Introduction and Overview (Preskills Notes)

Quantum Error Correction

Fundamental Problem
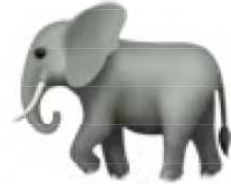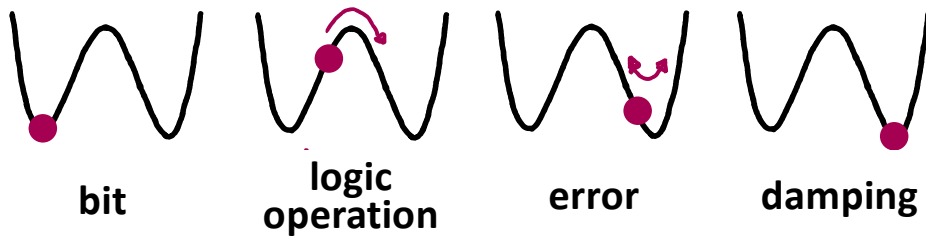
Quantum States are fragile, especially when entangled

Classical Computation **?** **Dissipation helps**

bit   logic operation   error   damping

No dissipation ➡ **Errors build up**

Quantum Computation ➡

* Cannot tolerate dissipation

* Destroys superposition and entanglement

What to do? **Error Correction!**

Classical Error Correction:

Simple example: **Redundancy** protects against bit flips

Encode:
$$0 \rightarrow (000)$$
$$1 \rightarrow (111)$$

Errors:
$$(000) \rightarrow (100)$$
$$(111) \rightarrow (011)$$

correct by majority vote

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Quantum Computation ➡️

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Von Neumann:

* **A classical computer w/faulty components can work, given enough redundancy**

* **Classical error correction is well developed and highly sophisticated...**

* **Quantum Errors**

1) Bit Flip $|0\rangle \rightarrow |1\rangle$, phase flip $|0\rangle \rightarrow |0\rangle$
$|1\rangle \rightarrow |0\rangle$ $|1\rangle \rightarrow -|1\rangle$

2) Small errors $a|0\rangle + b|1\rangle$ a, b can change by ε
errors accumulate

3) Measurement disturbs ➡️ collapse of quantum states

4) No cloning ➡️ Cannot protect by making copies

Introduction and Overview (Preskills Notes)

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* ## Quantum Errors

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4) No cloning ➡ Cannot protect by making copies

Example: **Peter Shor's code for bit flip error when $P(\text{error}) \ll 1$**

Encode: $|0\rangle \rightarrow |\bar{0}\rangle \cong |000\rangle$ (3 bit code)
$|1\rangle \rightarrow |\bar{1}\rangle \equiv |111\rangle$

⬇

$a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$

Single-qubit measurement ➡ collapse of state, destroys info, no majority voting !

Collective 2-qubit measurement:

- for $|x, y, z\rangle$ measure $y \oplus z$ (never measure individual bits)
  $x \oplus z$

- if $|000\rangle$, $|111\rangle$ these observables $= 0$

- if one bit-flip, at least one observable $= 1$

- easy to check that $(y \oplus z, x \oplus z) =$ binary address of qubit flip

$|000\rangle \rightarrow |010\rangle$ $(1, 0) =$ 2nd bit

Example: **Peter Shor's** code for bit flip error when P(error) << 1

Encode:

$$|0\rangle \rightarrow |\bar{0}\rangle \equiv |000\rangle$$
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$$|000\rangle \rightarrow |010\rangle \quad (1,0) = \text{2nd bit}$$

Small errors:

$$|000\rangle \rightarrow |000\rangle + \varepsilon|001\rangle$$
$$|111\rangle \rightarrow |111\rangle + \varepsilon|110\rangle$$

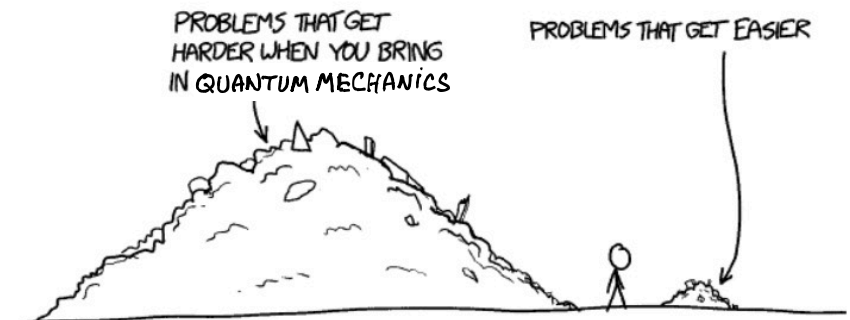**Quantum mechanics to the rescue !**

- mostly no error detected

  ⟹ collapse into $|000\rangle$ resp. $|111\rangle$

- sometime error detected

  ⟹ collapse into $|001\rangle$ resp. $|110\rangle$

  ⟹ full bit flip, correct as such

PROBLEMS THAT GET HARDER WHEN YOU BRING IN QUANTUM MECHANICS

PROBLEMS THAT GET EASIER

Source: xkcd.com

Introduction and Overview (Preskills Notes)

Example: **Peter Shor's** code for bit flip error when P(error) << 1

Encode:
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(3 bit code)

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Collective 2-qubit measurement:

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$$\begin{array}{l} y\oplus z \\ x\oplus z \end{array}$$
$\left(\begin{array}{c}\text{never measure}\\\text{individual bits}\end{array}\right)$

- if $|000\rangle$, $|111\rangle$ these observables $= 0$

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Quantum mechanics to the rescue !

- mostly no error detected
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How to implement ?

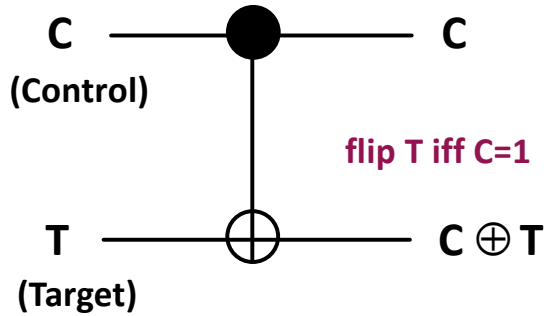Quantum circuit + single qubit measurement

Quantum Gates – work on superpositions, and entangled states

Introduction and Overview (Preskills Notes)

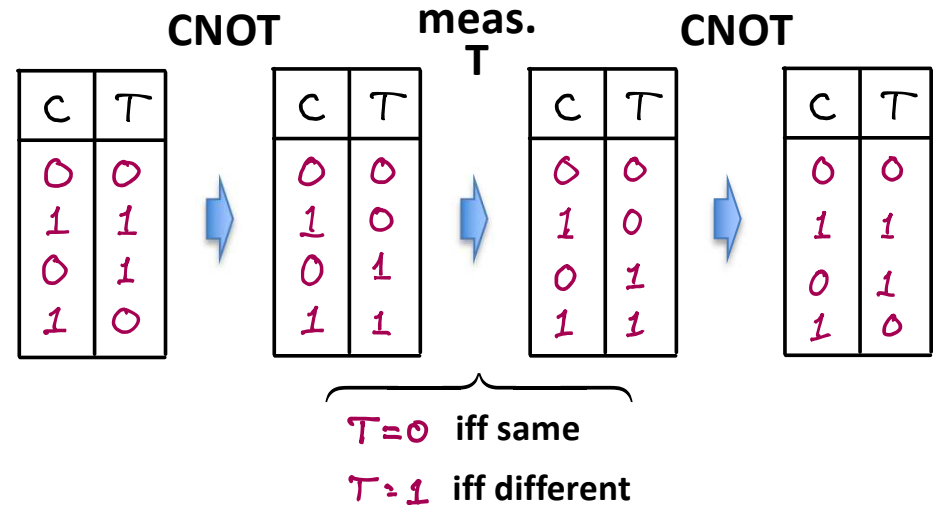## Controlled-NOT (CNOT)

C ———————●——————— C
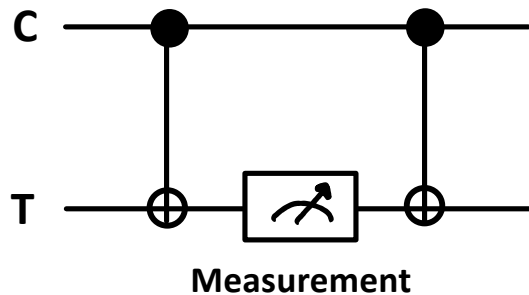(Control)

flip T iff C=1

T ———————⊕——————— C ⊕ T
(Target)

Truth Table

C	T	C ⊕ T
0	0	0
0	1	1
1	0	1
1	1	0

Quantum Circuit for joint measurement

C ———●————————●———

T ———⊕——[📐]——⊕———

Measurement

Measurement in $\{|0\rangle, |1\rangle\}$ basis

yields $C \oplus T$
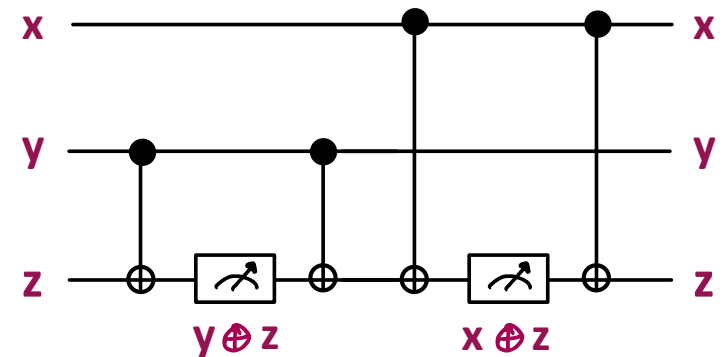
Circuit maps logical basis states as

CNOT   meas. T   CNOT

C	T
0	0
1	1
0	1
1	0

C	T
0	0
1	0
0	1
1	1

C	T
0	0
1	0
0	1
1	1

C	T
0	0
1	1
0	1
1	0

$T = 0$ iff same

$T = 1$ iff different

Full circuit to obtain Error Syndrome

x ————————————●————————●———— x

y ———●—————————●————————————— y

z ———⊕——[📐]——⊕———⊕——[📐]——⊕——— z

     y ⊕ z   x ⊕ z

✱ iff qubit flip, binary address = $(y \oplus z, x \oplus z)$

32

Circuit maps logical basis states as

CNOT meas. T CNOT

C	T
0	0
1	1
0	1
1	0

C	T
0	0
1	0
0	1
1	1

C	T
0	0
1	0
0	1
1	1

C	T
0	0
1	1
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1	0

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Full circuit to obtain Error Syndrome

x ———————————————●——————————●———— x

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z ——⊕—[meas]—⊕————⊕—[meas]—⊕—— z
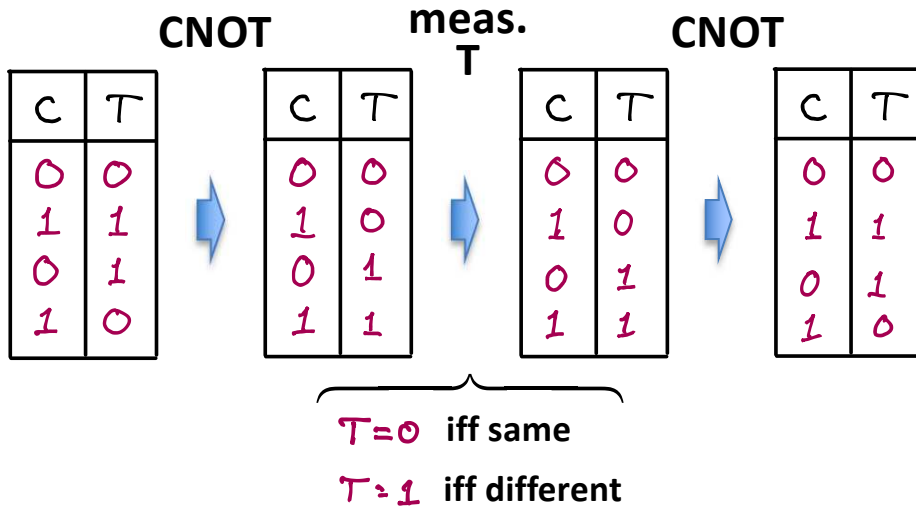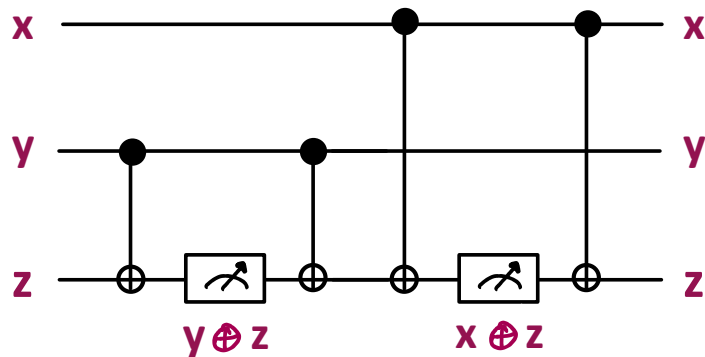
$y \oplus z$ $x \oplus z$

✳ iif qubit flip, binary address $= (y \oplus z, x \oplus z)$

Quantum Phase Error

$|0\rangle \rightarrow |0\rangle$

$|1\rangle \rightarrow -|1\rangle$

Encoding

$$|0\rangle \rightarrow |\bar{0}\rangle = \frac{1}{2^{3/2}} \overset{x'}{(|0\rangle+|1\rangle)} \overset{y'}{(|0\rangle+|1\rangle)} \overset{z'}{(|0\rangle+|1\rangle)}$$

$$|1\rangle \rightarrow |\bar{1}\rangle = \frac{1}{2^{3/2}} (|0\rangle-|1\rangle)(|0\rangle-|1\rangle)(|0\rangle-|1\rangle)$$

Relabel

$$\frac{1}{\sqrt{2}} (|0\rangle+|1\rangle) = |0'\rangle$$

$$\frac{1}{\sqrt{2}} (|0\rangle-|1\rangle) = |1'\rangle$$

Measure in basis

$$\{|0'\rangle, |1'\rangle\} \rightarrow y' \oplus z', \, x' \oplus z'$$

Error Syndrome

✳ Iff phase error, binary address $= (y' \oplus z', x' \oplus z')$

✳ Analogous to bit-flip code, just in different basis

Quantum Phase Error

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* Iff phase error, binary address $= \left(y' \oplus z', \; x' \oplus z'\right)$

* Analogous to bit-flip code, just in different basis

Shor's 9-bit code

* Combines flip/phase error correction

* Corrects one flip or phase error

General principle of error correction

* Encode p logical qubits in n physical qubits.

* Valid Logical States form 2^p–dimensional subspace $\mathcal{E}_p$ (code space) in n-qubit (2^n–dimensional) Hilbert space $\mathcal{E}_N$

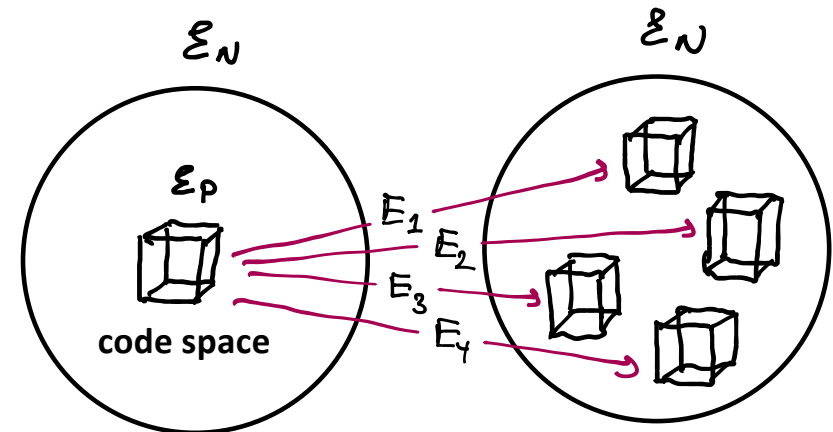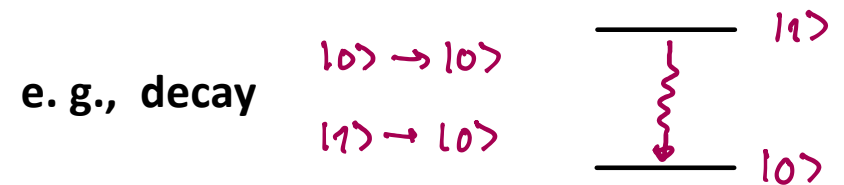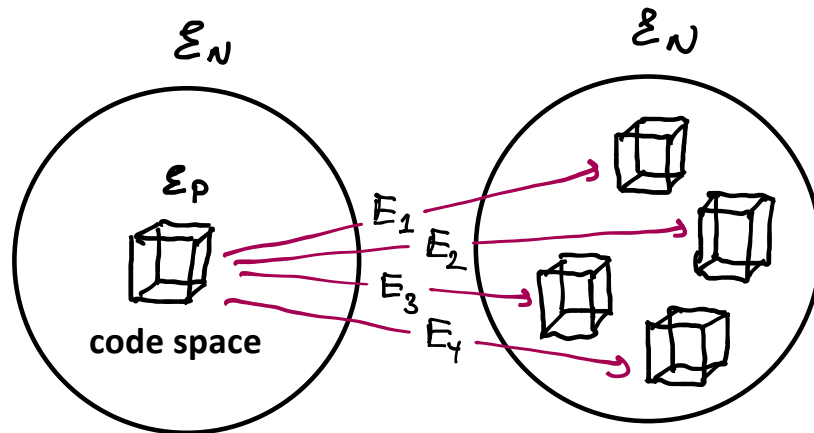* Errors displace system into orthogonal (distinguishable) subspaces.

Introduction and Overview (Preskills Notes)

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Geometric illustration

$\mathcal{E}_N$ $\mathcal{E}_N$

$\mathcal{E}_p$

E_1
E_2
E_3
E_4

code space

What about non-Unitary errors?

e. g., decay

$|0\rangle \to |0\rangle$

$|1\rangle \to |0\rangle$

$|1\rangle$

$|0\rangle$
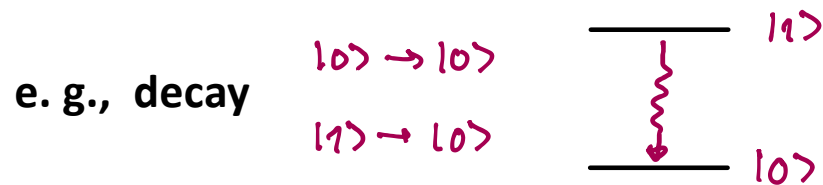
Problem: Errors not displaced into orthogonal subspaces

Solution: "Quantum jump codes", monitors the environment

Other kinds of errors?

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Geometric illustration

$\mathcal{E}_N$ $\mathcal{E}_N$

$\mathcal{E}_P$

E_1
E_2
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Other kinds of errors?

Catnip for Theoretical Physicists & Computer Scientists

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Catnip for Theoretical Physicists & Computer Scientists

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Quantum Hardware

**Physical Implementation is
Extremely demanding !**

Requirements

1. **Storage: Quantum memory.**

2. **Gates: We put computation U_f together
   from 1 and 2-qubit operations.**

3. **Readout: Method to measure qubits.**

4. **Isolation: No coupling to environment
   to avoid decoherence & errors**

5. **Precision: Gates, readouts must be
   highly accurate**

Inherent Contradictions

2. Gates   vs   4. Isolation

**coupling between
qubits**   **no coupling to
environment**
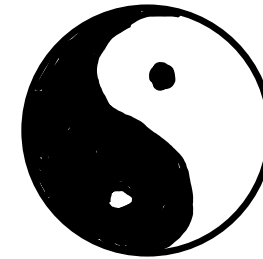
To build a Quantum Computer:

**Choose, find or invent a system
with acceptable tradeoffs.**

6. **Error Correction must not create
   more errors than it corrects.**

7. **Thresholds for Error Correction
   and Fault Tolerance**

Quantum Computing in the NISQ era and beyond

John Preskill

Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics,

California Institute of Technology, Pasadena CA 91125, USA

30 July 2018

Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

Introduction and Overview (Preskills Notes)

Inherent Contradictions

2. Gates   vs   **4. Isolation**

⬆   ⬆

coupling between qubits   no coupling to environment

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⬇
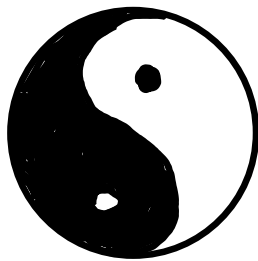
7. <u>Thresholds</u> for Error Correction and Fault Tolerance

Ion Trap Quantum Computing

First to demonstrate a Quantum Gate

✳ **Qubit is encoded in the electronic ground state of an atomic ion**

$|e\rangle$

Raman coupling

$|g_2\rangle$ $|1\rangle$

$|g_1\rangle$ $|0\rangle$
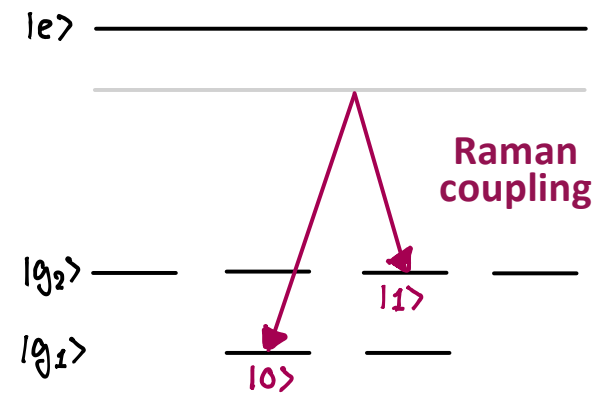
✳ **Early design with a few ions in large trap**

Ion Trap Quantum Computing

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Raman coupling

$|g_2\rangle$ ———   ———   ——— $|1\rangle$

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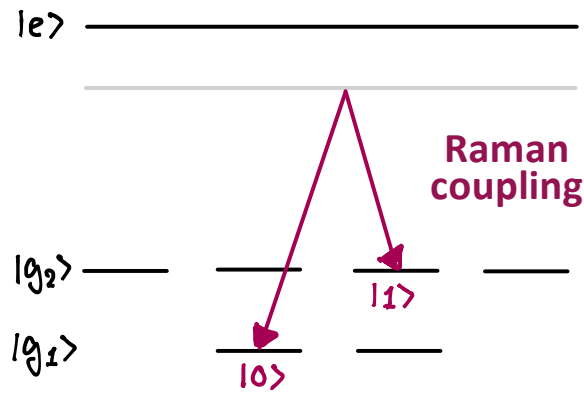
✻ **Early design with a few ions in large trap**

Requirements

1. **Storage:**   10s-100s coherence time

2. **Gates:**   Use collective vibrations as "quantum bus"

3. **Readout:**   Fluorescence

$|e\rangle$

"bright" —— $|g_2\rangle$

"dark" —— $|g_1\rangle$

Cirac & Zoller: 5 laser pulses

   CNOT gate between any 2 ions in linear array

Wineland: 3 laser pulses enough for CNOT

> **Use this example serves as conceptual template**

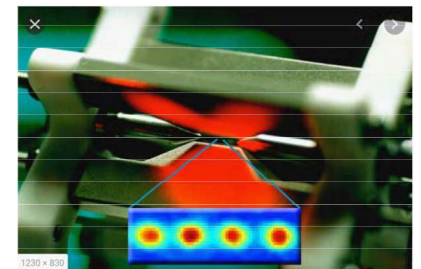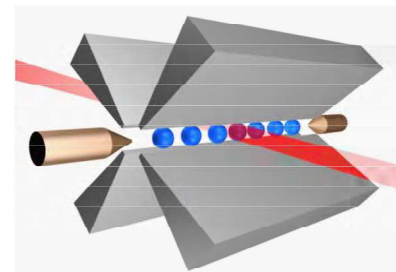## Ion Trap Quantum Computing

First to demonstrate a Quantum Gate

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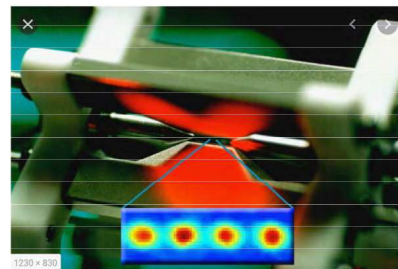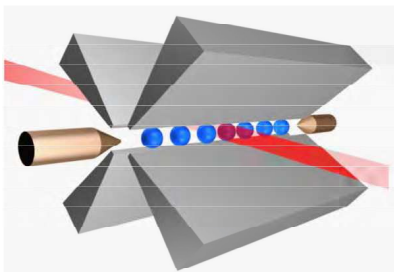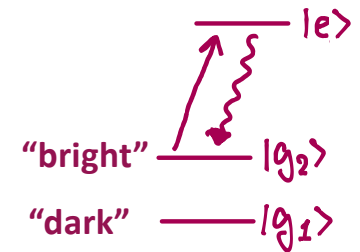
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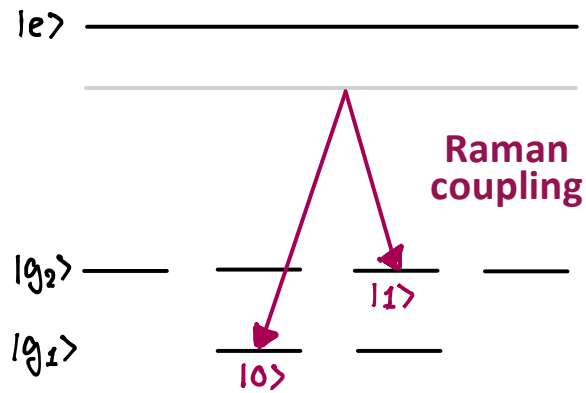
* **Early design with a few ions in large trap**
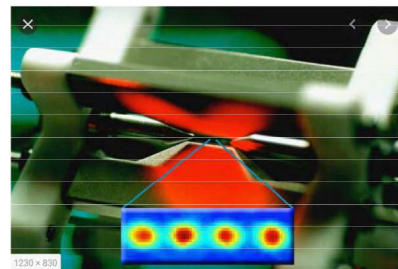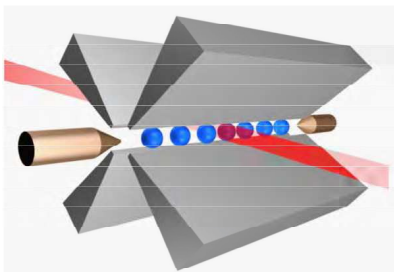
* **Scaling up in Linear Ion Traps**

15 µm

* **Limitations**
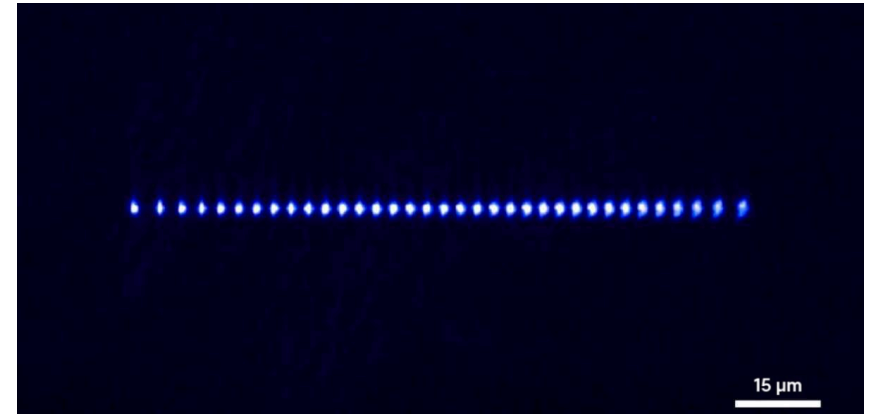
Introduction and Overview (Preskills Notes)
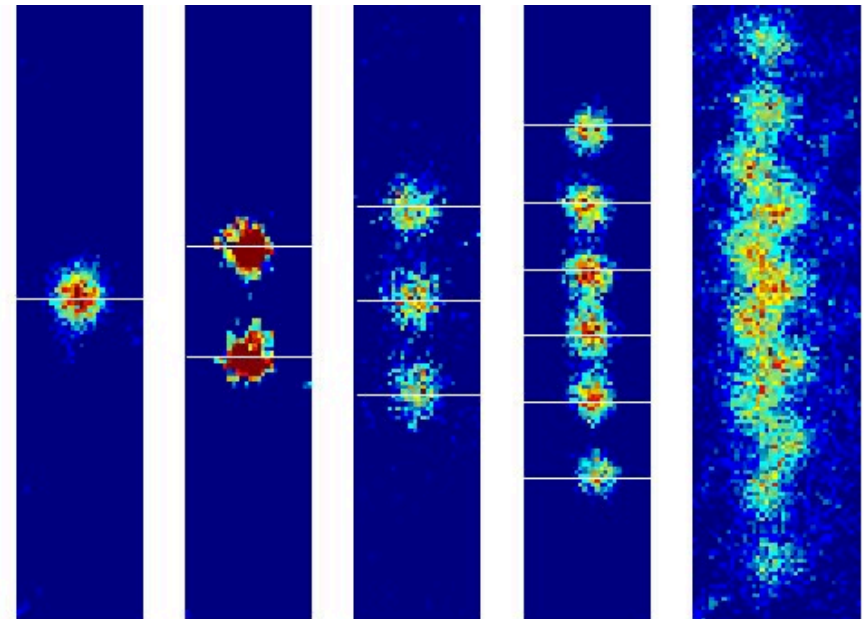
Status: Many important milestones achieved

* Entanglement of ≥ 20 ions (2018)

* Highest gate & readout fidelities, longest coherence times

* Error Correction, Fault Tolerance proof of principle demonstrations

* Complex algorithms on few ions, quantum simulations with ≥ 50

* Research groups in academia, National Labs, Industry

Some early leaders

NIST   Innsbruck   Quantinuum

Sandia NL   Duke U   IonQ

Many others these days

---

* Major challenges – same as late 2000's!

* "Clock speed" set by vibrational freqs microfabricated traps do better

* More ions -> harder to cool motion, harder to individually address ions in linear trap.

* Scaling up to 1000's of ions is an enormous challenge

Scalable Ion Trap Quantum Processor – one vision

to additional memory
or processor units

o o ● (p)

(r)

o o o o o o o o o o o o (m)

o   qubit-ion
●   cooling ion
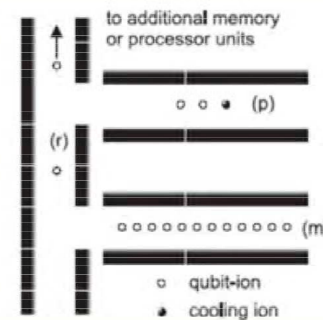
D. Kielpinski, C. Monroe, and D. J. Wineland, Nature 417, 709 (2002).

To appear in the 2005 International Symposium on Microarchitecture (MICRO-38)

A Quantum Logic Array Microarchitecture: Scalable Quantum Data Movement and Computation

Tzvetan S. Metodi[†], Darshan D. Thaker[†], Andrew W. Cross[‡]
Frederic T. Chong[§] and Isaac L. Chuang[‡]

Operation	Time	$P_{current}$	$P_{expected}$
Single Gate	$1\mu s$	0.0001	10^{-8}
Double Gate	$10\mu s$	0.03	10^{-7}
Measure	$100\mu s$	0.01	10^{-8}
Movement	$10 ns/\mu m$	$0.005/\mu m$	$10^{-6}/cell$
Split	$10\mu s$		
Cooling	$1\mu s$		
Memory time	$10 - 100\ sec$		

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NIST Group, Current as of 2023
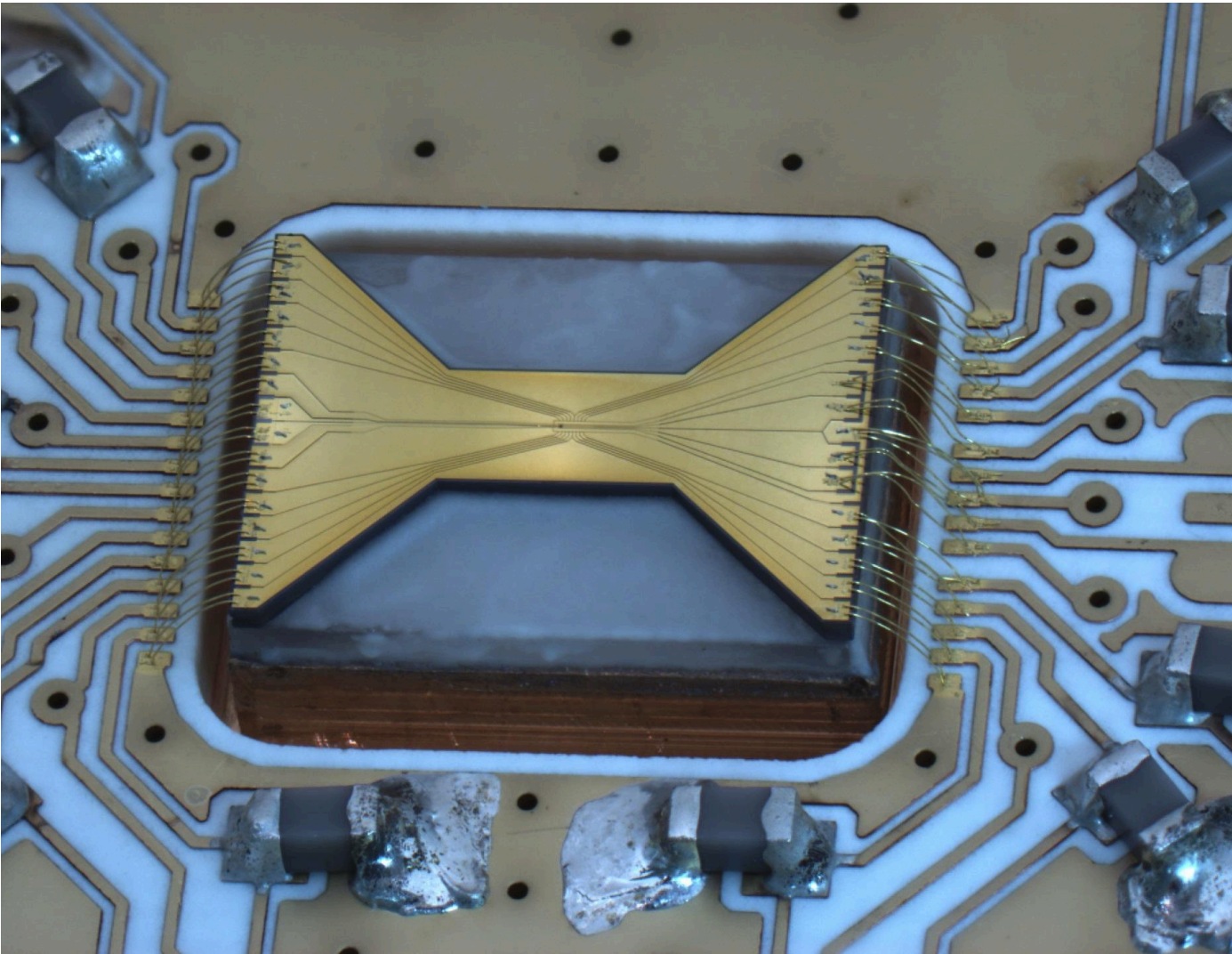
Demonstration of a Fundamental Quantum Logic Gate

C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland

National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 14 July 1995)

We demonstrate the operation of a two-bit "controlled-NOT" quantum logic gate, which, in conjunction with simple single-bit operations, forms a universal quantum logic gate for quantum computation. The two quantum bits are stored in the internal and external degrees of freedom of a single trapped atom, which is first laser cooled to the zero-point energy. Decoherence effects are identified for the operation, and the possibility of extending the system to more qubits appears promising.

FIG. 1. $^9Be^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ ($^2S_{1/2}|F = 2, m_F = 2\rangle$ and $^2S_{1/2}|F = 1, m_F = 1\rangle$ levels, respectively, separated by $\omega_0/2\pi \simeq 1.250$ GHz), and $|aux\rangle = {}^2S_{1/2}|F = 2, m_F = 0\rangle$ (separated from $|\downarrow\rangle$ by $\simeq 2.5$ MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \simeq 11.2$ MHz). Stimulated Raman transitions between $^2S_{1/2}$ hyperfine states are driven through the virtual $^2P_{1/2}$ level ($\Delta \simeq 50$ GHz) with a pair of $\simeq 313$ nm laser beams. Measurement of S is accomplished by driving the cycling $|\downarrow\rangle \rightarrow {}^2P_{3/2}|F = 3, m_F = 3\rangle$ transition with σ^+-polarized light and detecting the resulting ion fluorescence.

according to the following format:

(a) A $\pi/2$ pulse is applied on the carrier transition. The effect is described by the operator $V^{1/2}(\pi/2)$ in the notation of Ref. [1].

(b) A 2π pulse is applied on the blue sideband transition between $|\uparrow\rangle$ and an auxiliary atomic level $|aux\rangle$ (see Fig. 1). (1)

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is as follows:

$$\text{Input state} \longrightarrow \text{Output state}$$

$$|0\rangle|\downarrow\rangle \rightarrow |0\rangle|\downarrow\rangle$$
$$|0\rangle|\uparrow\rangle \rightarrow |0\rangle|\uparrow\rangle \qquad (2)$$
$$|1\rangle|\downarrow\rangle \rightarrow |1\rangle|\uparrow\rangle$$
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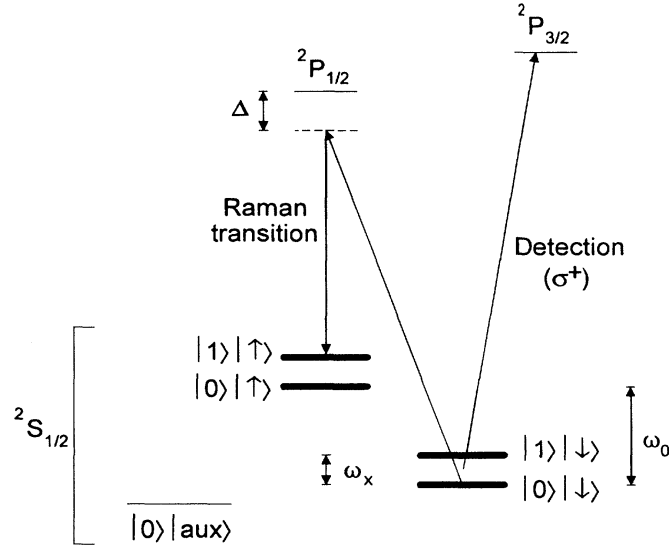Following Raman cooling to the $|0\rangle|\downarrow\rangle$ state, but before application of the CN operation, we apply appropriately

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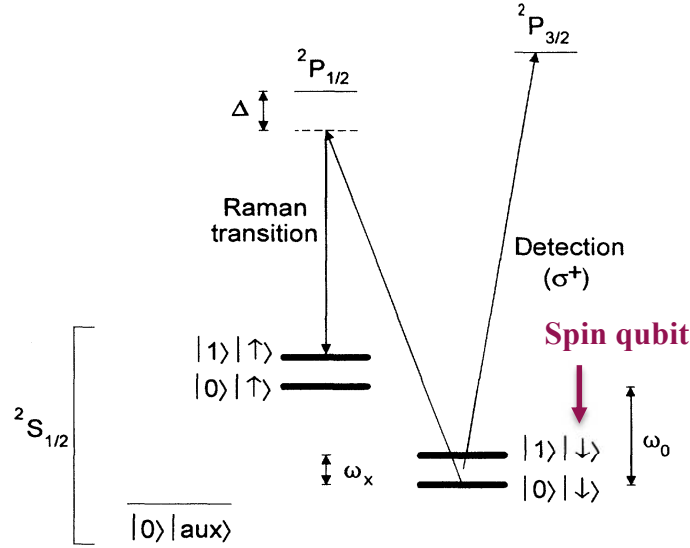
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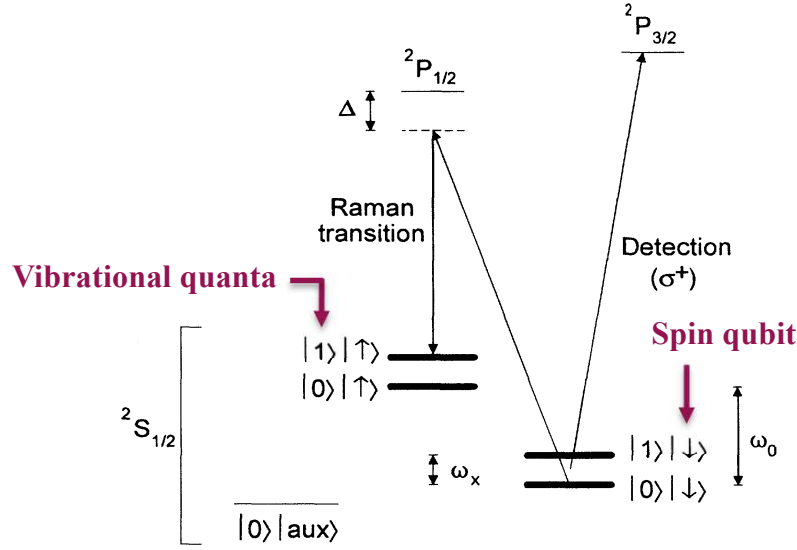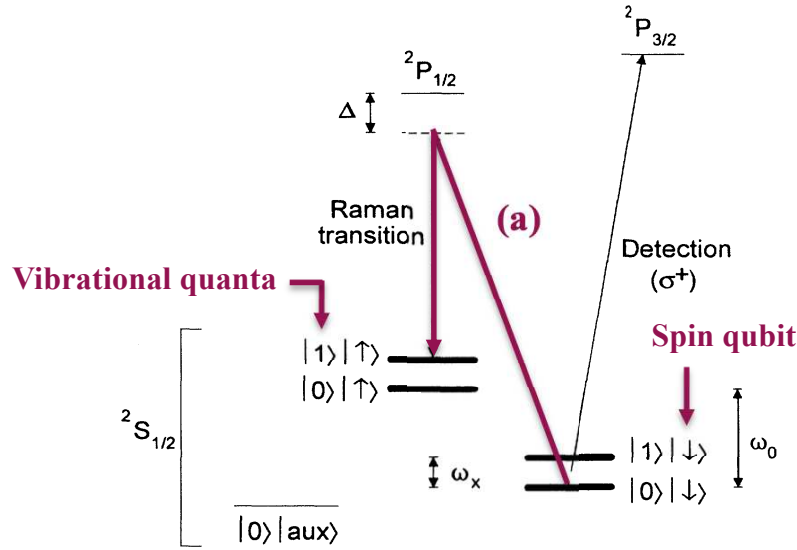
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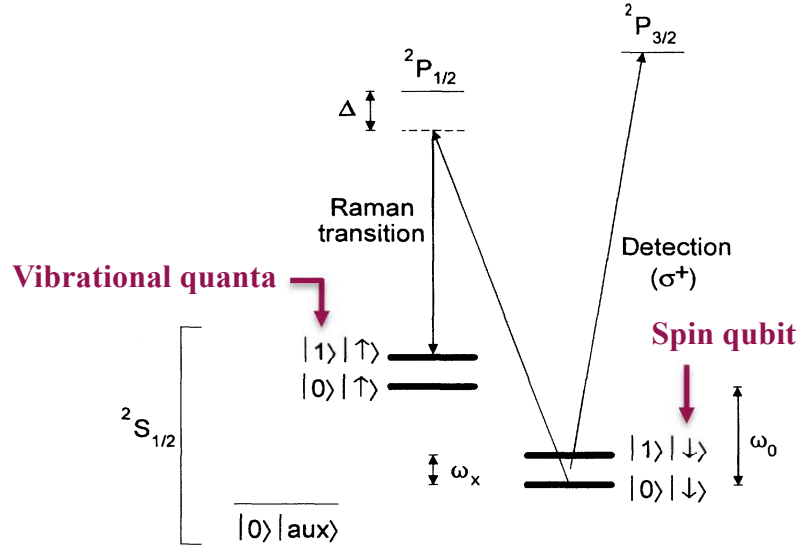
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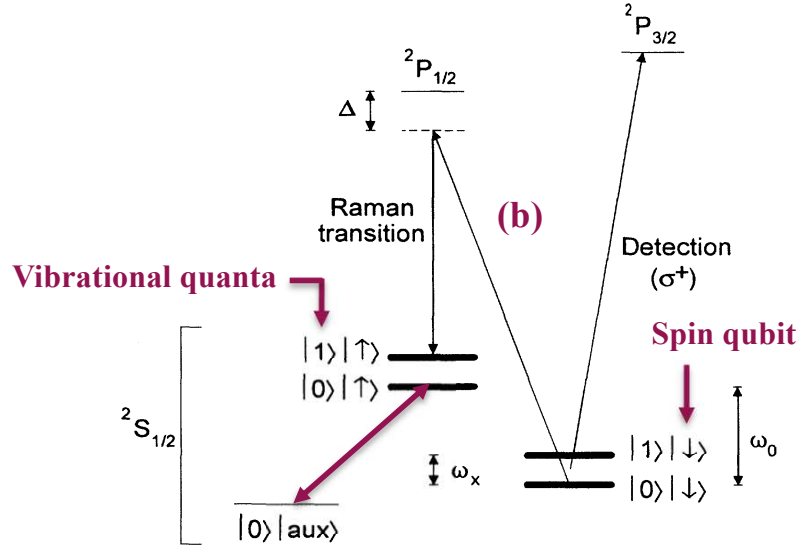
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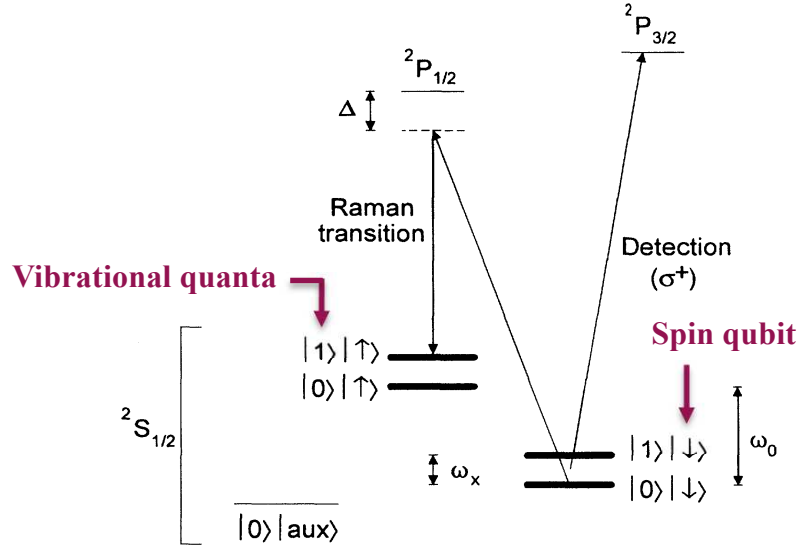
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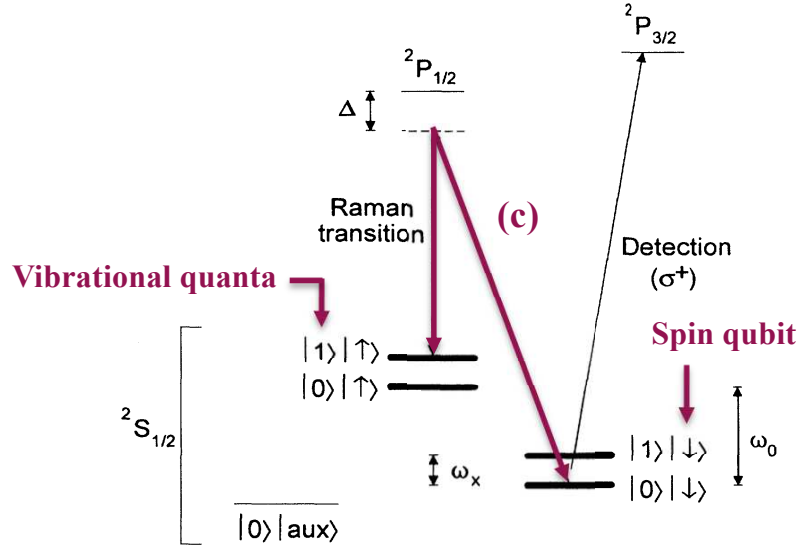
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FIG. 1. ^{9}Be$^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ ($^2S_{1/2}|F = 2, m_F = 2\rangle$ and $^2S_{1/2}|F = 1, m_F = 1\rangle$ levels, respectively, separated by $\omega_0/2\pi \simeq 1.250$ GHz), and $|aux\rangle = {}^2S_{1/2}|F = 2, m_F = 0\rangle$ (separated from $|\downarrow\rangle$ by $\simeq2.5$ MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \simeq 11.2$ MHz). Stimulated Raman transitions between $^2S_{1/2}$ hyperfine states are driven through the virtual $^2P_{1/2}$ level ($\Delta \simeq 50$ GHz) with a pair of $\simeq313$ nm laser beams. Measurement of S is accomplished by driving the cycling $|\downarrow\rangle \rightarrow {}^2P_{3/2}|F = 3, m_F = 3\rangle$ transition with σ^+-polarized light and detecting the resulting ion fluorescence.

according to the following format:

(a) A $\pi/2$ pulse is applied on the carrier transition. The effect is described by the operator $V^{1/2}(\pi/2)$ in the notation of Ref. [1].

(b) A 2π pulse is applied on the blue sideband transition between $|\uparrow\rangle$ and an auxiliary atomic level $|aux\rangle$ (see Fig. 1). (1)

(c) A $\pi/2$ pulse is applied on the carrier transition, with a π phase shift relative to (a), leading to the operator $V^{1/2}(-\pi/2)$ of Ref. [1].

is as follows:

$$\text{Input state} \longrightarrow \text{ Output state}$$

$$|0\rangle|\downarrow\rangle \rightarrow |0\rangle|\downarrow\rangle$$
$$|0\rangle|\uparrow\rangle \rightarrow |0\rangle|\uparrow\rangle \qquad (2)$$
$$|1\rangle|\downarrow\rangle \rightarrow |1\rangle|\uparrow\rangle$$
$$|1\rangle|\uparrow\rangle \rightarrow |1\rangle|\downarrow\rangle.$$

The experiment apparatus is described elsewhere [16,17]. A single ^{9}Be$^+$ ion is stored in a coaxial-resonator rf-ion trap [17], which provides pseudopotential oscillation frequencies of $(\omega_x, \omega_y, \omega_z)/2\pi \simeq (11.2, 18.2, 29.8)$ MHz along the principal axes of the trap. We cool the ion so that the $n_x = 0$ vibrational ground state is occupied $\simeq95\%$ of the time by employing resolved-sideband stimulated Raman cooling in the x dimension, exactly as in Ref. [16]. The two Raman beams each contain $\simeq1$ mW of power at $\simeq313$ nm and are detuned $\simeq50$ GHz red of the $^2P_{1/2}$ excited state. The Raman beams are applied to the ion in directions such that their wave-vector difference $\boldsymbol{\delta k}$ points nearly along the x axis of the trap; thus the Raman transitions are highly insensitive to motion in the other two dimensions. The Lamb-Dicke parameter is $\eta_x = \delta k\, x_0 \simeq 0.2$, where $x_0 \simeq 7$ nm is the spread of the $n_x = 0$ wave function. The carrier $(|n\rangle|\downarrow\rangle \rightarrow |n\rangle|\uparrow\rangle)$ Rabi frequency is $\Omega_0 2\pi \simeq 140$ kHz, the red $(|1\rangle|\downarrow\rangle \rightarrow |0\rangle|\uparrow\rangle)$ and blue $(|0\rangle|\downarrow\rangle \rightarrow |1\rangle|\uparrow\rangle)$ sideband Rabi frequencies are $\eta_x\Omega_0/2\pi \simeq 30$ kHz, and the auxiliary transition $(|1\rangle|\uparrow\rangle \rightarrow |0\rangle|\downarrow\rangle)$ Rabi frequency is $\eta_x\Omega_{aux}/2\pi \simeq 12$ kHz. The difference frequency of the Raman beams is tunable from 1200 to 1300 MHz with the use of a double pass acousto-optic modulator (AOM), and the Raman pulse durations are controlled with additional switching AOMs. Since the Raman beams are generated from a single laser and an AOM, broadening of the Raman transitions due to a finite laser linewidth is negligible [18].

Following Raman cooling to the $|0\rangle|\downarrow\rangle$ state, but before application of the CN operation, we apply appropriately
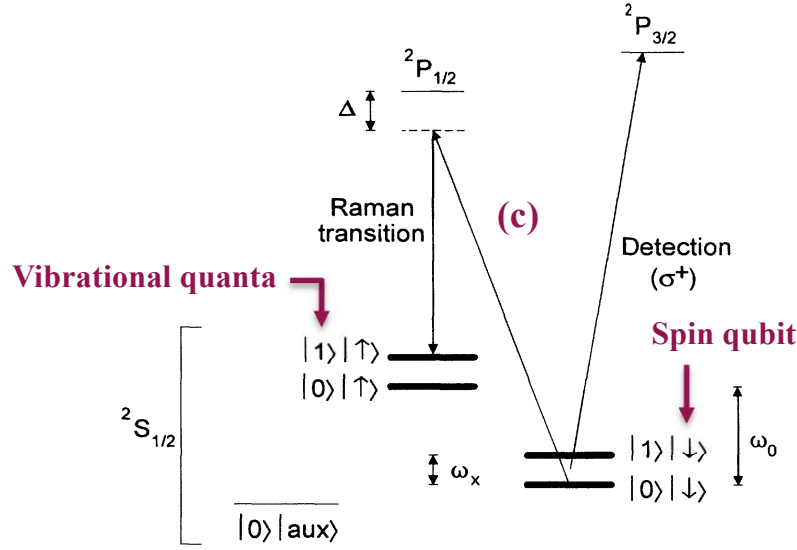
54

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Following Raman cooling to the $|0\rangle|\downarrow\rangle$ state, but before application of the CN operation, we apply appropriately

55

Some Benchmarks for Ion Traps

TABLE I. Selected state-of-the-art gate demonstrations.

Gate Type	Gate Method	Fidelity	Gate Time (μs)	Ion Species	Ref.
Single-Qubit					
	Optical	0.99995	5	^{40}Ca$^+$	[28]
	Raman	0.99993	7.5	^{43}Ca$^+$	[27]
	Raman	0.99996	2	^{9}Be$^+$	[37]
	Raman	0.99	0.00005	^{171}Yb$^+$	[163]
	Raman	0.999	8	^{88}Sr$^+$	[113]
	Microwave	0.999999	12	^{43}Ca$^+$	[22]
	Microwave		0.0186	^{25}Mg$^+$	[164]
Two-Qubit (1 species)					
	Optical	0.996	–	^{40}Ca$^+$	[38]
	Optical	0.993	50	^{40}Ca$^+$	[8]
	Raman	0.9991(6)	30	^{9}Be$^+$	[37]
	Raman	0.999	100	^{43}Ca$^+$	[27]
	Raman	0.998	1.6	^{43}Ca$^+$	[40]
	Raman	0.60	0.5	^{43}Ca$^+$	[40]
	Microwave (AC B-field gradient)	0.997	3250	^{43}Ca$^+$	[165]
	Microwave (DC B-field gradient)	0.985	2700	^{171}Yb$^+$	[166]
Two-Qubit (2 species)					
	Raman/Raman	0.998(6)	27.4	^{40}Ca$^+$/^{43}Ca$^+$	[167]
	Raman/Raman	0.979(1)	35	^{9}Be$^+$/^{25}Mg$^+$	[168]

larization (or frequency) does not allow it to couple to e ground state, and it cannot be rotated in $|0\rangle|n = 0\rangle$ cause the drive is red-detuned. A final π-pulse on the ntrol ion will return the control ion to its initial state. e resulting state transformation looks like:

$$|0\rangle_c|0\rangle_t \to |0\rangle_c|0\rangle_t \tag{1}$$
$$|0\rangle_c|1\rangle_t \to |0\rangle_c|1\rangle_t \tag{2}$$
$$|1\rangle_c|0\rangle_t \to |1\rangle_c|0\rangle_t \tag{3}$$
$$|1\rangle_c|1\rangle_t \to -|1\rangle_c|1\rangle_t \tag{4}$$

e gate thus inverts the phase of only the $|1\rangle|1\rangle$ state, alizing an entangling controlled-phase interaction. Be- es cooling to the motional ground state, the CZ gate quires individual addressing of each ion and multiple larizations for the drive laser. Despite these limita- ns, a modified CZ interaction was demonstrated the ne year it was proposed [4], entangling the internal te and motional state of a single ^{9}Be$^+$ ion. In 1998, wo-ion entangling gate with fidelity of 0.7 was demon- ated between two Be$^+$ ions with gate time of $\sim 10\,\mu s$, while a Cirac-Zoller gate and single-qubit rotations re used to implement the CNOT operations on two

The requirement that the ions remain in the motiona ground state is a significant limitation on the origina Cirac-Zoller proposal. As discussed in Sec. II C 2, eve when the ions have been cooled to the motional groun state, they can be subsequently heated by electric-fiel noise. In 1999, Mølmer and Sørensen introduced controlled-phase gate which could be implemented with out the need to be in the motional ground state [25]. Th Mølmer-Sørensen (MS) gate generates a state-depender force with bichromatic laser fields tuned near first-orde sideband transitions. The motional-state wavepacket e ecutes a closed trajectory in phase space, giving rise t a state-dependent geometric phase. At the conclusion the gate, internal and motional states are disentangle for all values of n. Hence, the MS gate can be used f ions that are not cooled to the motional ground stat An additional feature of the MS interaction is that er tanglement among multiple ions can be generated usin only global control lasers (that is, it does not requir lasers independently focused on each ion). The MS er tangling gate was first demonstrated for chains of 2 an 4 Be$^+$ ions in 2000 [6]. To date, the highest-achieve fidelities in both optical and hyperfine two-qubit gate

Some links to get started

Amazon Braket   (IonQ, other Technologies)
   https://aws.amazon.com/braket/

Quantinuum   (Ion Trap Quantum Computing
   https://www.quantinuum.com

IonQ   https://ionq.com

NIST   https://www.nist.gov/pml/time-and-frequency-division/ion-storage
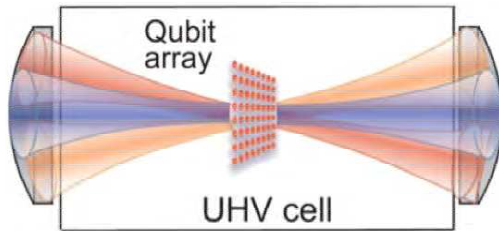
Challenge: Do a web search and look for the largest GHZ state made in the lab

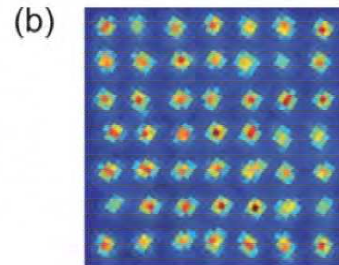$$|GHz\rangle = \frac{1}{\sqrt{N}} \left(|00...00\rangle + |11...11\rangle \right)$$

Note: What is the fidelity of the state ?

Introduction and Overview (Preskills Notes)

Neutral Atom based Quantum Processors

2-photon transition to a Rydberg state

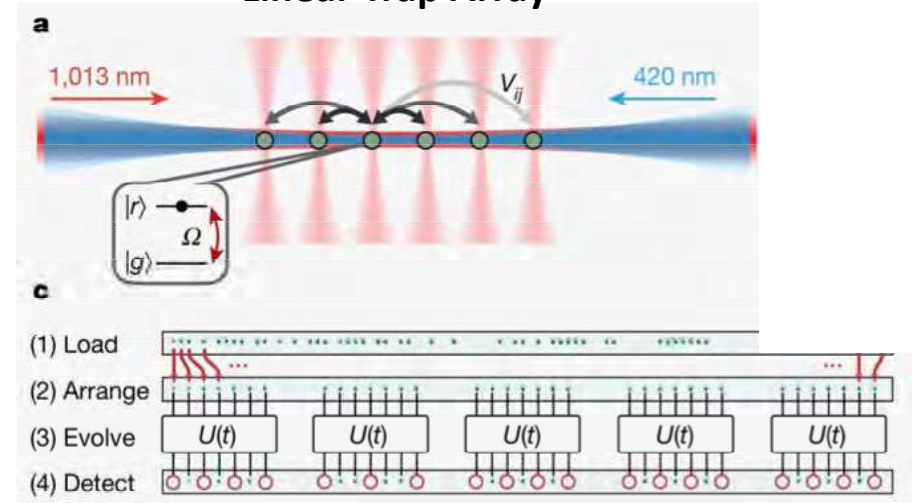**Linear Trap Array**

Optical tweezer array
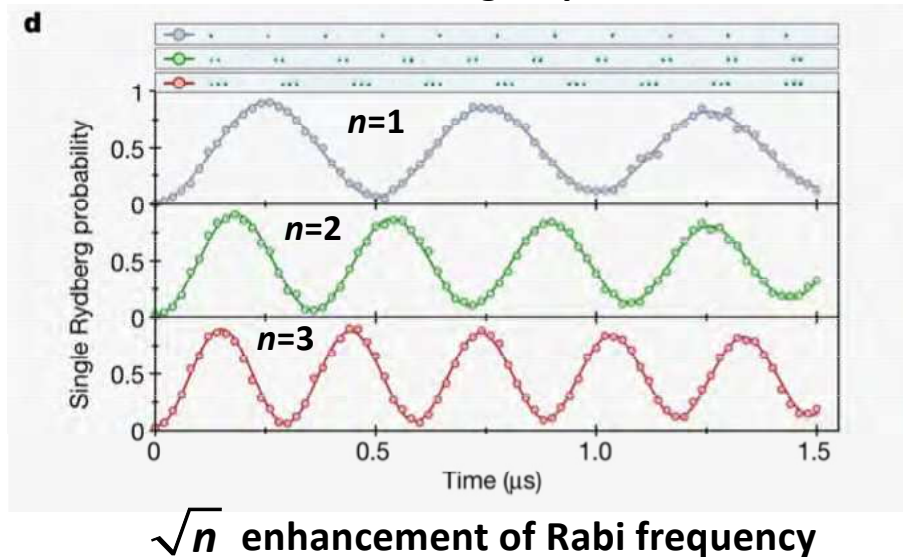
Fluorescing atoms

* **Large numbers of non-interacting qubits, (~ 256) trapped in 2D or 3D arrays.**

* **Qubits interact when excited into Rydberg states with large dipole moments**

* **Major advantage: Weak coupling to the environment when not doing gates**
  ➡ **excellent quantum memory**

* **Favorite platform for quantum simulation of quantum manybody physics**

* **BEC's in optical lattices as analog simulators of superconductivity, quantum magnetism and more**

Sorted into groups of n

$\sqrt{n}$ **enhancement of Rabi frequency**

Introduction and Overview (Preskills Notes)

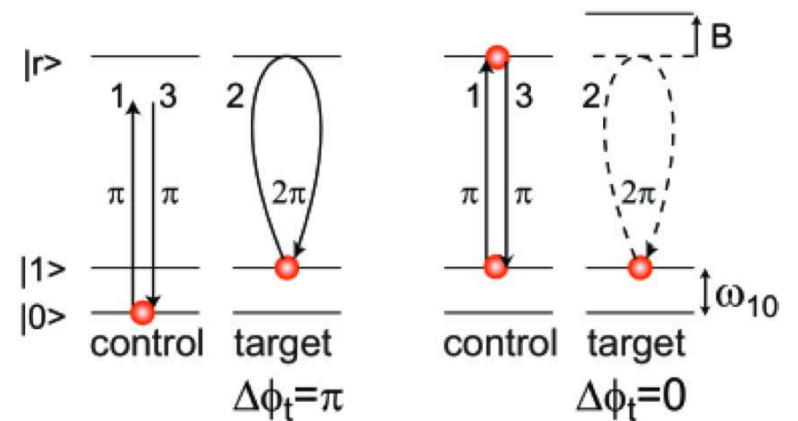## Linear Trap Array

Sorted into groups of _n_
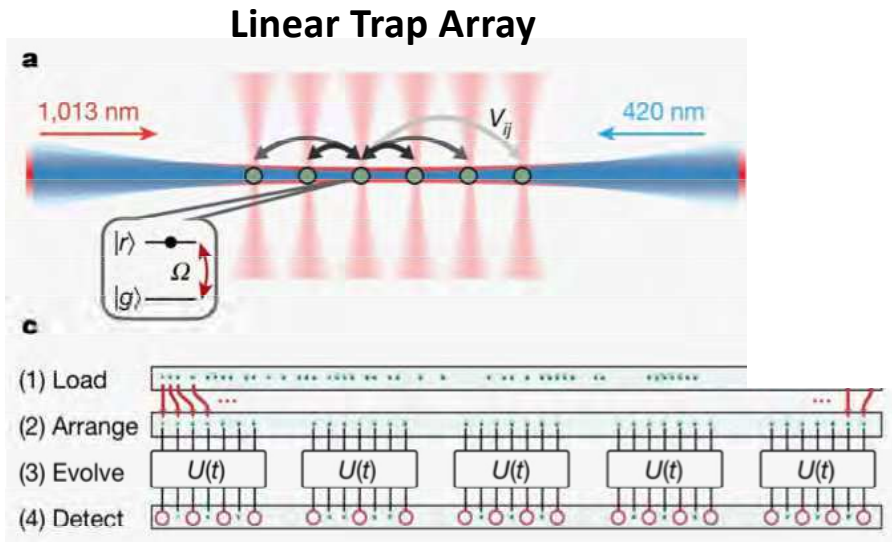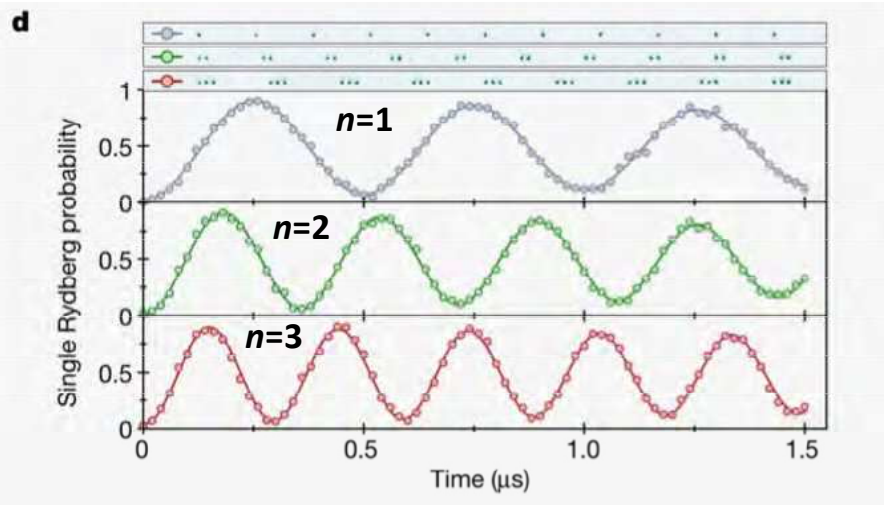
$\sqrt{n}$ **enhancement of Rabi frequency**
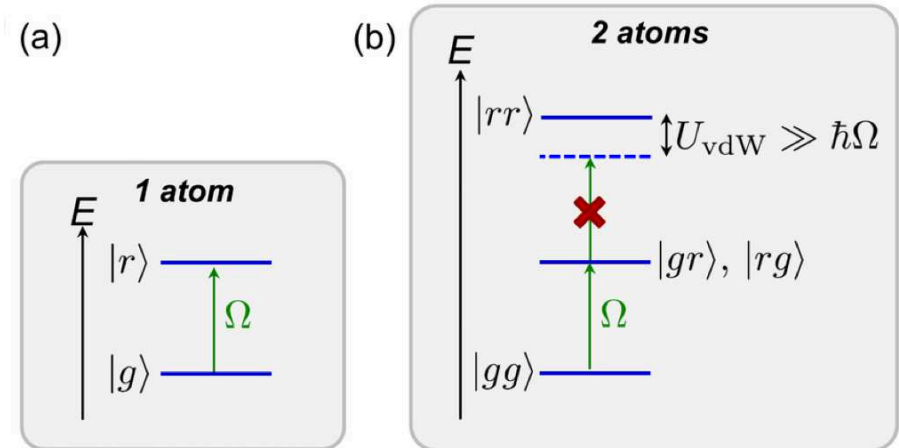
Rydberg Blockade
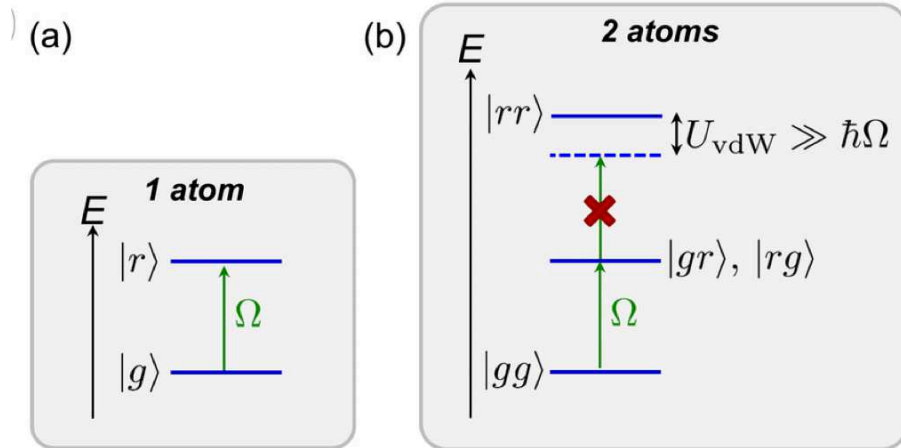

Principle of the Rydberg blockade. (a) A resonant laser couples, with strength Ω, the Rydberg state |r and the ground state |g of an atom. (b) For two nearby atoms, interactions U vdW shift the doubly excited state |rr, preventing the double excitation of the atom pair when U vdW Ω.

Controlled-Phase Gate

π phase shift of Target conditioned on Control

Introduction and Overview (Preskills Notes)

Rydberg Blockade

(a)

(b) E **2 atoms**

$|rr\rangle$ ——— $\updownarrow U_{vdW} \gg \hbar\Omega$

$|gr\rangle, |rg\rangle$

Ω

$|gg\rangle$

1 atom

E

$|r\rangle$ ———

Ω

$|g\rangle$

Principle of the Rydberg blockade. (a) A resonant laser couples, with strength Ω, the Rydberg state |r and the ground state |g of an atom. (b) For two nearby atoms, interactions U vdW shift the doubly excited state |rr, preventing the double excitation of the atom pair when U vdW Ω.

Controlled-Phase Gate
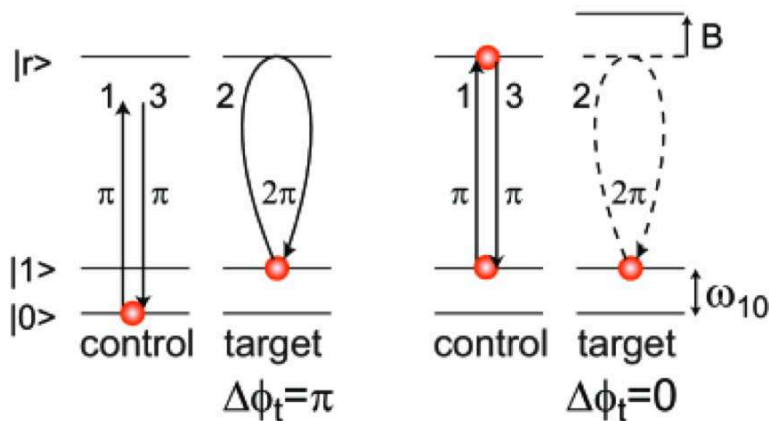
π phase shift of Target conditioned on Control

Some links to get started

QuEra   https://www.quera.com/aquila

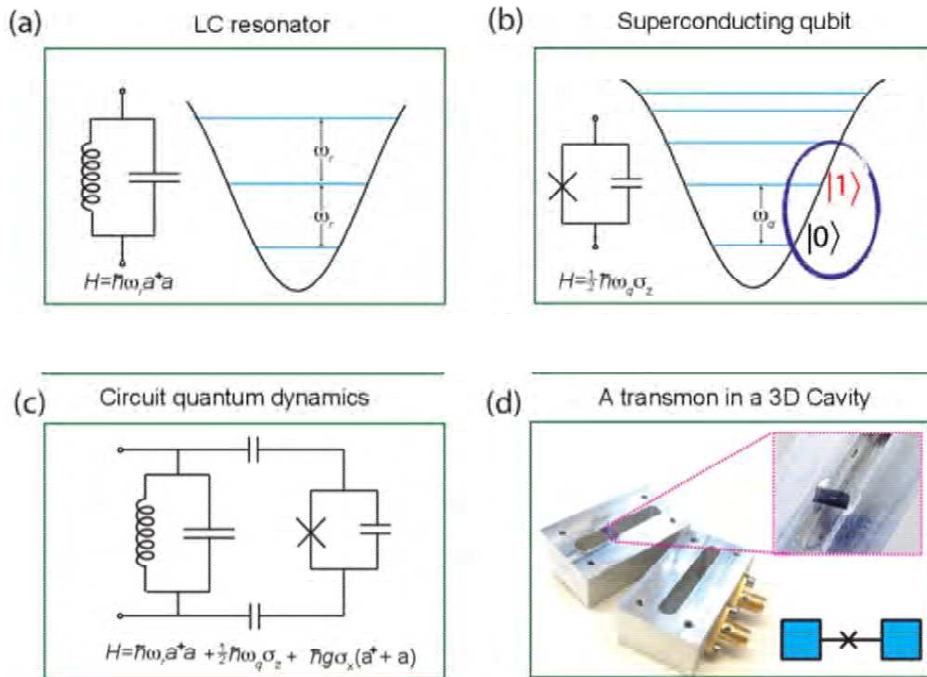**Cold Quanta**   https://coldquanta.com

Sandia, Los Alamos National Labs

Individual PI's – Quantum Simulation

Lukin, Vuletic, Greiner, Endres, Bloch, Saffman, Biederman, Browaeys, Weiss, and many, many others…
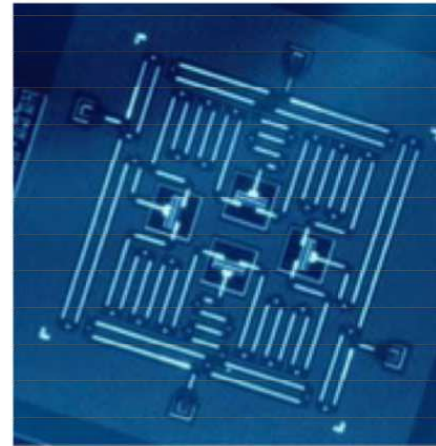
Superconducting Qubits

❋ The basic building block is the so-called Transmon Qubit

**❋ A Transmon is a nonlinear oscillator made from a Josephson Junction and other circuit elements**

(a) LC resonator
$H = \hbar\omega_r a^\dagger a$

(b) Superconducting qubit
$H = \frac{1}{2}\hbar\omega_q \sigma_z$
$|1\rangle$
$|0\rangle$

(c) Circuit quantum dynamics
$H = \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\omega_q \sigma_z + \hbar g \sigma_x (a^\dagger + a)$

(d) A transmon in a 3D Cavity

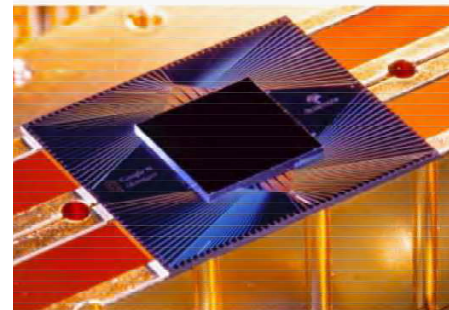**Jaynes-Cummings Hamiltonian**

Superconducting Qubits

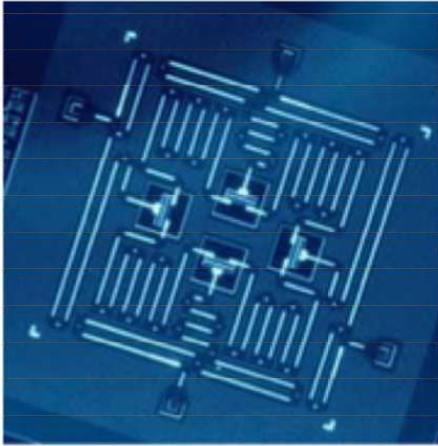### IBM 4–Transmon device (2017)

A device consisting of four transmon qubits, four quantum busses, and four readout resonators fabricated by IBM and published in npj Quantum Information in January 2017.[4]

Google 54–Transmon device (2019)
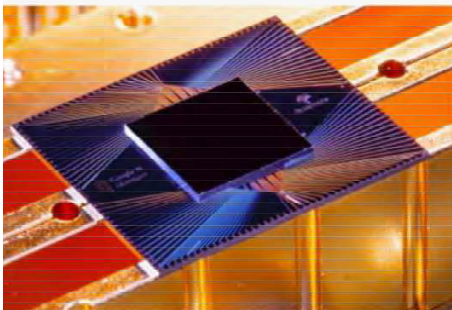

Qubit Adjustable coupler
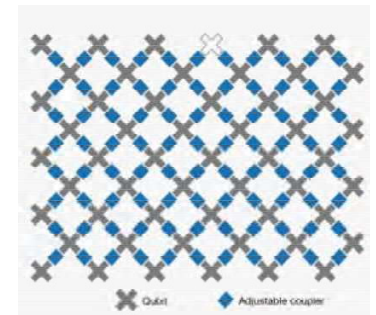
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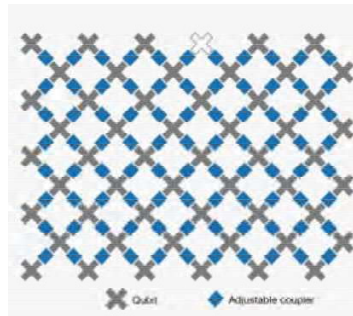
Advantages

* Solid State platform, looks like electronics

* Clearer path to scale up to many qubits?

Challenges

* Gates, coherence times not as good as atomic platforms, but gap is closing

* Requires dilution refrigerator

Industry Favorite

* Large efforts at IBM, Google, Rigetti

Cloud Quantum Computing

* **Amazon Braket**   (IonQ, other Technology)

https://aws.amazon.com/braket/

61