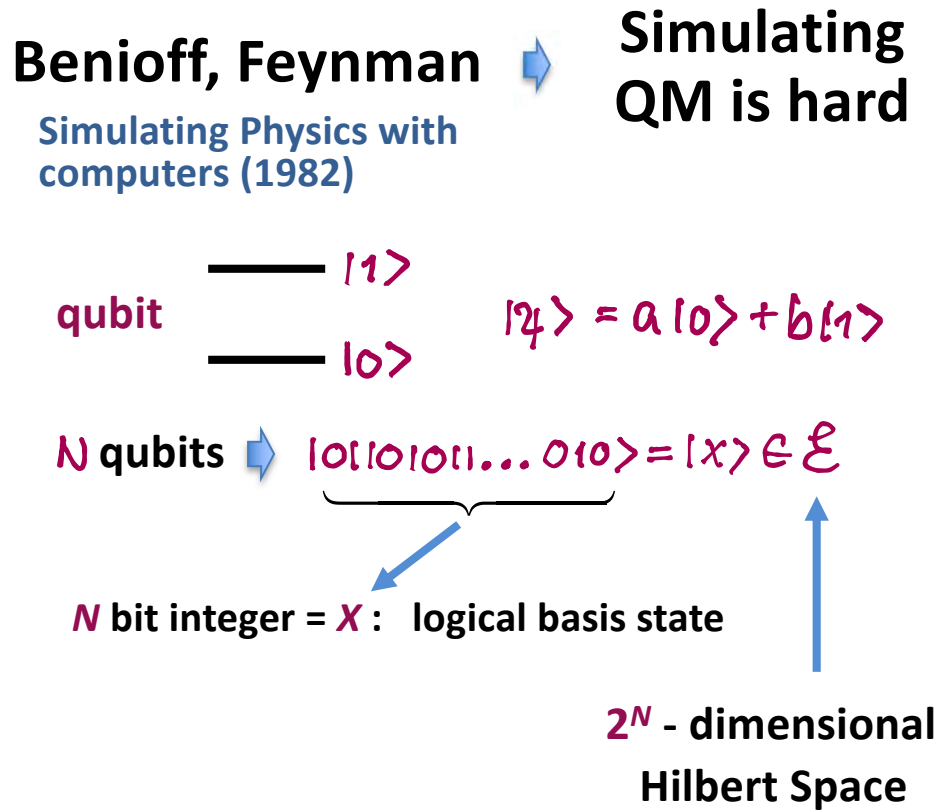
Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain. The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics. If we suppose that we know all the physical laws perfectly, of course we don't have to pay any attention to computers. It's interesting anyway to entertain oneself with the idea that we've got something to learn about physical laws; and if I take a relaxed view here (after all I'm here and not at home) I'll admit that we don't understand everything.

The first question is, What kind of computer are we going to use to simulate physics? Computer theory has been developed to a point where it realizes that it doesn't make any difference; when you get to a *universal computer*, it doesn't matter how it's manufactured, how it's actually made. Therefore my question is, Can physics be simulated by a universal computer? I would like to have the elements of this computer *locally interconnected*, and therefore sort of think about cellular automata as an example (but I don't want to force it). But I do want something involved with the

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- * Is it possible for a classical computer to efficiently simulate QM ?
- * Use probabilistic local algorithm (the most general kind)

John Bell

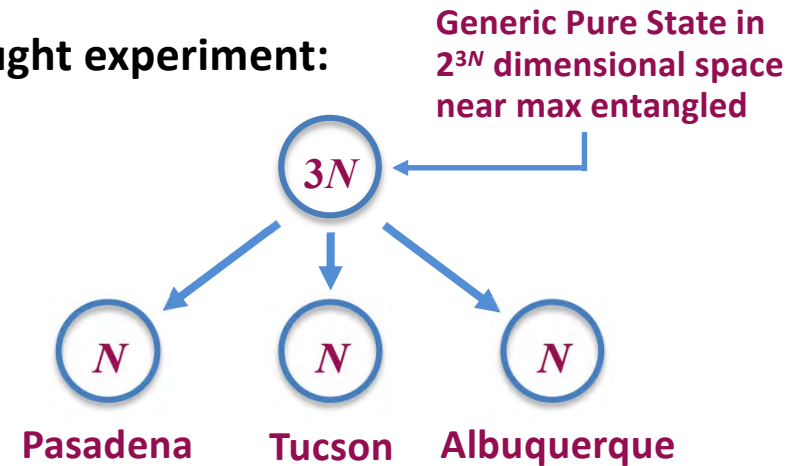
Bell's Theorem:

No local probabilistic theory can reproduce all of QM

Non-Local Correlations

Key to Quantum Information

Thought experiment:



Local state close to random $\rho \sim \frac{1}{2^N} \mathbb{I}$

Shannon Info $S = -\sum_{i=1}^{2^N} p_i \log p_i$

Von Neuman Info $S = -\text{Tr}[\rho \log \rho]$

entropy \uparrow

max value of S
 $\sim N - 2^{-(N+1)}$

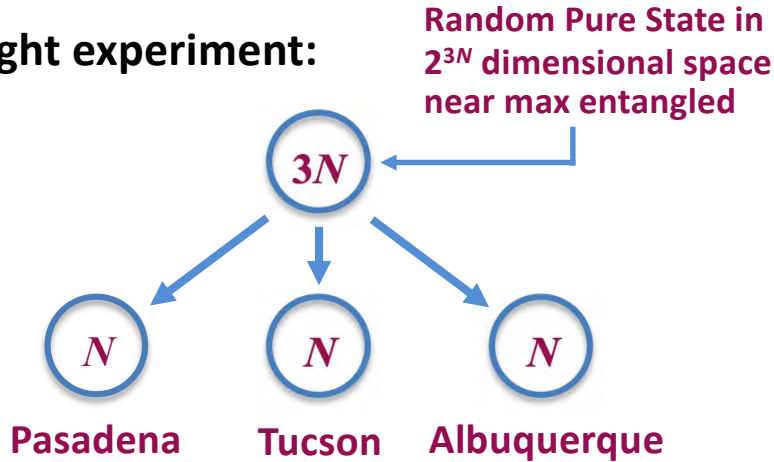
Almost all info in N - body state in Non-Local correlations

Introduction and Overview (Preskills Notes)

Non-Local Correlations

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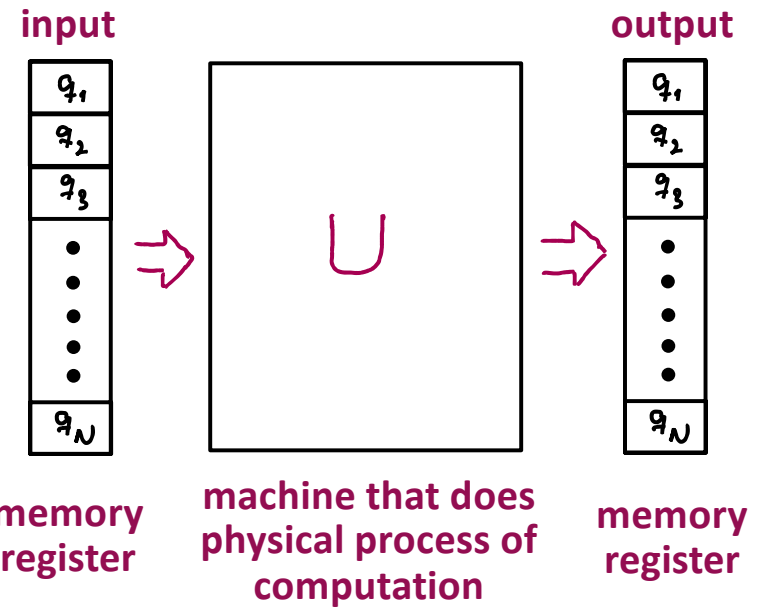
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entropy

Almost all info in N - body state in Non-Local correlations

OK – Plausible QM can do more
Where does the QC's power come from?

Visualization of Computation



Classical: Register is in one of the logical states

$$x = q_1 q_2 q_3 \dots q_N$$

binary #

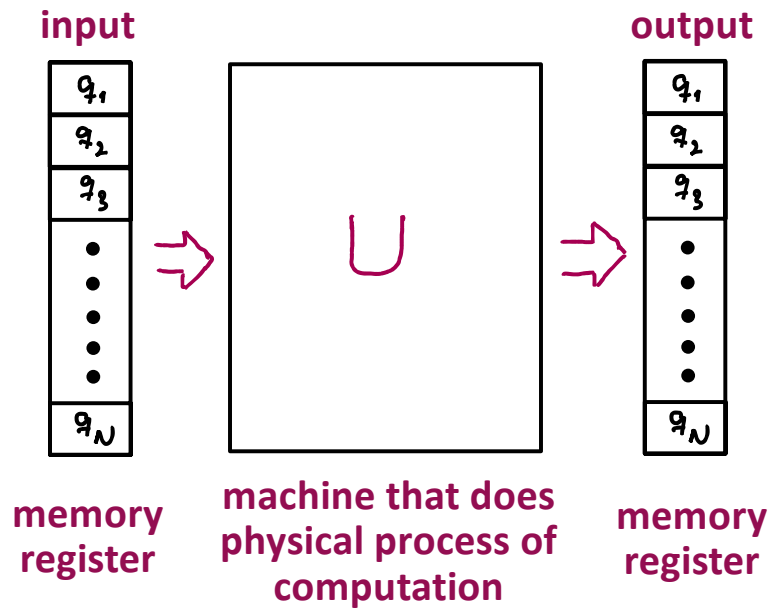
Reversible transformation

$$U: x \rightarrow y$$

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Quantum: Register can be in any coherent superposition of logical states $|x\rangle$

Unitary transformation $U: |x\rangle \rightarrow |y\rangle$

Maps basis to basis $U: \{|x\rangle\} \rightarrow \{|y\rangle\}$

Quantum Parallelism

$$|\psi_{in}\rangle \rightarrow \sum_x a_x |x\rangle \rightarrow |$$

$$\rightarrow |\psi_{out}\rangle = U|\psi_{in}\rangle = \sum_x a_x |y\rangle = \sum_x b_x |x\rangle$$

Machine processes 2^N inputs “in parallel” !

Beware: measurement collapses Q. Register into a single basis state at random

We get one random result out of 2^N

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Quantum Sampling Problem

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Quantum Algorithms look for global properties of functions – symmetry, periodicity, etc.

- * Classical -> requires many function evaluations
- * Quantum -> design **U** so measurement gives answer with high probability
- * \exists classes of problems (**sampling problems**) which are classically hard but quantum “easy”

Google “Quantum Supremacy”

Expert insight into current research

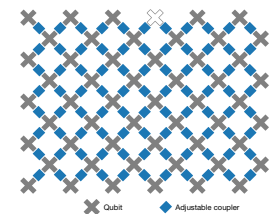
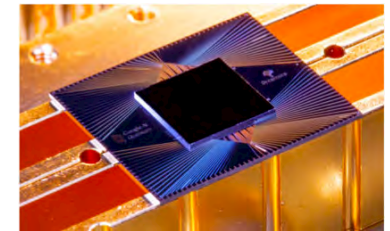
News & views

Quantum information

Quantum computing takes flight

William D. Oliver

A programmable quantum computer has been reported to outperform the most powerful conventional computers in a specific task – a milestone in computing comparable in importance to the Wright brothers’ first flights. See p.505



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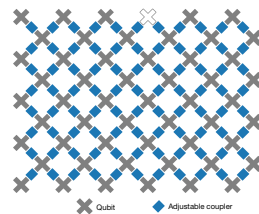
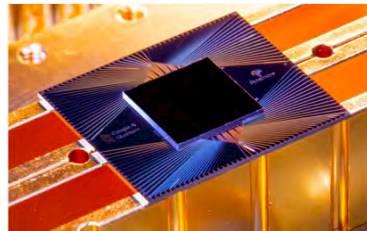
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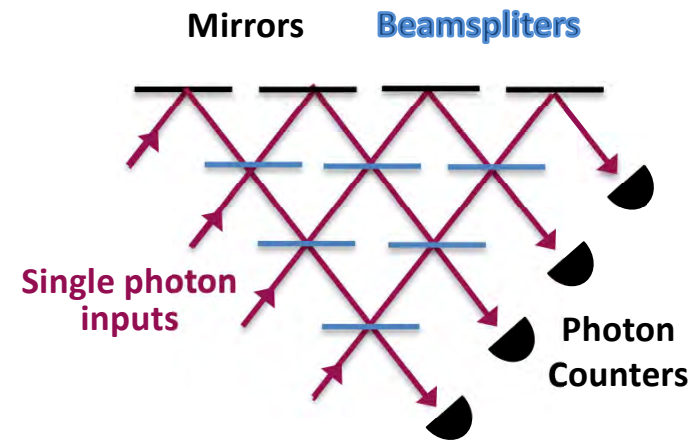
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Boson Sampling

An example from Optics/Photonics Setup



Beware: Boson behavior at Beamsplitters hard to predict photon statistics across outputs.

Exponential in #'s of Beamsplitters

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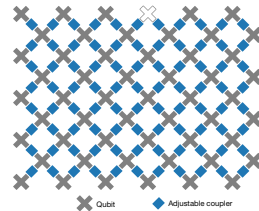
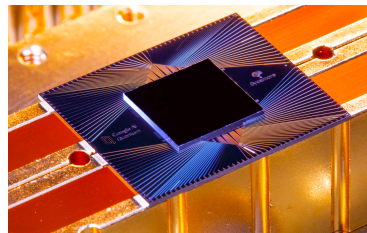
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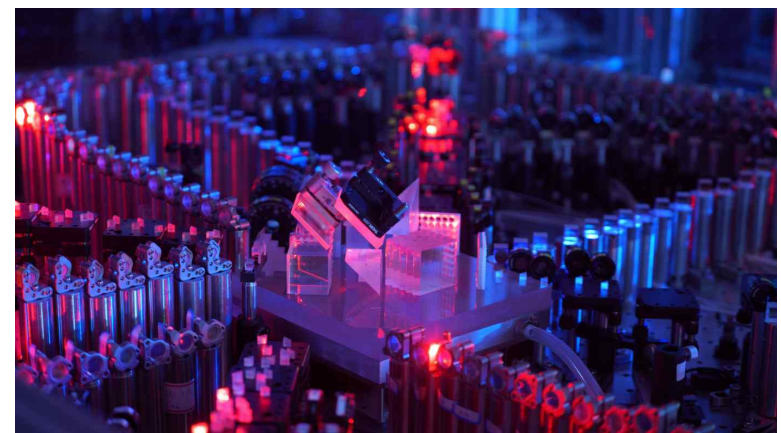
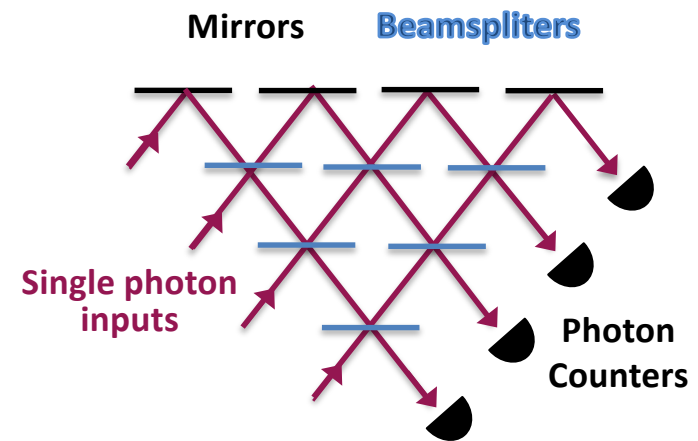
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An example from Optics/Photonics Setup



An optical quantum computer developed by a team of Chinese researchers including those from the University of Science and Technology of China. (courtesy of Han-Sen Zhong of the research group)

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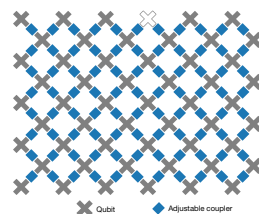
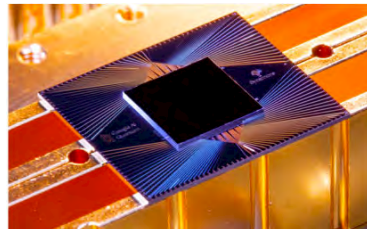
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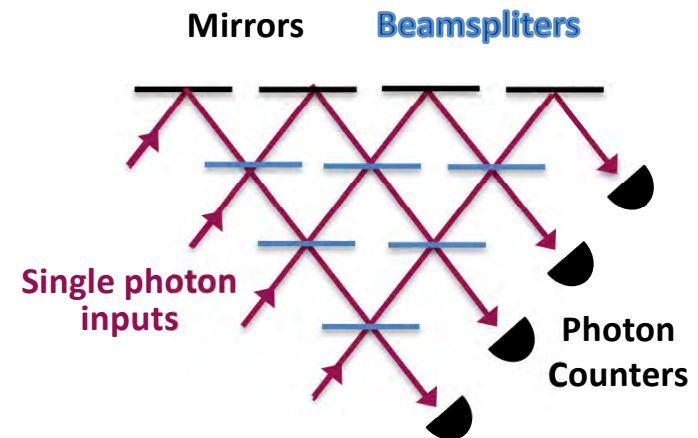
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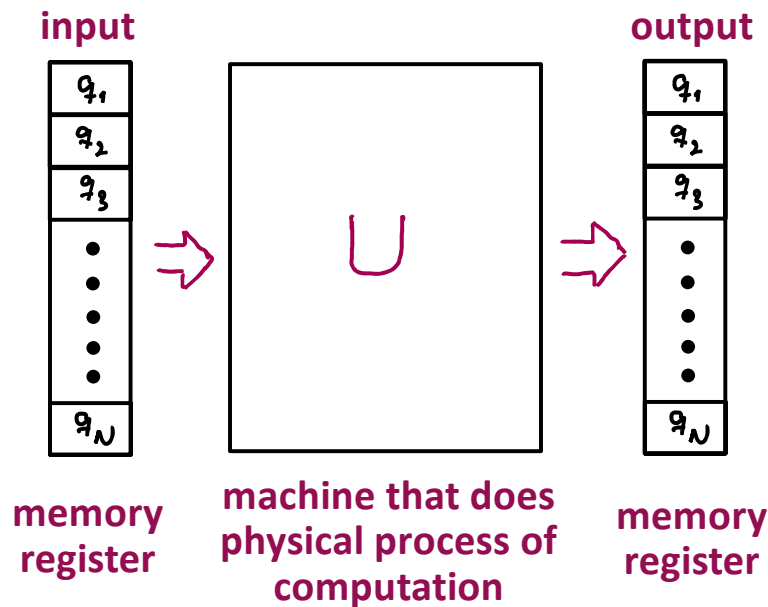
An example from Optics/Photonics Setup



Imagine aligning that thing...!

Back to Universal Computation

Visualization of Computation



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$$x = q_1 q_2 q_3 \dots q_N$$

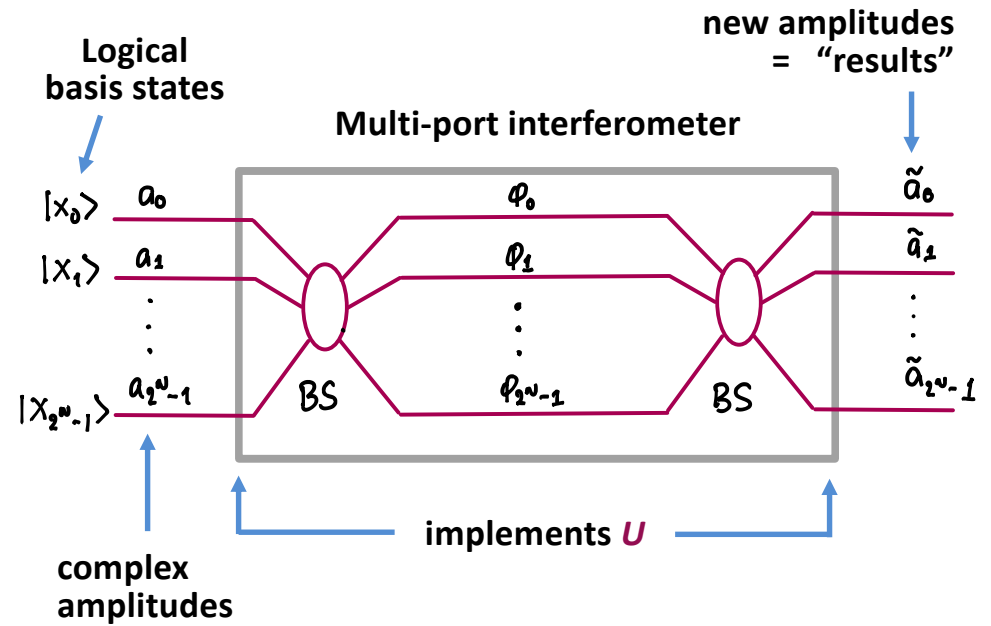
binary #

Reversible transformation

$$U: x \rightarrow y$$

What might be inside the machine ?

Wave interference w/classical fields ?

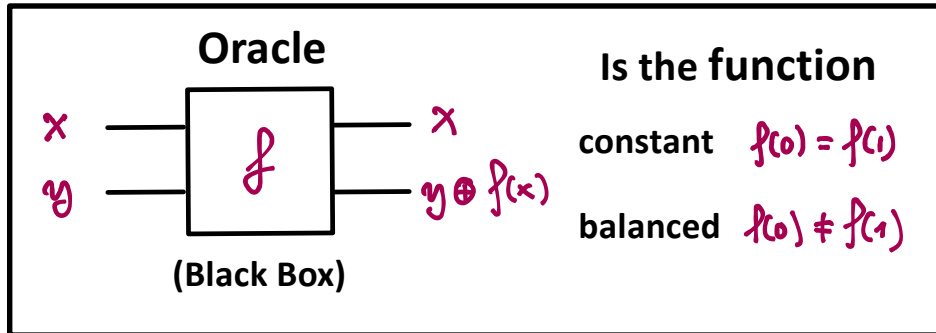


Note: N - qubit register $\rightarrow 2^N$ "paths"

Beware of Resource Scaling !

Quantum Advantage

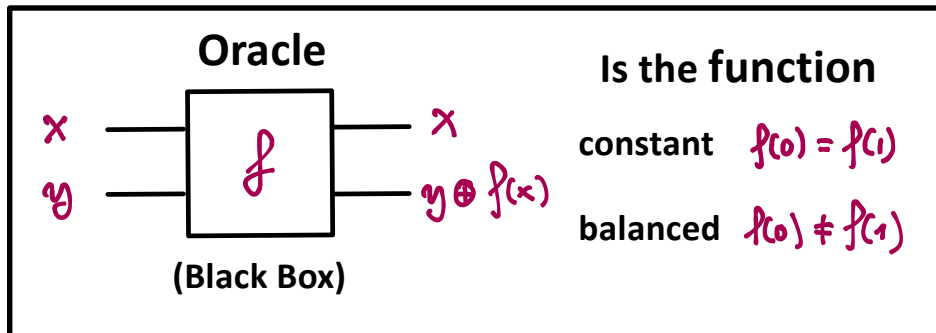
David Deutsch: Toy problem that shows Quantum Advantage



Classical Box: Need 2 queries $f(0)$ & $f(1)$

Quantum Advantage

David Deutsch: Toy problem that shows Quantum Advantage



Quantum Box: In 3 steps can show that

$$(1) U_f : |x\rangle|y\rangle \rightarrow |x\rangle|y \oplus f(x)\rangle$$



$$(2) U_f : |x\rangle \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \rightarrow |x\rangle \frac{1}{\sqrt{2}} (|f(x)\rangle - |1 \oplus f(x)\rangle)$$

$$= |x\rangle \frac{1}{\sqrt{2}} (-1)^{f(x)} (|0\rangle - |1\rangle)$$

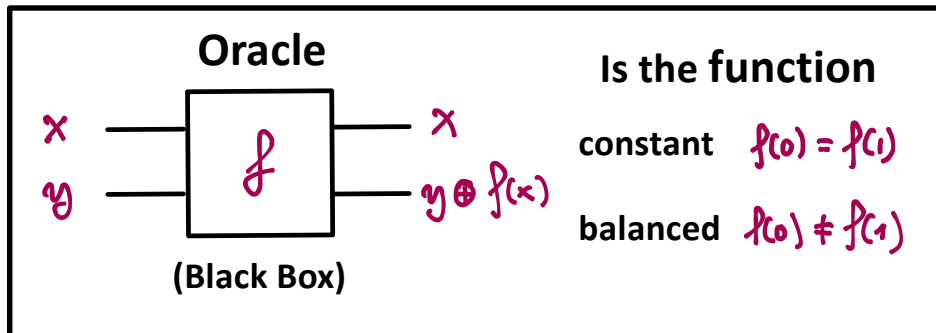


$$(3) U_f : \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

$$\rightarrow \frac{1}{2} \left((-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle \right) (|0\rangle - |1\rangle)$$

Quantum Advantage

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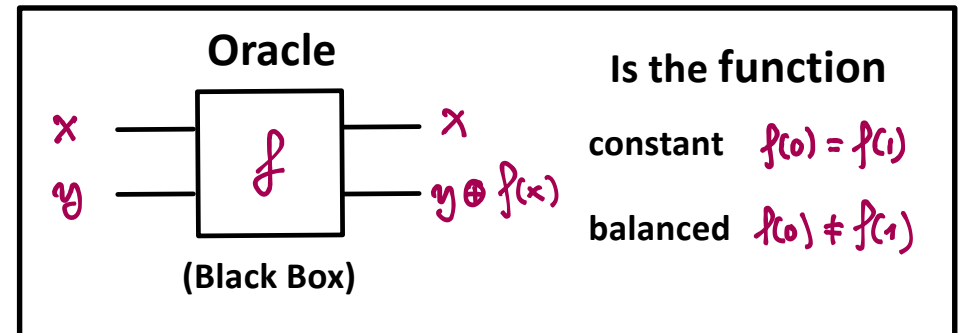
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Quantum Computation:

$$\text{Input } |x\rangle|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$U_f \rightarrow \left[(-1)^{f(1)} |0\rangle + (-1)^{f(0)} |1\rangle \right] \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

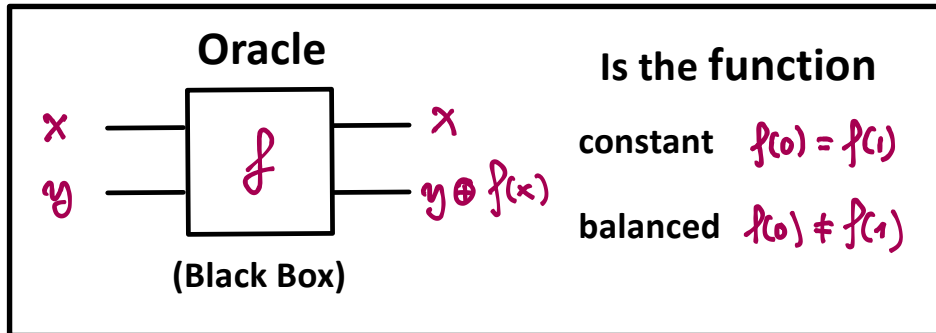
$$\text{Measure 1st qubit in basis } | \pm \rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$\rightarrow |+\rangle$ if constant, $|-\rangle$ if balanced

Quantum Speedup: can solve w/1 query

Quantum Advantage

David Deutsch: Toy problem that shows Quantum Advantage



Quantum Computation:

Input $|x\rangle|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

$$\xrightarrow{U_f} [(-1)^{f(1)}|0\rangle + (-1)^{f(0)}|1\rangle] \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

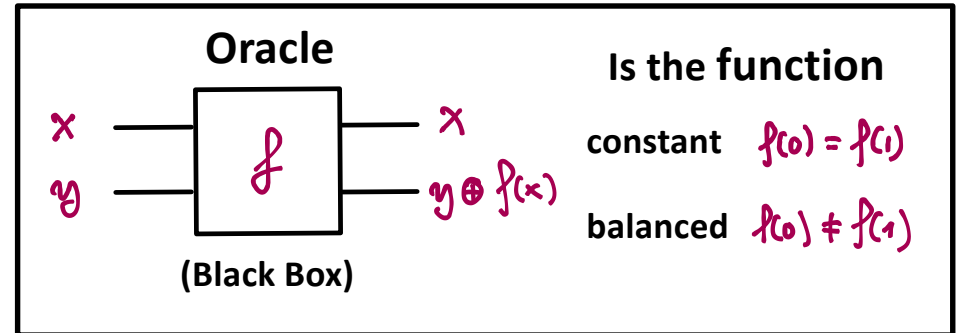
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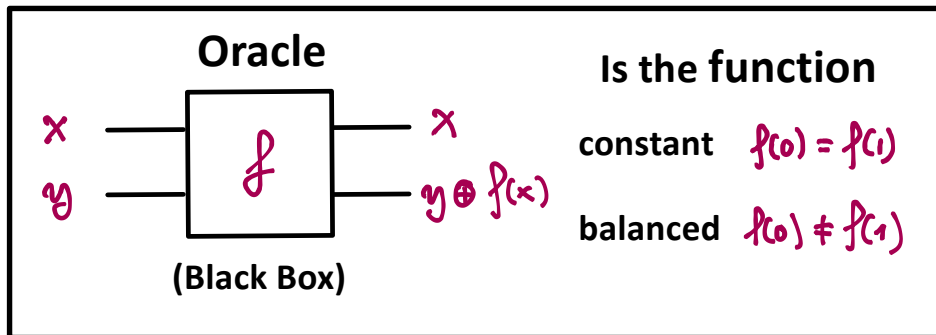
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Quantum Speedup: can solve w/1 query

Key aspect of Deutsch's algorithm:

We are looking for a global property of the function f

Generally: $U_f: |x\rangle|0\rangle \rightarrow |x\rangle|f(x)\rangle$

Input $|q_{in}\rangle = \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \right]^{\otimes N} |0\rangle$

$$= \frac{1}{2^{N/2}} \sum_{x=0}^{2^N-1} |x\rangle|0\rangle$$

compute once

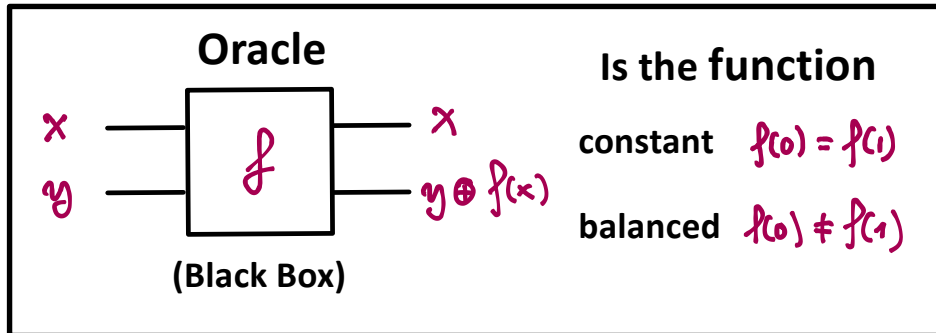
Output $|q_{out}\rangle = \frac{1}{2^{N/2}} \sum_{x=0}^{2^N-1} |x\rangle|f(x)\rangle$

Global properties encoded in state, trick is to extract desired information

Quantum Advantage

David Deutsch:

Toy problem that shows Quantum Advantage



Quantum Computation:

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Peter Shor: Period finding, QFT, Factoring

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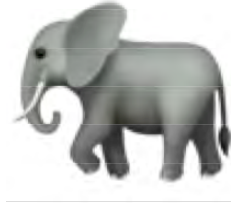
Peter Shor: Period finding,
QFT, Factoring

Next: Will this work with real-world
Quantum Hardware ?

Faulty gates, decoherence !

Quantum Error Correction

Fundamental Problem



Quantum States are fragile, especially when entangled

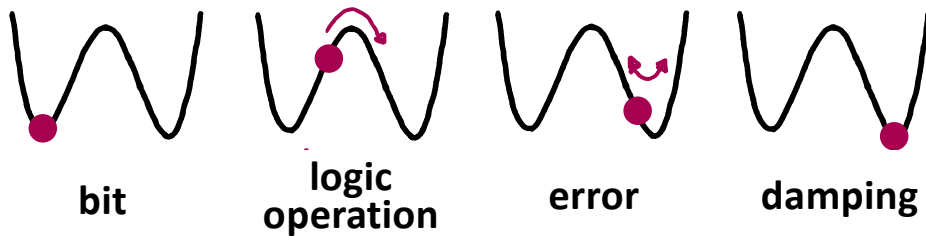
Quantum Computation →

- * Cannot tolerate dissipation
- * Destroys superposition and entanglement

What to do? **Error Correction!**

Classical Computation ?

Dissipation helps



No dissipation →

Errors build up

Classical Error Correction:

Simple example: Redundancy protects against bit flips

Encode:
 $0 \rightarrow (000)$
 $1 \rightarrow (111)$

Errors:
 $(000) \rightarrow (100)$
 $(111) \rightarrow (011)$ correct by majority vote

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Von Neumann:

- * A classical computer w/faulty components can work, given enough redundancy
- * Classical error correction is well developed and highly sophisticated...

* Quantum Errors

1) Bit Flip $|0\rangle \rightarrow |1\rangle$, phase flip $|0\rangle \rightarrow |0\rangle$
 $|1\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow -|1\rangle$

2) Small errors $a|0\rangle + b|1\rangle$ a, b can change by ϵ
errors accumulate

3) Measurement disturbs  collapse of quantum states

4) No cloning  Cannot protect by making copies

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 errors accumulate
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- 4) No cloning \Rightarrow Cannot protect by making copies

Example: Peter Shor's code for bit flip error when $P(\text{error}) \ll 1$

Encode: $|0\rangle \rightarrow |0\rangle \equiv |000\rangle$ (3 bit code)
 $|1\rangle \rightarrow |1\rangle \equiv |111\rangle$

$$a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

Single-qubit measurement \Rightarrow

collapse of state, destroys info, no majority voting!

Collective 2-qubit measurement:

- for $|x, y, z\rangle$ measure $y \oplus z$ (never measure individual bits)
 $x \oplus z$
- if $|000\rangle, |111\rangle$ these observables = 0
- if one bit-flip, at least one observable = 1
- easy to check that $(y \oplus z, x \oplus z) =$ binary address of qubit flip

$$|000\rangle \rightarrow |010\rangle \quad (1, 0) = \text{2nd bit}$$

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Example: Peter Shor's code for bit flip error when $P(\text{error}) \ll 1$

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Quantum mechanics to the rescue !

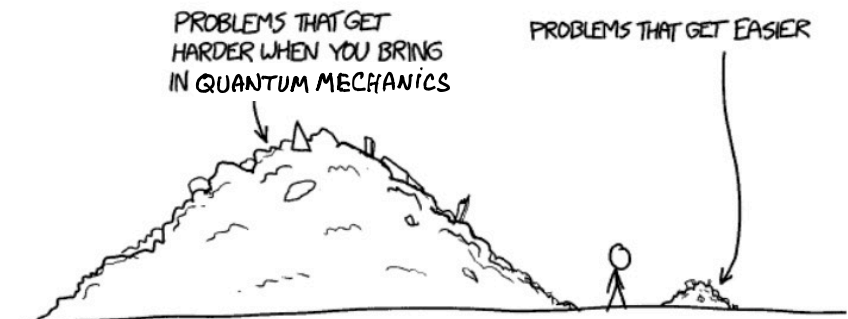
- mostly no error detected

\rightarrow collapse into $|000\rangle$ resp. $|111\rangle$

- sometime error detected

\rightarrow collapse into $|001\rangle$ resp. $|110\rangle$

\rightarrow full bit flip, correct as such



Source: xkcd.com

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How to implement ?

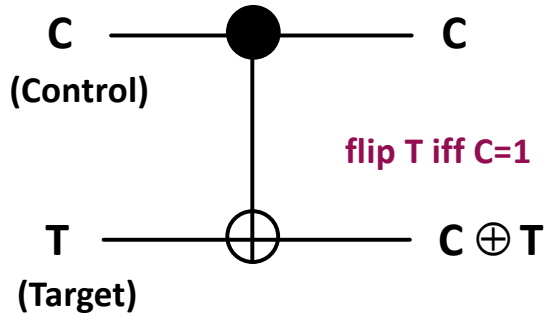
Quantum circuit + single qubit measurement

Quantum Gates – work on superpositions, and entangled states

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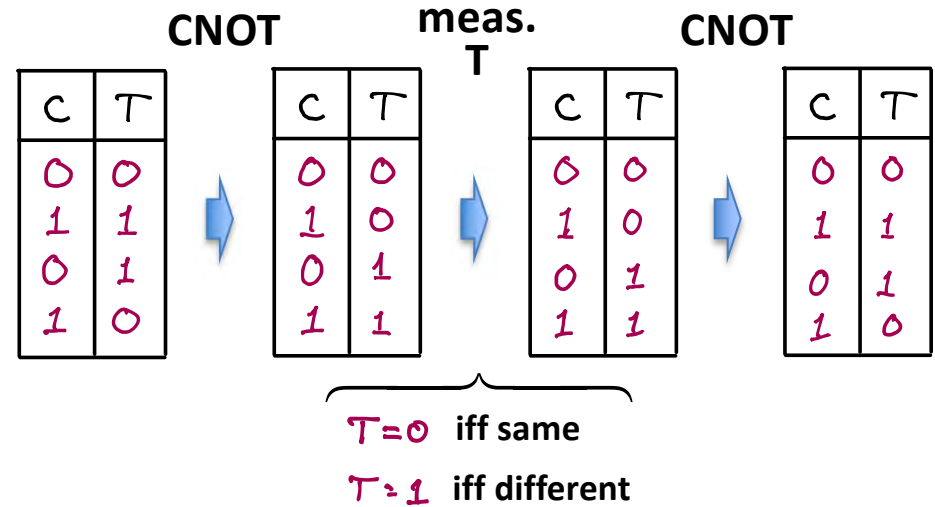
Controlled-NOT (CNOT)

Truth Table

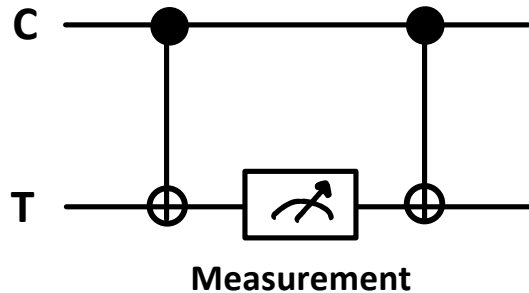


C	T	$C \oplus T$
0	0	0
0	1	1
1	0	1
1	1	0

Circuit maps logical basis states as

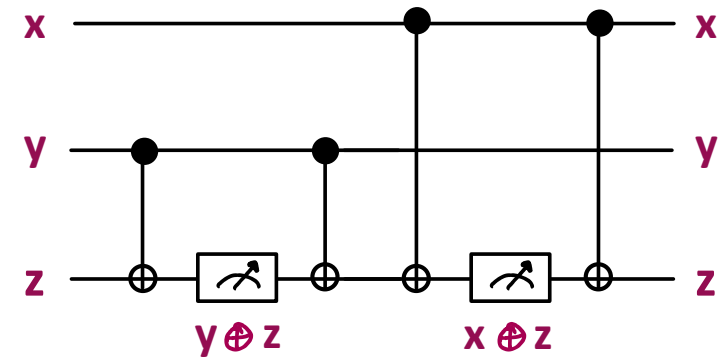


Quantum Circuit for joint measurement



Measurement in $\{|0\rangle, |1\rangle\}$ basis yields $C \oplus T$

Full circuit to obtain Error Syndrome

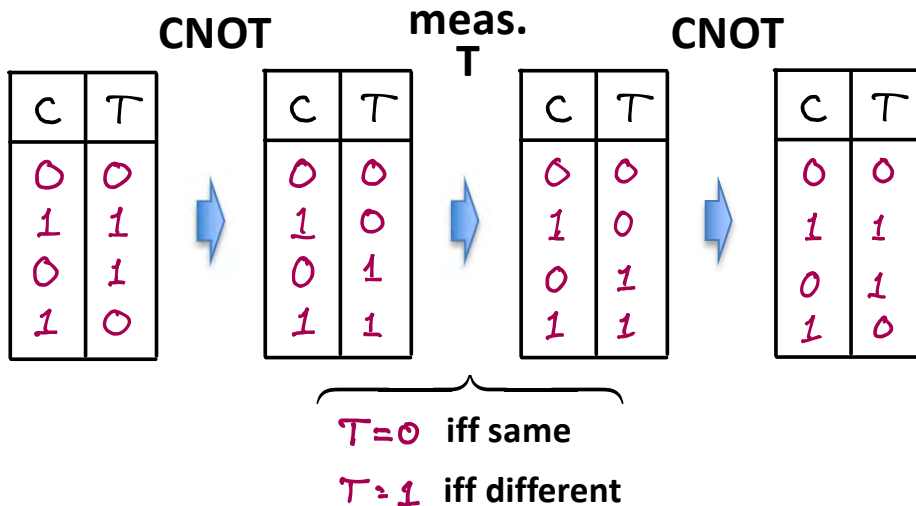


* iff qubit flip, binary address = $(y \oplus z, x \oplus z)$

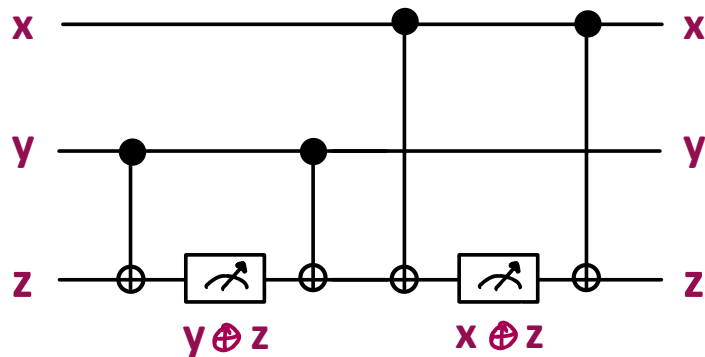
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Full circuit to obtain Error Syndrome



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Quantum Phase Error

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow -|1\rangle$$

Encoding

$$|0\rangle \rightarrow |\bar{0}\rangle = \frac{1}{2^{3/2}} (|0\rangle + |1\rangle)^{x'} (|0\rangle + |1\rangle)^{y'} (|0\rangle + |1\rangle)^{z'}$$

$$|1\rangle \rightarrow |\bar{1}\rangle = \frac{1}{2^{3/2}} (|0\rangle - |1\rangle)^{x'} (|0\rangle - |1\rangle)^{y'} (|0\rangle - |1\rangle)^{z'}$$

Relabel

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |0'\rangle$$

$$\frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |1'\rangle$$

Measure in basis

$$\{|0'\rangle, |1'\rangle\} \rightarrow y' \oplus z', x' \oplus z'$$

Error Syndrome

* Iff phase error, binary address = $(y' \oplus z', x' \oplus z')$

* Analogous to bit-flip code, just in different basis

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Shor's 9-bit code

- * Combines flip/phase error correction
- * Corrects one flip or phase error

General principle of error correction

- * Encode p logical qubits in n physical qubits.
- * Valid Logical States form 2^p -dimensional subspace \mathcal{E}_p (code space) in n -qubit (2^n -dimensional) Hilbert space \mathcal{E}_n
- * Errors displace system into orthogonal (distinguishable) subspaces.

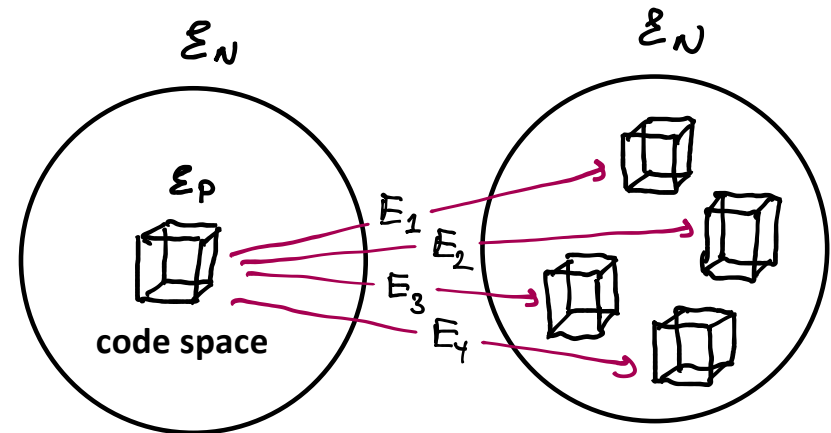
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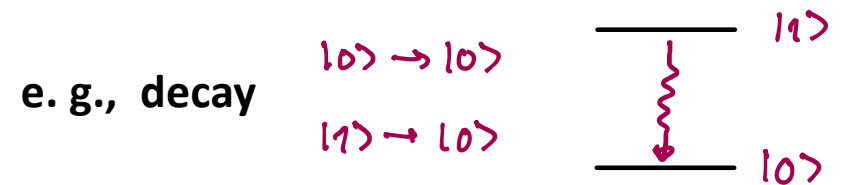
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Geometric illustration



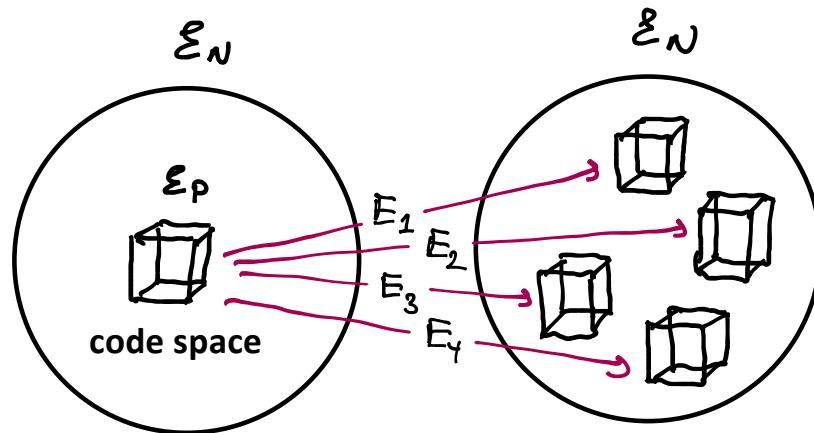
What about non-Unitary errors?



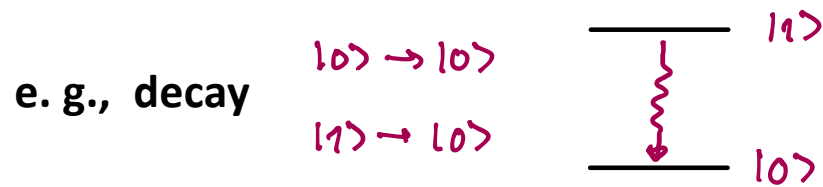
- Problem: Errors not displaced into orthogonal subspaces
- Solution: "Quantum jump codes", monitors the environment

Other kinds of errors?

Geometric illustration



What about non-Unitary errors?



Problem: Errors not displaced into orthogonal subspaces

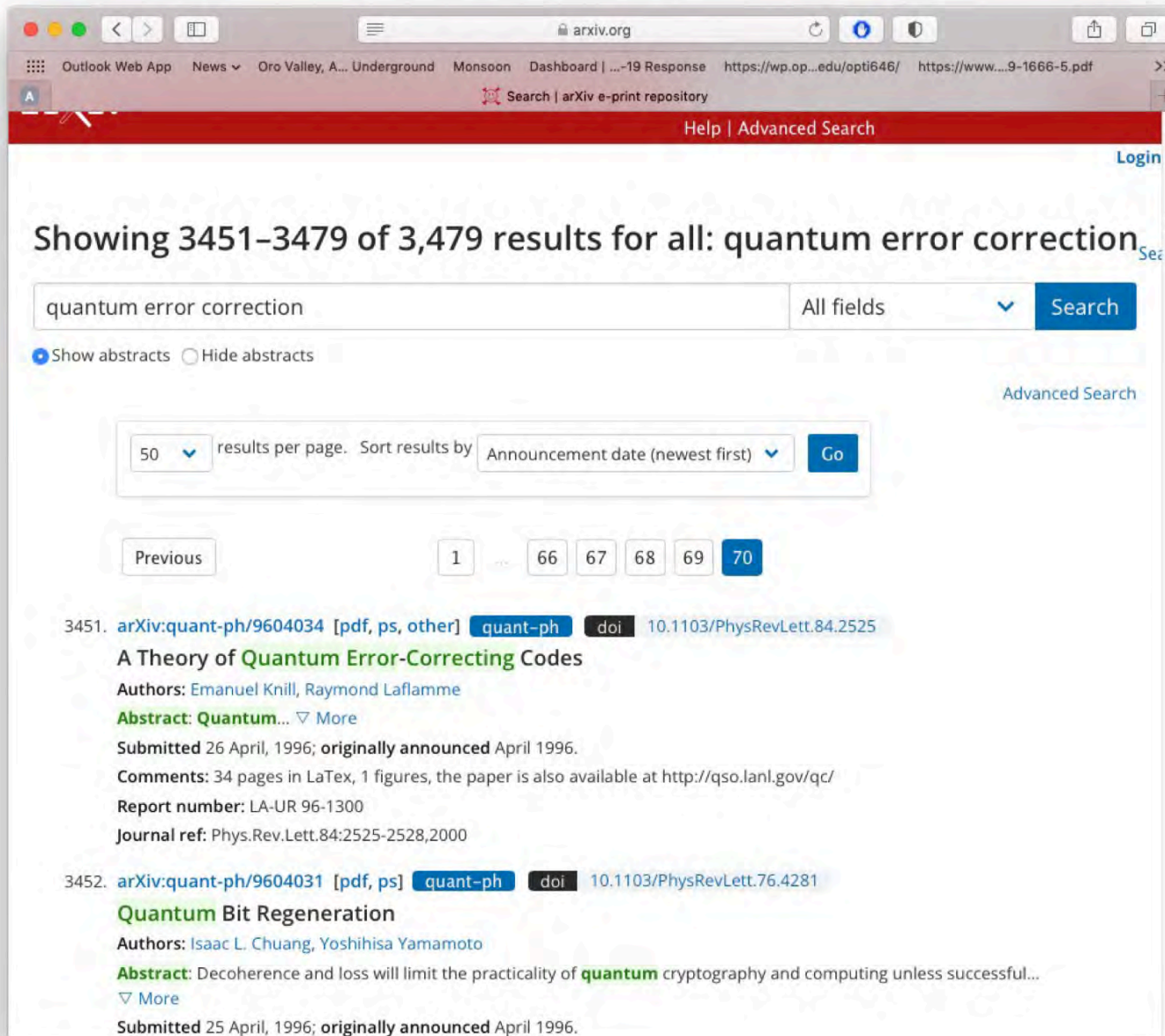
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Other kinds of errors?

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9-05-2024

Catnip for Theoretical Physicists & Computer Scientists



The screenshot shows the arXiv search results page for the query "quantum error correction". The page displays a list of search results, with the first two entries visible. The search interface includes a search bar, a "Search" button, and options to show or hide abstracts. The results are sorted by "Announcement date (newest first)" and are displayed in a list format with pagination controls.

Showing 3451–3479 of 3,479 results for all: quantum error correction

quantum error correction All fields Search

Show abstracts Hide abstracts

50 results per page. Sort results by Announcement date (newest first) Go

Previous 1 66 67 68 69 70

3451. arXiv:quant-ph/9604034 [pdf, ps, other] quant-ph doi 10.1103/PhysRevLett.84.2525
A Theory of Quantum Error-Correcting Codes
Authors: Emanuel Knill, Raymond Laflamme
Abstract: Quantum... More
Submitted 26 April, 1996; originally announced April 1996.
Comments: 34 pages in LaTeX, 1 figures, the paper is also available at <http://qso.lanl.gov/qc/>
Report number: LA-UR 96-1300
Journal ref: Phys.Rev.Lett.84:2525-2528,2000

3452. arXiv:quant-ph/9604031 [pdf, ps] quant-ph doi 10.1103/PhysRevLett.76.4281
Quantum Bit Regeneration
Authors: Isaac L. Chuang, Yoshihisa Yamamoto
Abstract: Decoherence and loss will limit the practicality of quantum cryptography and computing unless successful... More
Submitted 25 April, 1996; originally announced April 1996.

Introduction and Overview (Preskills Notes)

Catnip for Theoretical Physicists & Computer Scientists

The screenshot shows the arXiv search results page for the query "quantum error correction". The page displays 4,633 results, with the first 50 shown. The search results are sorted by "Announcement date (newest first)". The first two results are highlighted.

Showing 1–50 of 4,633 results for all: Quantum Error Correction

quantum error correction All fields Search

Show abstracts Hide abstracts

50 results per page. Sort results by Announcement date (newest first) Go

Previous 1 66 67 68 69 70

3451. [arXiv:quant-ph/9604034](#) [pdf, ps, other] [quant-ph](#) [doi](#) 10.1103/PhysRevLett.84.2525
A Theory of Quantum Error-Correcting Codes
 Authors: Emanuel Knill, Raymond Laflamme
 Abstract: Quantum... More
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 Comments: 34 pages in LaTeX, 1 figures, the paper is also available at <http://qso.lanl.gov/qc/>
 Report number: LA-UR 96-1300
 Journal ref: Phys.Rev.Lett.84:2525-2528,2000

3452. [arXiv:quant-ph/9604031](#) [pdf, ps] [quant-ph](#) [doi](#) 10.1103/PhysRevLett.76.4281
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Quantum Hardware

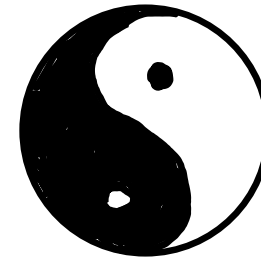
Physical Implementation is
Extremely demanding !

Requirements

1. Storage: Quantum memory.
2. Gates: We put computation U_f together from 1 and 2-qubit operations.
3. Readout: Method to measure qubits.
4. Isolation: No coupling to environment to avoid decoherence & errors
5. Precision: Gates, readouts must be highly accurate

Inherent Contradictions

2. Gates **vs** 4. Isolation
- ↑ ↑
- coupling between qubits no coupling to environment



To build a Quantum Computer:

Choose, find or invent a system with acceptable tradeoffs.

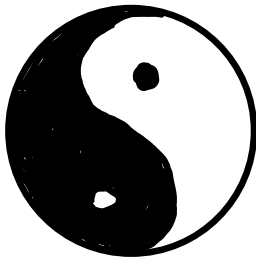
6. Error Correction must not create more errors than it corrects.



7. Thresholds for Error Correction and Fault Tolerance

Inherent Contradictions

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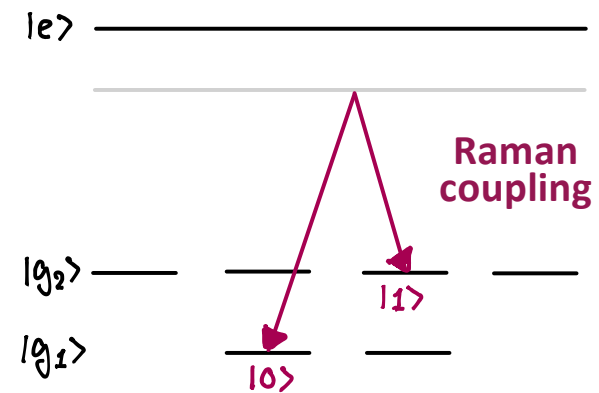


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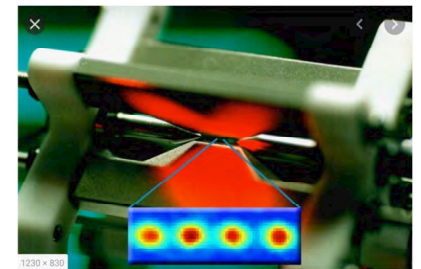
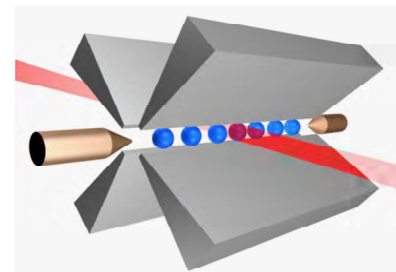
Ion Trap Quantum Computing

First to demonstrate a Quantum Gate

* Qubit is encoded in the electronic ground state of an atomic ion



* Early design with a few ions in large trap

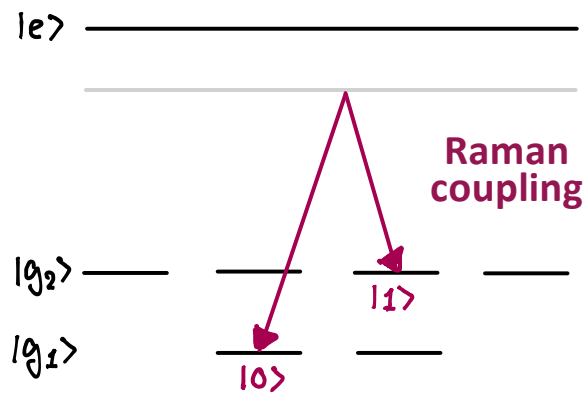


Introduction and Overview (Preskills Notes)

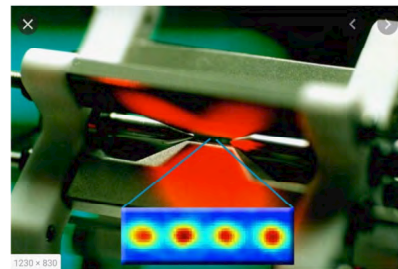
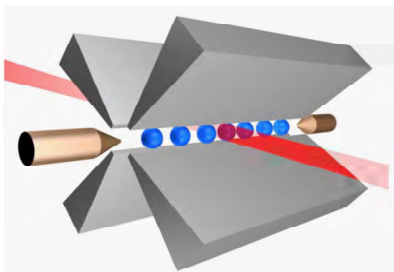
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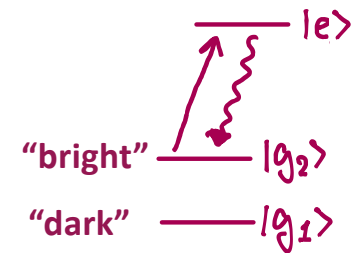
- * Early design with a few ions in large trap



Requirements

1. Storage: 10s-100s coherence time
2. Gates: Use collective vibrations as "quantum bus"

3. Readout: Fluorescence



Cirac & Zoller: 5 laser pulses

CNOT gate between any 2 ions in linear array

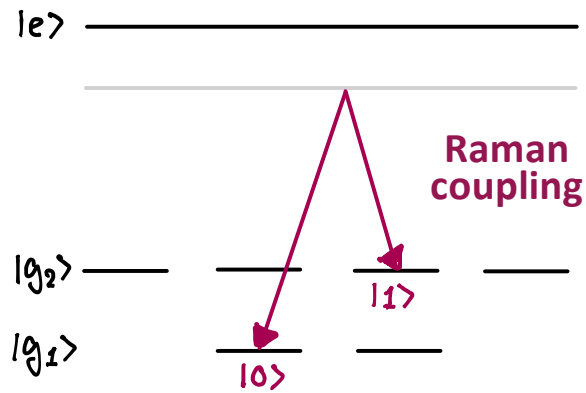
Wineland: 3 laser pulses enough for CNOT

Use this example serves as conceptual template

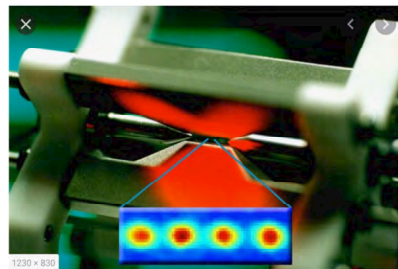
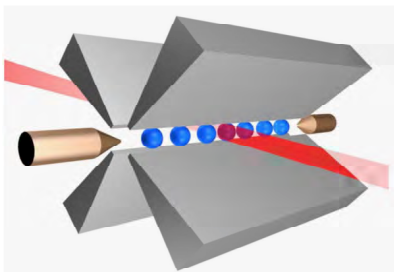
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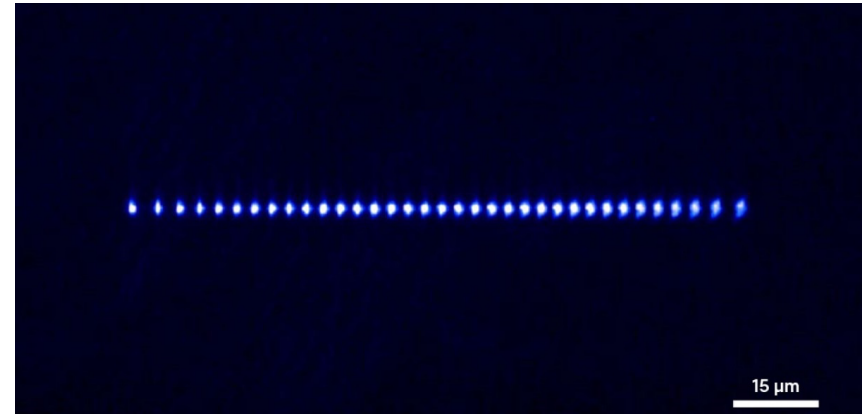
- * Qubit is encoded in the electronic ground state of an atomic ion



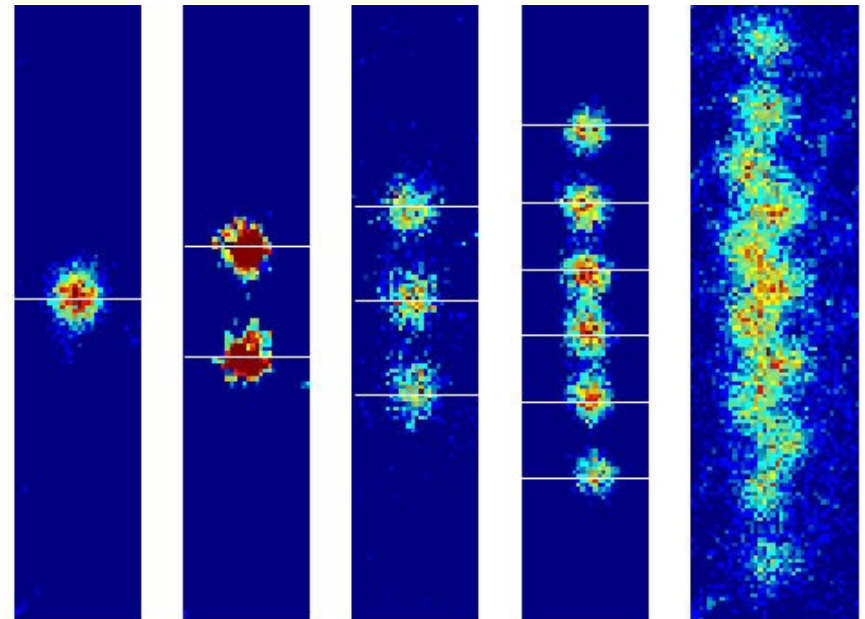
- * Early design with a few ions in large trap



- * Scaling up in Linear Ion Traps



- * Limitations



Status: Many important milestones achieved

- * Entanglement of ≥ 20 ions (2018)
- * Highest gate & readout fidelities, longest coherence times
- * Error Correction, Fault Tolerance proof of principle demonstrations
- * Complex algorithms on few ions, quantum simulations with ≥ 50
- * Research groups in academia, National Labs, Industry

Some early leaders

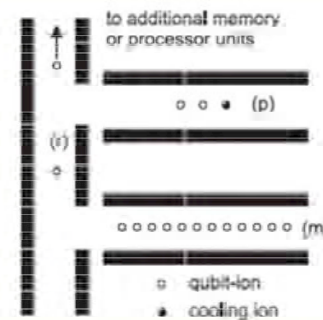
NIST Innsbruck Quantinuum

Sandia NL Duke U IonQ

Many others these days

- * Major challenges – same as late 2000’s!
- * “Clock speed” set by vibrational freqs microfabricated traps do better
- * More ions -> harder to cool motion, harder to individually address ions in linear trap.
- * Scaling up to 1000’s of ions is an enormous challenge

Scalable Ion Trap Quantum Processor – one vision



D. Klepinski, C. Monroe, and D. J. Wineland, Nature 417, 709 (2002).

To appear in the 2007 International Symposium on Microstructure (MICRO-35)
A Quantum Logic Array Microarchitecture: Scalable Quantum Data Movement and Computation

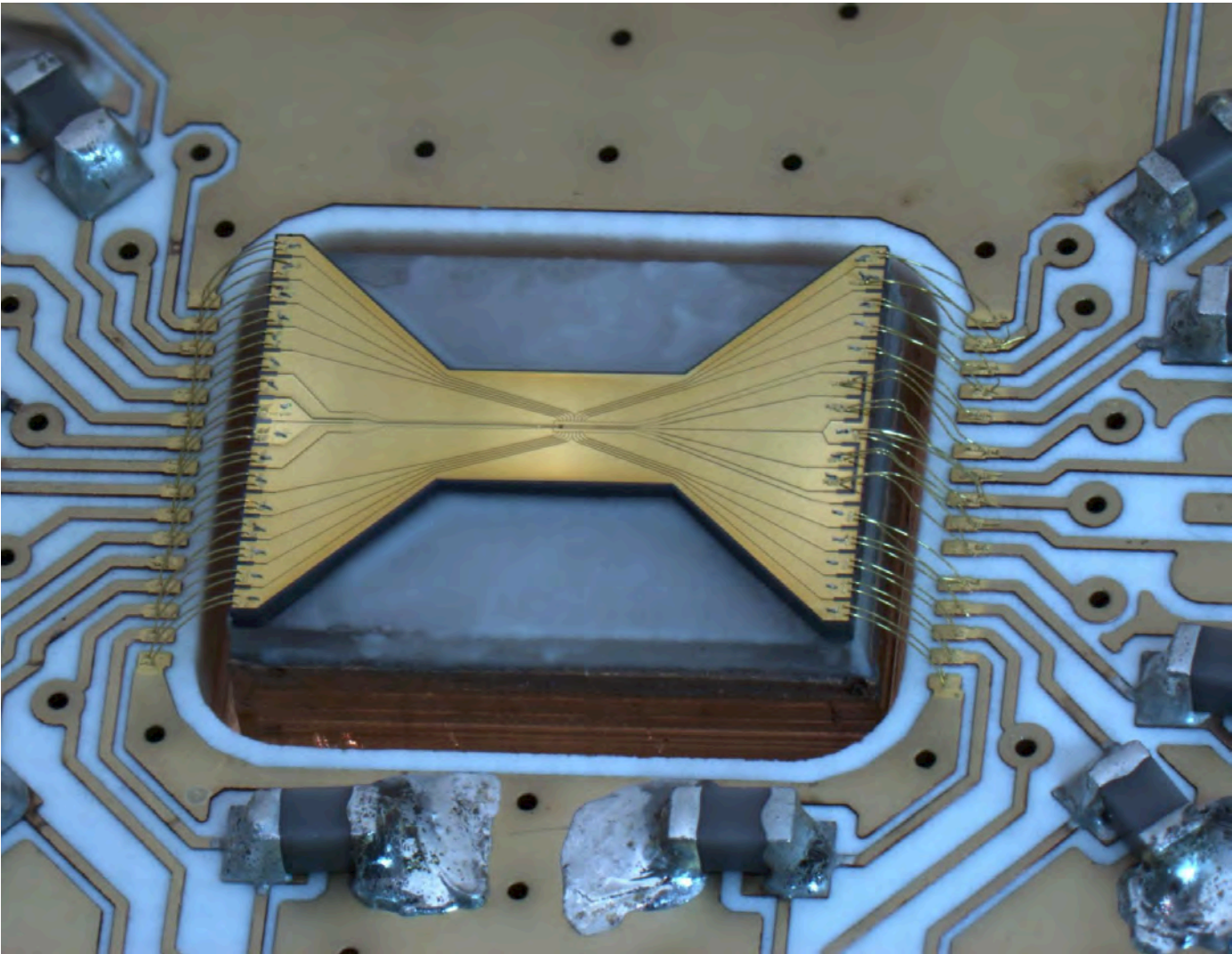
Tzvetan S. Metodi¹, Darshan D. Thaker¹, Andrew W. Cross²
Frederic T. Cheng¹ and Isaac L. Chuang²

Operation	Time	$P_{current}$	$P_{expected}$
Single Gate	1 μ s	0.0001	10^{-5}
Double Gate	10 μ s	0.03	10^{-7}
Measure	100 μ s	0.01	10^{-8}
Movement	10ns/ μ m	0.005/ μ m	10^{-6} /cell
Split	10 μ s		
Cooling	1 μ s		
Memory time	10 – 100 sec		

Introduction and Overview (Preskills Notes)

9-10-2024

NIST Group, Current as of 2023



Demonstration of a Fundamental Quantum Logic Gate

C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland

National Institute of Standards and Technology, Boulder, Colorado 80303

(Received 14 July 1995)

We demonstrate the operation of a two-bit “controlled-NOT” quantum logic gate, which, in conjunction with simple single-bit operations, forms a universal quantum logic gate for quantum computation. The two quantum bits are stored in the internal and external degrees of freedom of a single trapped atom, which is first laser cooled to the zero-point energy. Decoherence effects are identified for the operation, and the possibility of extending the system to more qubits appears promising.

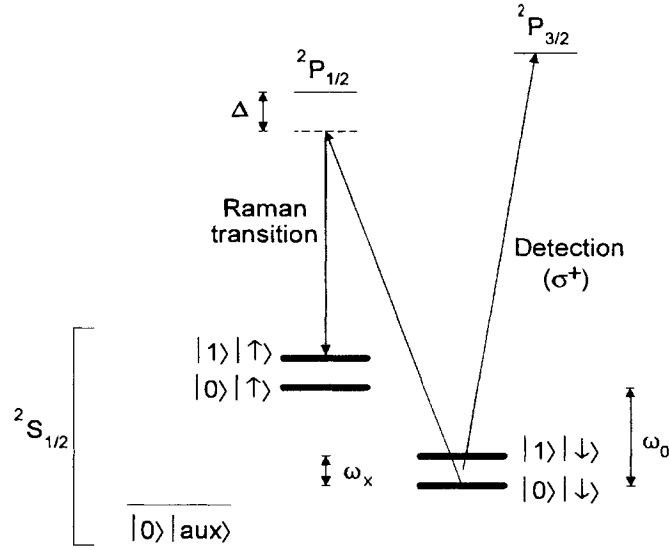


FIG. 1. ${}^9\text{Be}^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ (${}^2S_{1/2}|F=2, m_F=2\rangle$ and ${}^2S_{1/2}|F=1, m_F=1\rangle$ levels, respectively, separated by $\omega_0/2\pi \approx 1.250$ GHz), and $|\text{aux}\rangle = {}^2S_{1/2}|F=2, m_F=0\rangle$ (separated from $|\downarrow\rangle$ by ≈ 2.5 MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \approx 11.2$ MHz). Stimulated Raman transitions between ${}^2S_{1/2}$ hyperfine states are driven through the virtual ${}^2P_{1/2}$ level ($\Delta \approx 50$ GHz) with a pair of ≈ 313 nm laser beams. Measurement of S is accomplished by driving the cycling $|\downarrow\rangle \rightarrow {}^2P_{3/2}|F=3, m_F=3\rangle$ transition with σ^+ -polarized light and detecting the resulting ion fluorescence.

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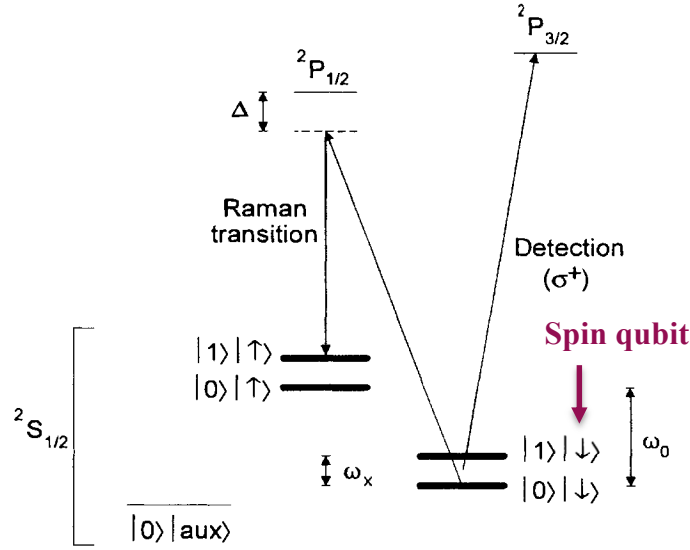


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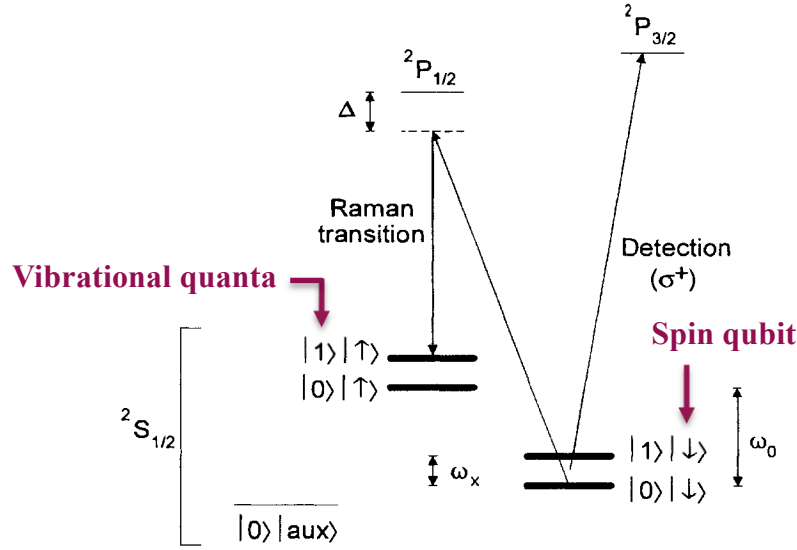


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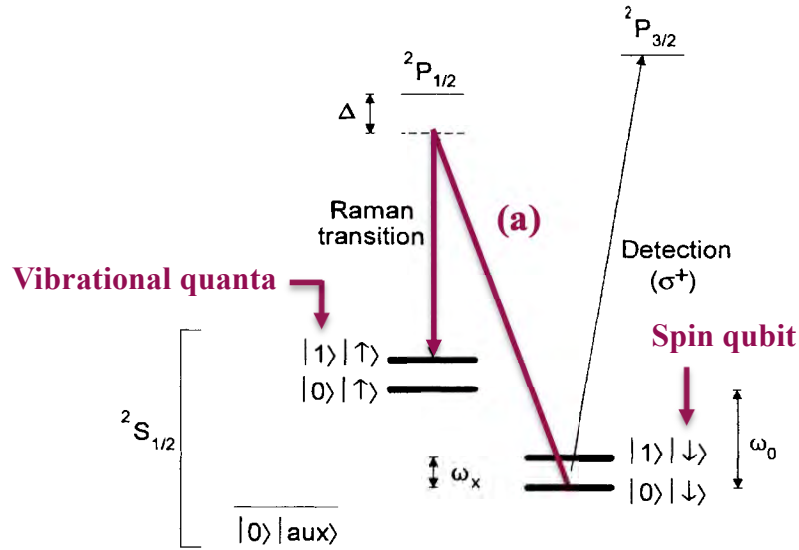


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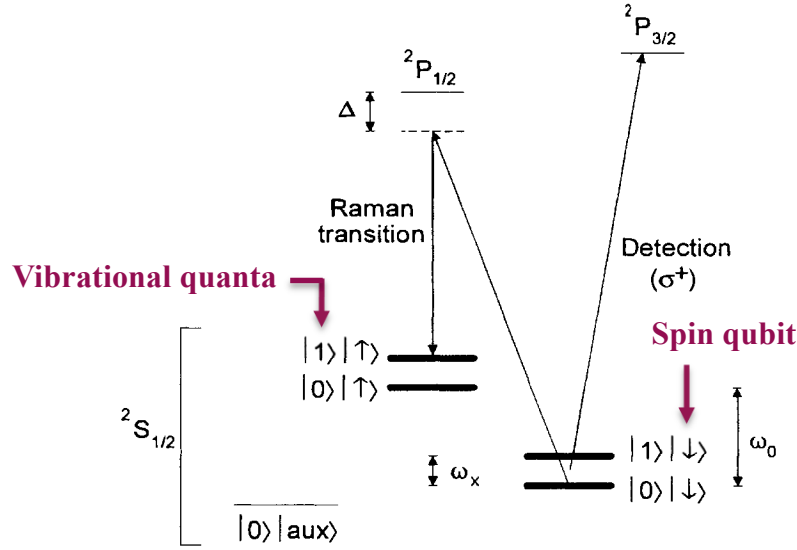


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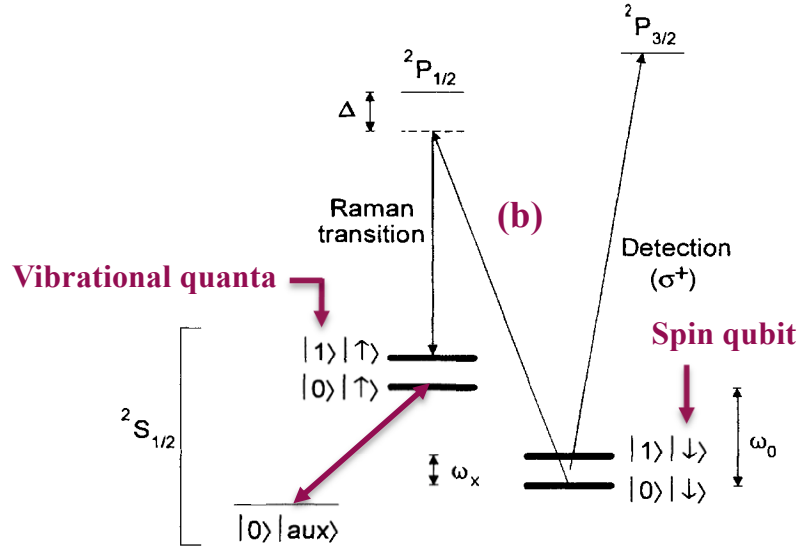


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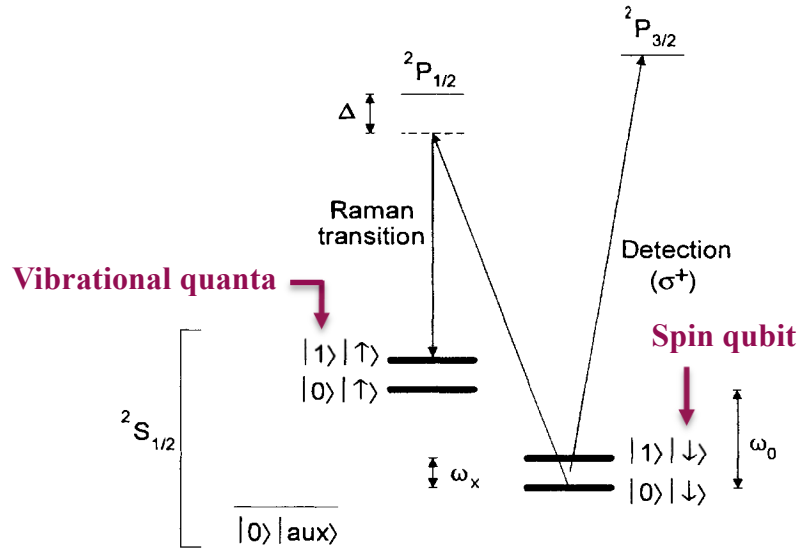


FIG. 1. ${}^9\text{Be}^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ (${}^2S_{1/2}|F=2, m_F=2\rangle$ and ${}^2S_{1/2}|F=1, m_F=1\rangle$ levels, respectively, separated by $\omega_0/2\pi \approx 1.250$ GHz), and $|\text{aux}\rangle = {}^2S_{1/2}|F=2, m_F=0\rangle$ (separated from $|\downarrow\rangle$ by ≈ 2.5 MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \approx 11.2$ MHz). Stimulated Raman transitions between ${}^2S_{1/2}$ hyperfine states are driven through the virtual ${}^2P_{1/2}$ level ($\Delta \approx 50$ GHz) with a pair of ≈ 313 nm laser beams. Measurement of S is accomplished by driving the cycling $|\downarrow\rangle \rightarrow {}^2P_{3/2}|F=3, m_F=3\rangle$ transition with σ^+ -polarized light and detecting the resulting ion fluorescence.

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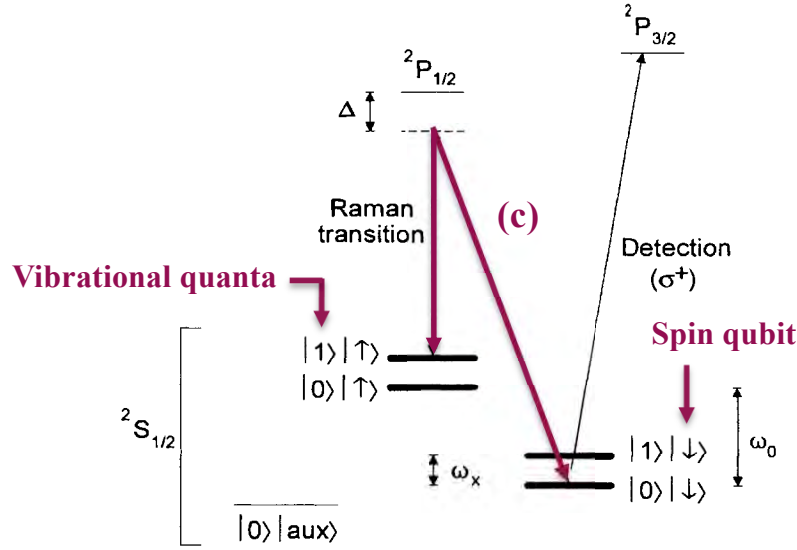


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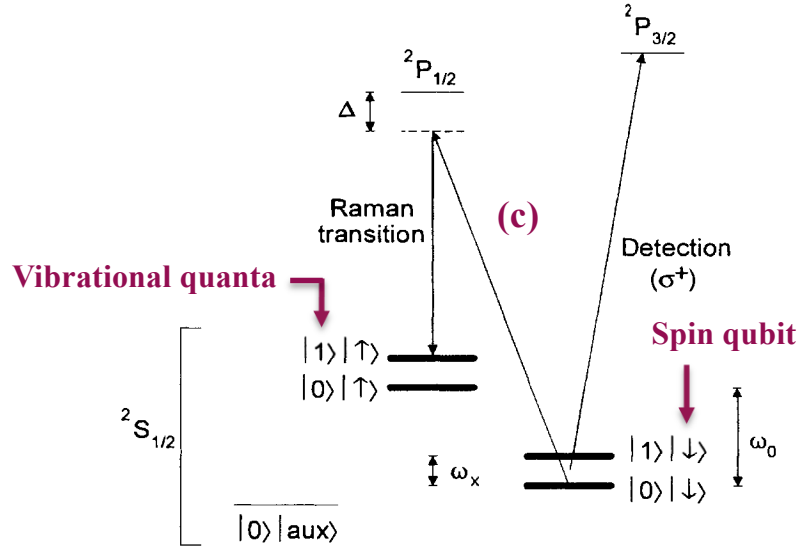


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Introduction and Overview (Preskills Notes)

Some Benchmarks for Ion Traps

TABLE I. Selected state-of-the-art gate demonstrations.

Gate Type	Gate Method	Fidelity	Gate Time (μ s)	Ion Species	Ref.
Single-Qubit	Optical	0.99995	5	$^{40}\text{Ca}^+$	[28]
	Raman	0.99993	7.5	$^{43}\text{Ca}^+$	[27]
	Raman	0.99996	2	$^9\text{Be}^+$	[37]
	Raman	0.99	0.00005	$^{171}\text{Yb}^+$	[163]
	Raman	0.999	8	$^{88}\text{Sr}^+$	[113]
	Microwave	0.999999	12	$^{43}\text{Ca}^+$	[22]
Two-Qubit (1 species)	Microwave		0.0186	$^{25}\text{Mg}^+$	[164]
	Optical	0.996	-	$^{40}\text{Ca}^+$	[38]
	Optical	0.993	50	$^{40}\text{Ca}^+$	[8]
	Raman	0.9991(6)	30	$^9\text{Be}^+$	[37]
	Raman	0.999	100	$^{43}\text{Ca}^+$	[27]
	Raman	0.998	1.6	$^{43}\text{Ca}^+$	[40]
	Raman	0.60	0.5	$^{43}\text{Ca}^+$	[40]
	Microwave	0.997	3250	$^{43}\text{Ca}^+$	[165]
	(AC B-field gradient) Microwave	0.985	2700	$^{171}\text{Yb}^+$	[166]
	(DC B-field gradient)				
Two-Qubit (2 species)	Raman/Raman	0.998(6)	27.4	$^{40}\text{Ca}^+ / ^{43}\text{Ca}^+$	[167]
	Raman/Raman	0.979(1)	35	$^9\text{Be}^+ / ^{25}\text{Mg}^+$	[168]

larization (or frequency) does not allow it to couple to $n \geq 1$ ground state, and it cannot be rotated in $|0\rangle_c |n=0\rangle_t$ because the drive is red-detuned. A final π -pulse on the control ion will return the control ion to its initial state. The resulting state transformation looks like:

$$\begin{aligned}
 |0\rangle_c |0\rangle_t &\rightarrow |0\rangle_c |0\rangle_t & (1) \\
 |0\rangle_c |1\rangle_t &\rightarrow |0\rangle_c |1\rangle_t & (2) \\
 |1\rangle_c |0\rangle_t &\rightarrow |1\rangle_c |0\rangle_t & (3) \\
 |1\rangle_c |1\rangle_t &\rightarrow -|1\rangle_c |1\rangle_t & (4)
 \end{aligned}$$

The gate thus inverts the phase of only the $|1\rangle_c |1\rangle_t$ state, realizing an entangling controlled-phase interaction. Besides cooling to the motional ground state, the CZ gate requires individual addressing of each ion and multiple laserizations for the drive laser. Despite these limitations, a modified CZ interaction was demonstrated the same year it was proposed [4], entangling the internal state and motional state of a single $^9\text{Be}^+$ ion. In 1998, a two-ion entangling gate with fidelity of 0.7 was demonstrated between two Be^+ ions with gate time of $\sim 10 \mu\text{s}$, while a Cirac-Zoller gate and single-qubit rotations were used to implement the CNOT operations on two

The requirement that the ions remain in the motional ground state is a significant limitation on the original Cirac-Zoller proposal. As discussed in Sec. II C 2, even when the ions have been cooled to the motional ground state, they can be subsequently heated by electric-field noise. In 1999, Mølmer and Sørensen introduced a controlled-phase gate which could be implemented without the need to be in the motional ground state [25]. The Mølmer-Sørensen (MS) gate generates a state-dependent force with bichromatic laser fields tuned near first-order sideband transitions. The motional-state wavepacket executes a closed trajectory in phase space, giving rise to a state-dependent geometric phase. At the conclusion of the gate, internal and motional states are disentangled for all values of n . Hence, the MS gate can be used for ions that are not cooled to the motional ground state. An additional feature of the MS interaction is that entanglement among multiple ions can be generated using only global control lasers (that is, it does not require lasers independently focused on each ion). The MS entangling gate was first demonstrated for chains of 2 and 4 Be^+ ions in 2000 [6]. To date, the highest-achieved fidelities in both optical and hyperfine two-qubit gates

Some links to get started

Amazon Braket (IonQ, other Technologies)
<https://aws.amazon.com/braket/>

Quantinuum (Ion Trap Quantum Computing)
<https://www.quantinuum.com>

IonQ <https://ionq.com>

NIST <https://www.nist.gov/pml/time-and-frequency-division/ion-storage>

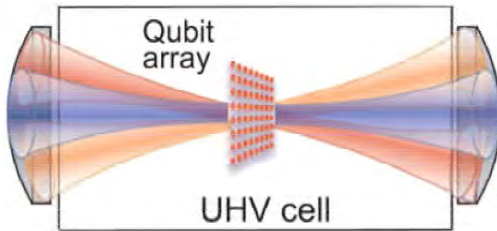
Challenge: Do a web search and look for the largest GHZ state made in the lab

$$|GHZ\rangle = \frac{1}{\sqrt{2}} (|00\dots 00\rangle + |11\dots 11\rangle)$$

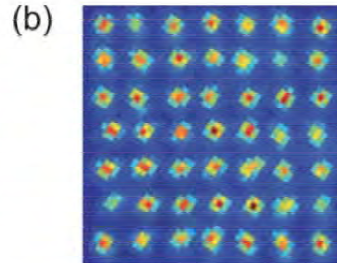
Note: What is the fidelity of the state ?

Neutral Atom based Quantum Processors

Optical tweezer array



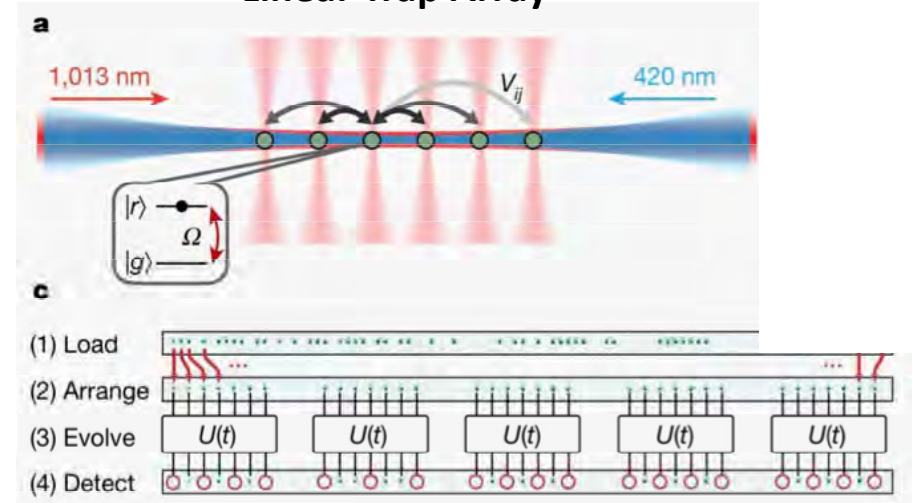
Fluorescing atoms



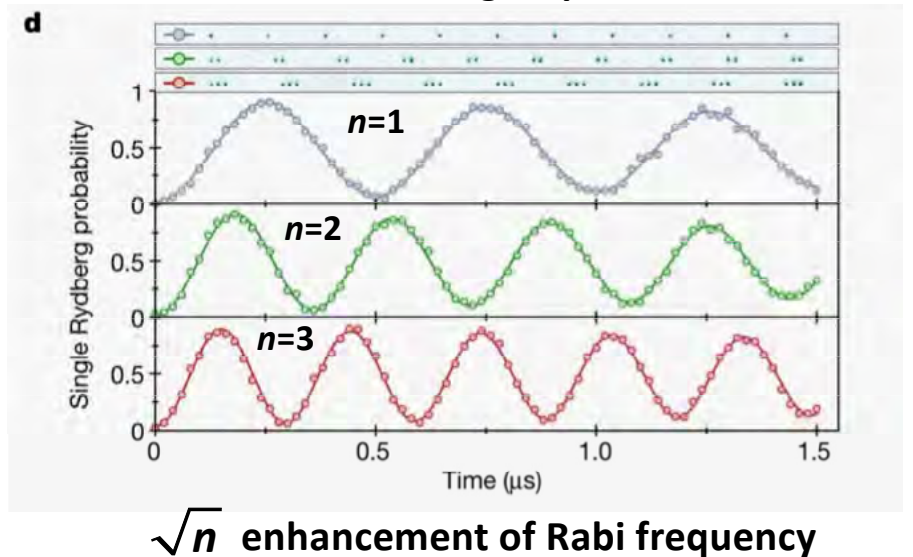
- * Large numbers of non-interacting qubits, (~ 256) trapped in 2D or 3D arrays.
- * Qubits interact when excited into Rydberg states with large dipole moments
- * Major advantage: Weak coupling to the environment when not doing gates
 - ➡ excellent quantum memory
- * Favorite platform for quantum simulation of quantum manybody physics
- * BEC's in optical lattices as analog simulators of superconductivity, quantum magnetism and more

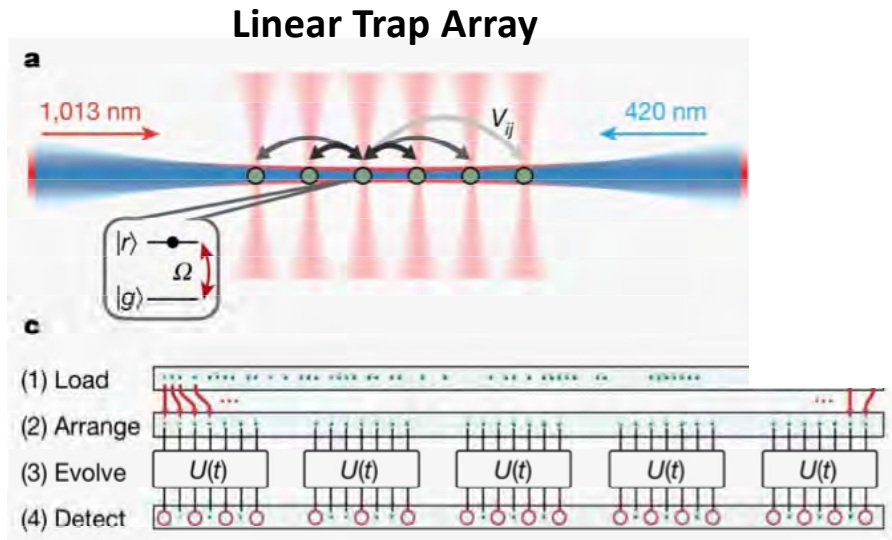
2-photon transition to a Rydberg state

Linear Trap Array

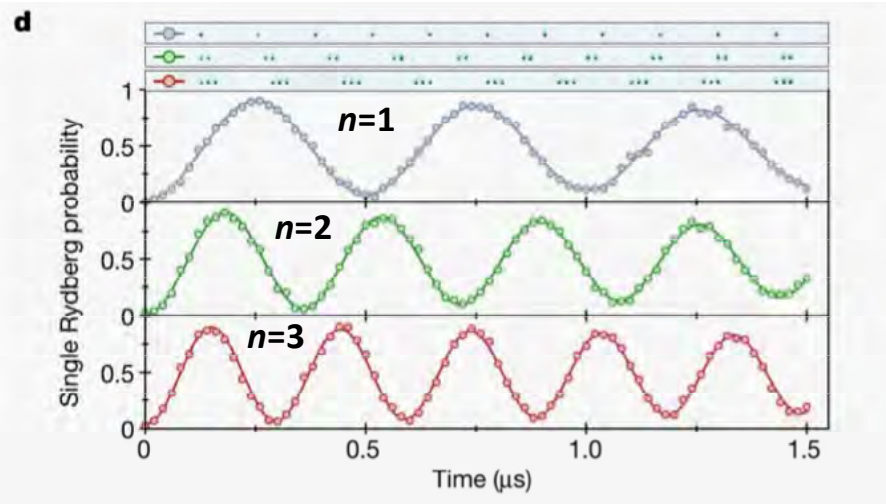


Sorted into groups of n



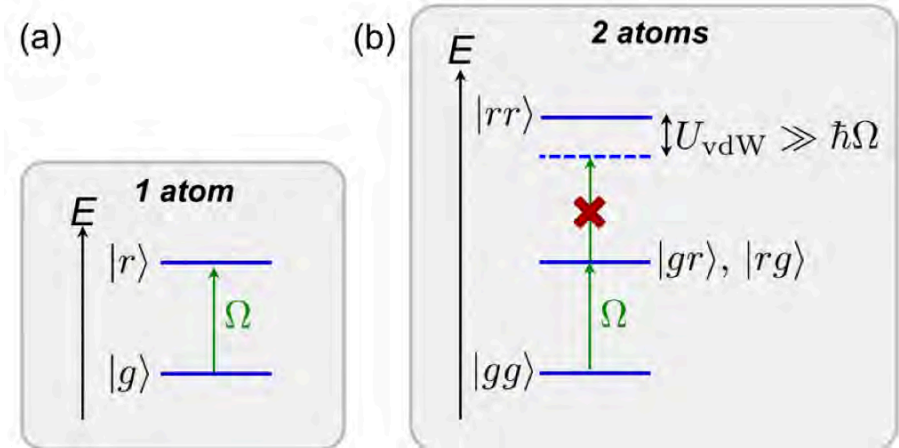


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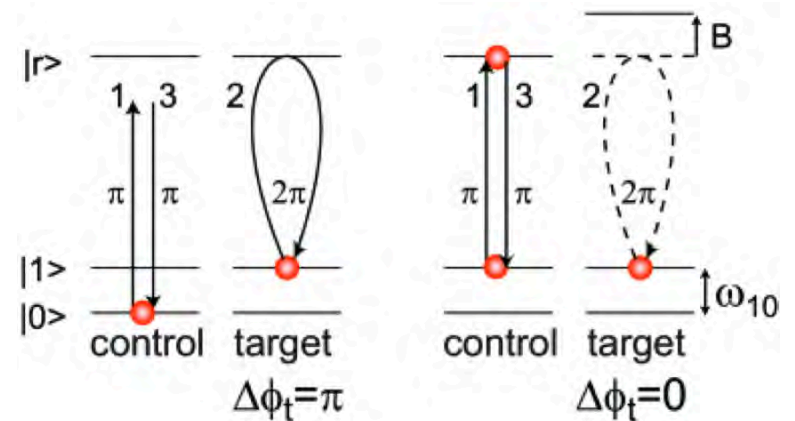
\sqrt{n} enhancement of Rabi frequency

Rydberg Blockade



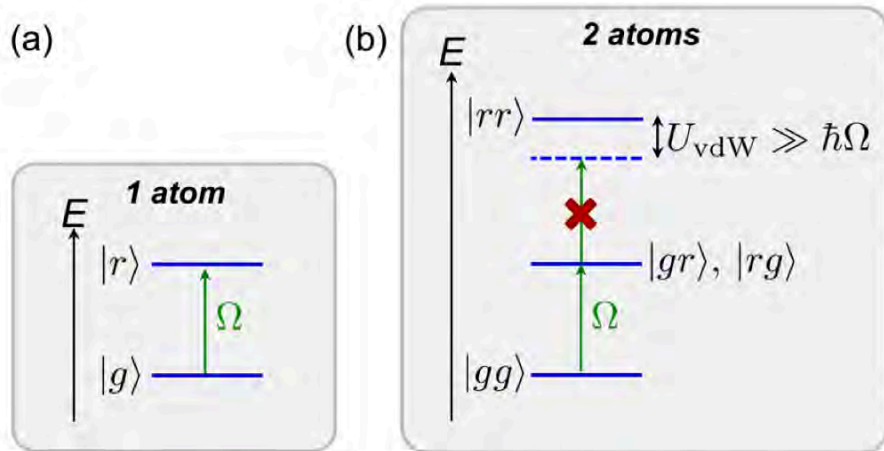
Principle of the Rydberg blockade. (a) A resonant laser couples, with strength Ω , the Rydberg state $|r\rangle$ and the ground state $|g\rangle$ of an atom. (b) For two nearby atoms, interactions U_{vdW} shift the doubly excited state $|rr\rangle$, preventing the double excitation of the atom pair when $U_{\text{vdW}} \gg \hbar\Omega$.

Controlled-Phase Gate



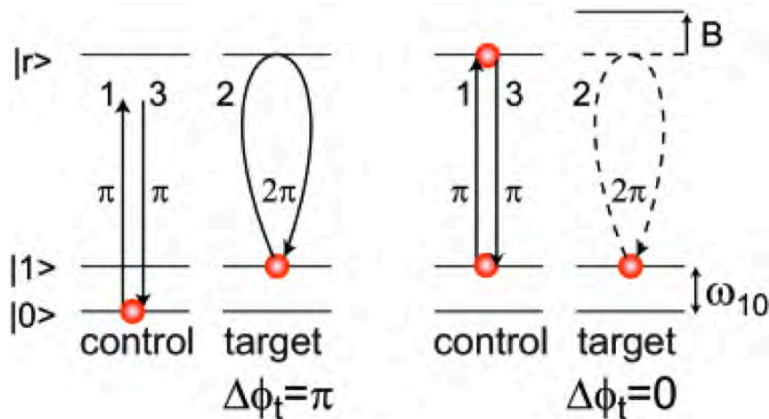
π phase shift of Target conditioned on Control

Rydberg Blockade



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Controlled-Phase Gate



π phase shift of Target conditioned on Control

Some links to get started

QuEra <https://www.quera.com/aquila>

Cold Quanta <https://coldquanta.com>

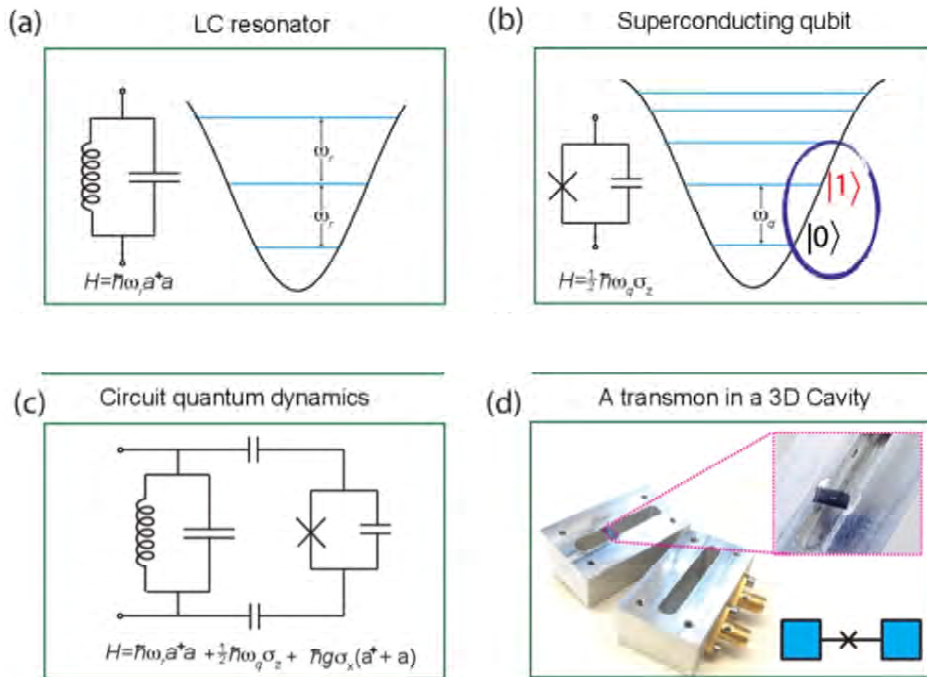
Sandia, Los Alamos National Labs

Individual PI's – Quantum Simulation

Lukin, Vuletic, Greiner, Endres, Bloch, Saffman, Biederman, Browaeys, Weiss, and many, many others...

Superconducting Qubits

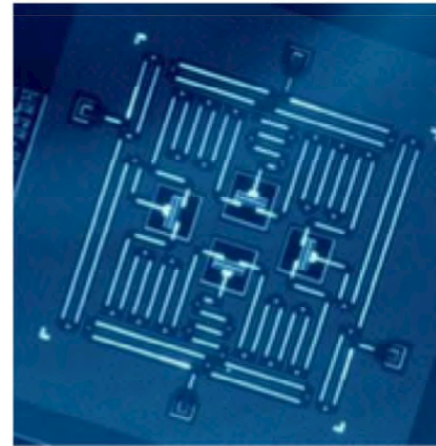
- * The basic building block is the so-called Transmon Qubit
- * A Transmon is a nonlinear oscillator made from a Josephson Junction and other circuit elements



Jaynes-Cummings Hamiltonian

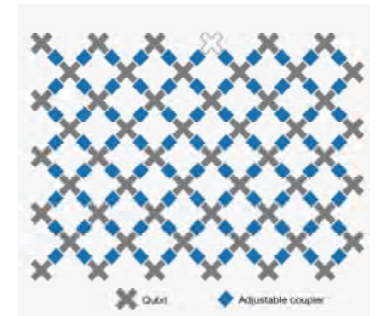
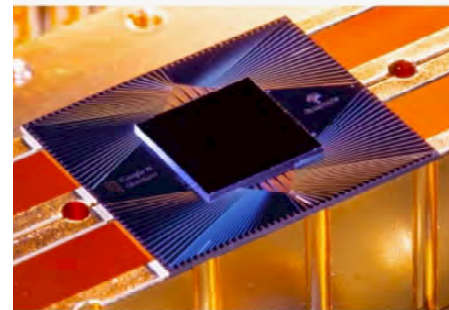
Superconducting Qubits

IBM 4-Transmon device (2017)



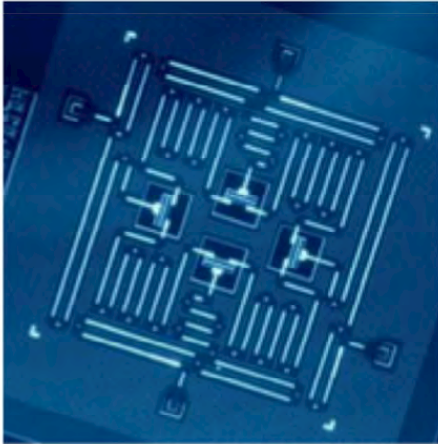
A device consisting of four transmon qubits, four quantum busses, and four readout resonators fabricated by IBM and published in *npj Quantum Information* in January 2017. [4]

Google 54-Transmon device (2019)



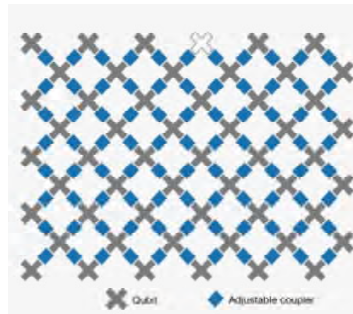
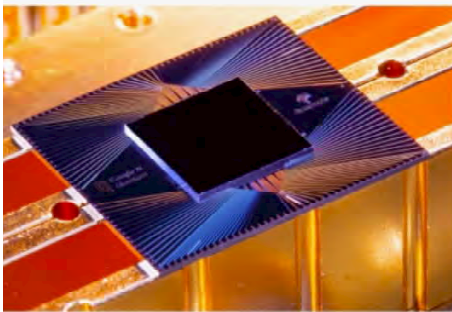
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Advantages

- * Solid State platform, looks like electronics
- * Clearer path to scale up to many qubits?

Challenges

- * Gates, coherence times not as good as atomic platforms, but gap is closing
- * Requires dilution refrigerator

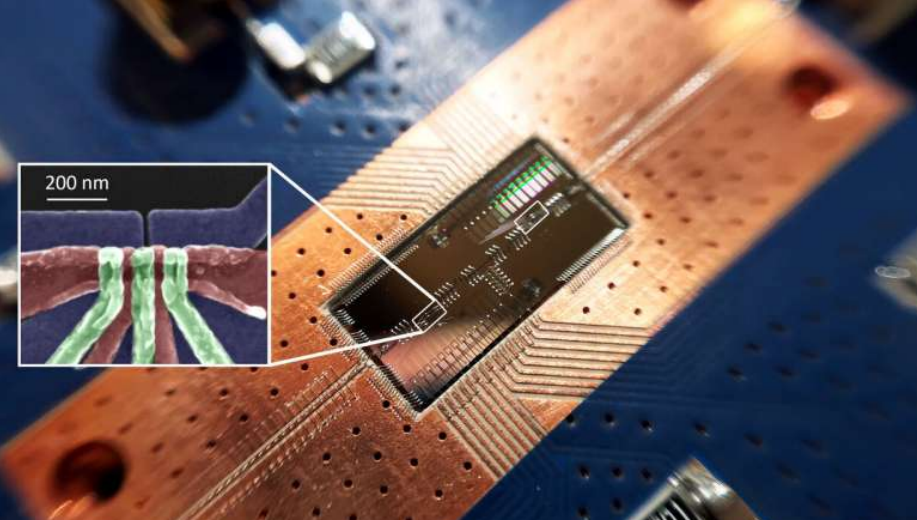
Industry Favorite

- * Large efforts at IBM, Google, Rigetti

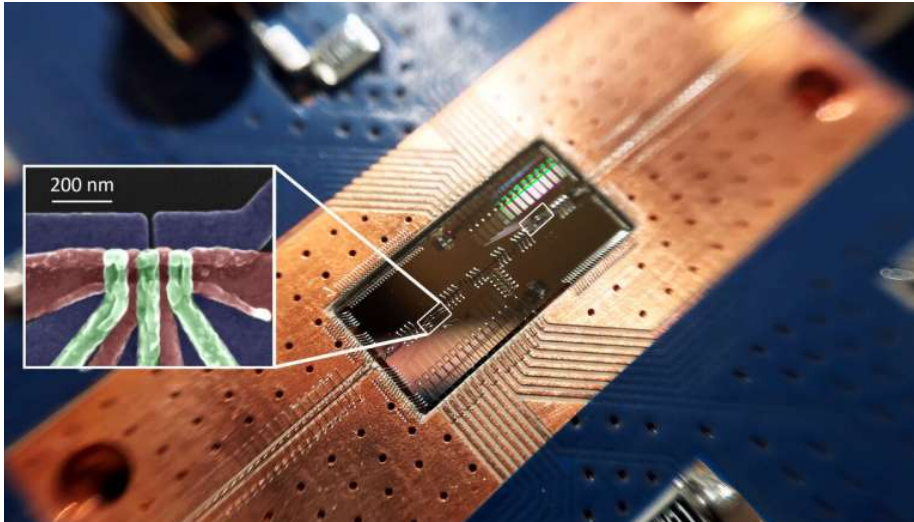
↑
Cloud Quantum Computing

- * **Amazon Braket** (IonQ, other Technology)
<https://aws.amazon.com/braket/>

Spins in Silicon Quantum Dots

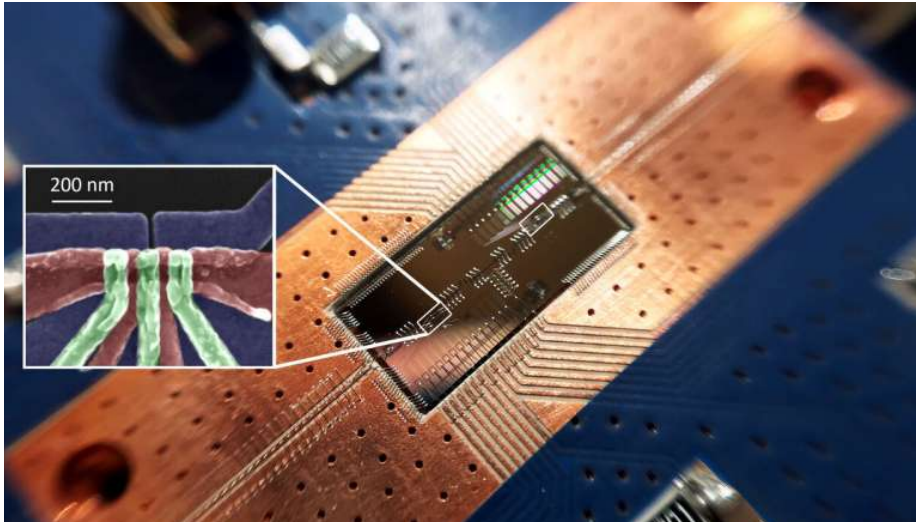


Spins in Silicon Quantum Dots



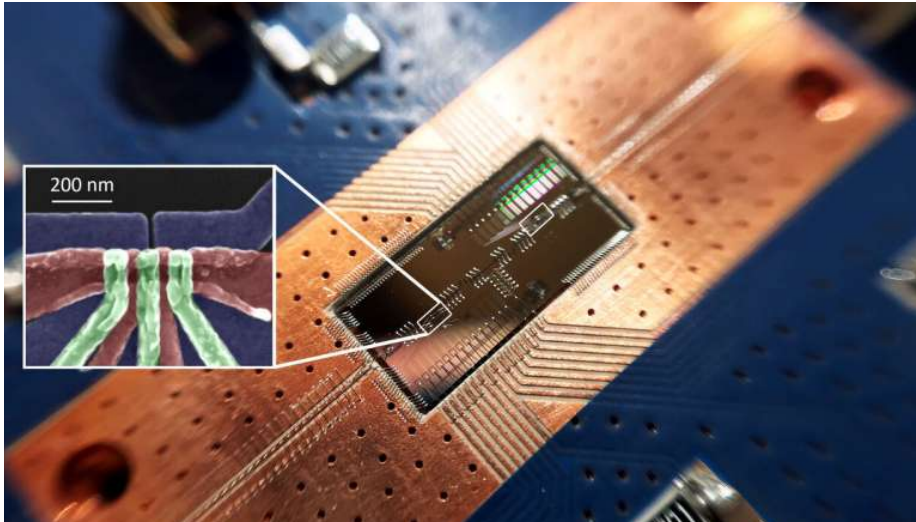
* Si is highly scalable

Spins in Silicon Quantum Dots



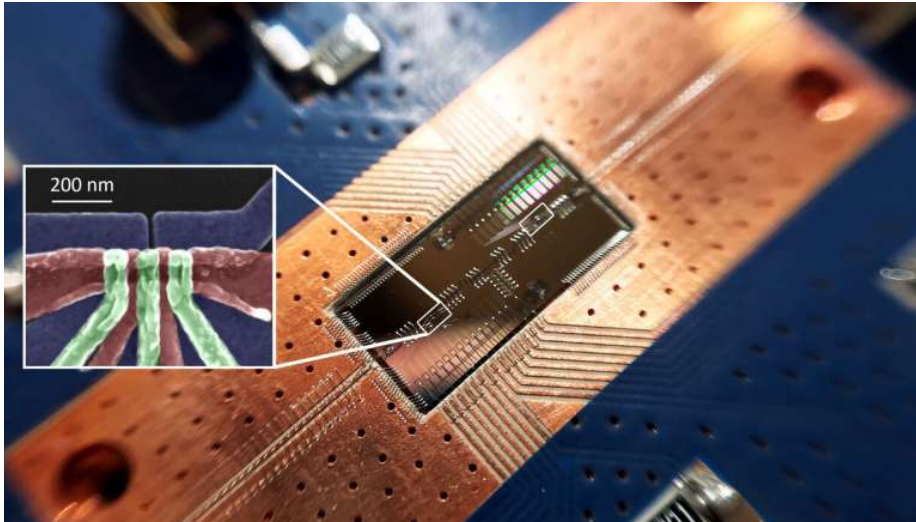
- * Si is highly scalable
- * Qubits and Quantum Gates demonstrated

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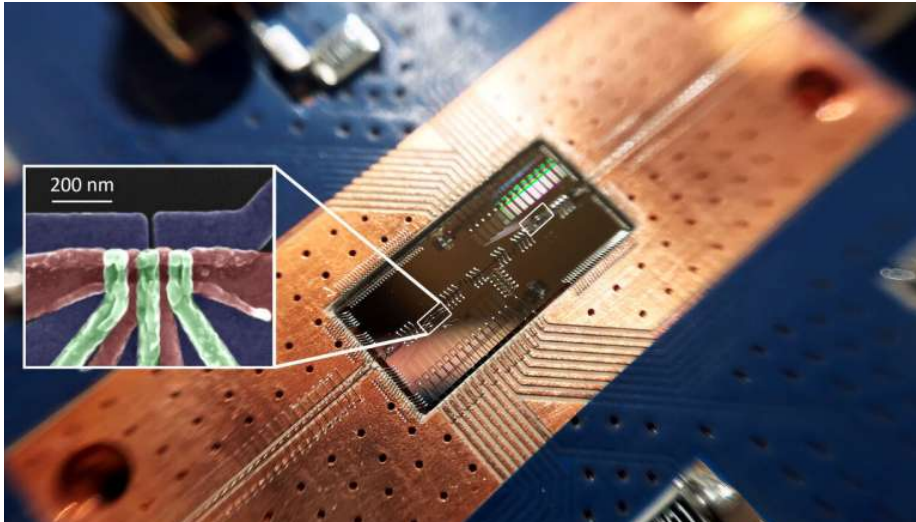
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- * HRL Laboratories, UNSW (Australia) group, Princeton Group, many others...

Comparing Physical Platforms: Status as of 2022

Table 1. Comparison of the achievable performances between three types of systems regarding QC. Numbers shown above are representative data. For the number N of qubits that can be prepared in one register, other notable results include $N = 40$ for trapped ions [44], and $N \sim 50$ in references [45, 46, 47], $N \sim 150$ in reference [48], $N = 184$ in reference [49], and $N = 200$ in reference [50] for neutral atoms; for fidelities $\mathcal{F}_{1(2)}$ of single(two)-qubit gates, other notable results include $\mathcal{F}_2 = 0.991$ [51] and 0.9944 [52] for SC, and $\mathcal{F}_1 = 0.998$ [72, 77] and $\mathcal{F}_2 = 0.974$ [53] for neutral atoms. Here, results with larger fidelities are shown. Faster gates based on a similar mechanism can have smaller fidelities as studied in reference [54]; take trapped ions as example, reference [38] studied single-qubit gates of duration $2 \mu\text{s}$ and fidelity $0.999\,96$, and reference [55] studied an entangling gate of duration $1.6 \mu\text{s}$ and fidelity 0.9982 .

	Number of qubits	Coherence time	Fidelity and duration of quantum operations	
			One-qubit gate ^a	Two-qubit gate or Bell state ^b
SC	53 [56]; 54 [57]	$70 \mu\text{s}$ [58] ^c	0.9992 ; 10 ns [52] ^c	0.997 ; 60 ns [59] (C_Z gates)
Trapped ions	53 [60]	50 s [61, 62]	$0.999\,999$; $12 \mu\text{s}$ [61]	0.9992 ; $30 \mu\text{s}$ [38] (Bell states)
Neutral atoms	209 [63]; 219 [64]; 256 [65]	7 s [66]; 48 s [67] ^d	$0.999\,86$; $31 \mu\text{s}$ [66]	0.991 ; 59 ns [67] ^e (Bell states)

^aThe duration for single-qubit gates refers to that of a Clifford gate such as a $\pi/2$ rotation between the two states of a qubit.

^bThe time here refers to the duration of either implementing a controlled-Z (C_Z) gate or creating a Bell state from a product state.

^cThe coherence time for superconducting qubits refers to the smaller one among the relaxation time (T_1) and the decoherence time (T_2^*) of reference [51]; the single-qubit gate data are taken from table S2 of the supplementary information of reference [52].

^dUnlike that in reference [72] which studied qubits defined by ground states, the coherence time in reference [73] refers to that of the optical clock state $(5s5p)^3P_0$ of ^{88}Sr . Reference [73] reported an atomic coherence time up to 48 s .

^eA Rabi frequency $\Omega = 2\pi \times 6\text{--}7 \text{ MHz}$ was used in reference [67] so that a π pulse for exciting the ground to Rydberg states has a duration $\pi/(\sqrt{2}\Omega) \sim 51 - 59 \text{ ns}$ with $\sqrt{2}$ a many-body enhancement factor.

Introduction and Overview (Preskills Notes)

Other Platforms

Nuclear Magnetic Resonance

- * Qubits encoded in spin-1/2 nuclei in a single molecule.
- * Mature technology, many early proof of principle demonstrations
- * Fundamentally not scalable, many early demonstrations, largely abandoned

Photonics

- * Photons can carry QI in, e. g., their polarization state.
- * Great for transmitting quantum info
- * Easy to make, transmit and detect
- * Difficult to store → work on photon Quantum Memories
- * Photon-photon gates in cavities, mediated By Rydberg polaritons, One-Way Q.C., Measurement based Quantum Computing
- * PsiQuantum, <https://www.psiquantum.com>

Other Platforms

- * NV Centers in Diamond – Good for Quantum Sensing
- * Electrons floating on liquid Helium



ARTICLE

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OPEN

Coupling a single electron on superfluid helium to a superconducting resonator

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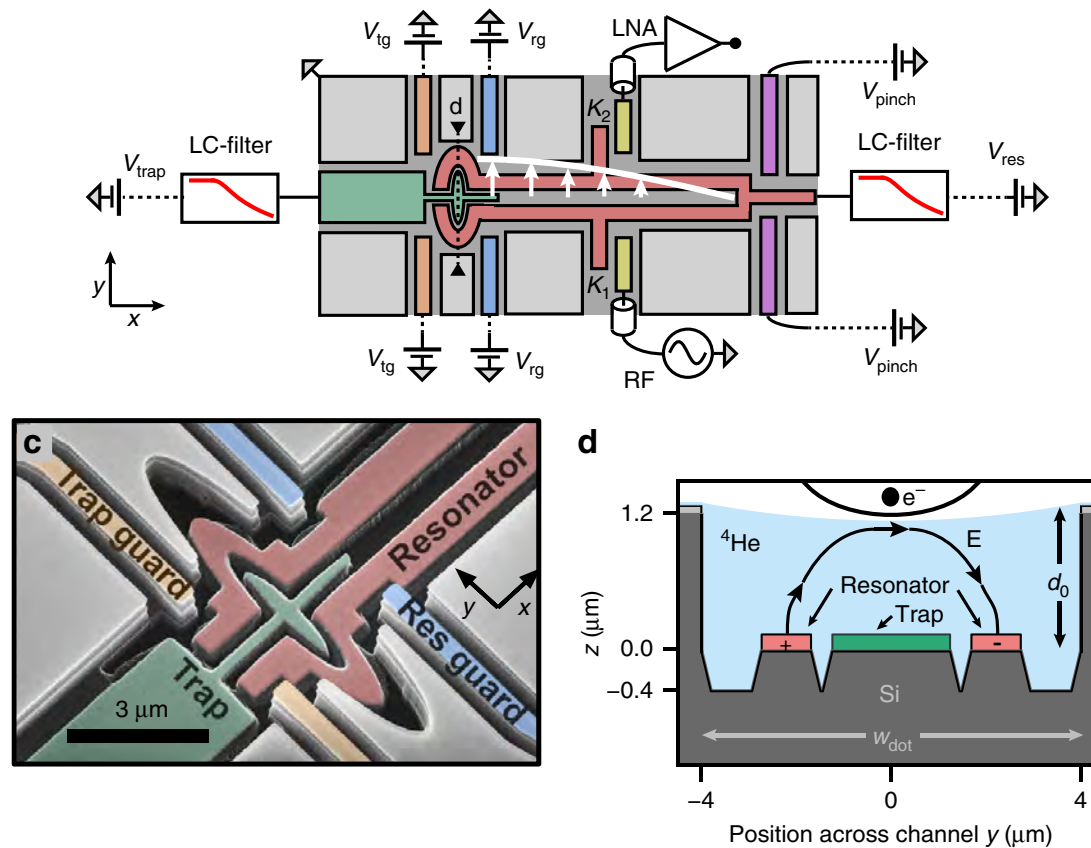


Fig. 1 An electron-on-helium dot **a** Optical micrograph and **b** schematic of the device. The resonator (red) can be probed with microwaves via coplanar waveguides

Let's go look at some websites...

Quantum Computing in the NISQ era and beyond

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Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.