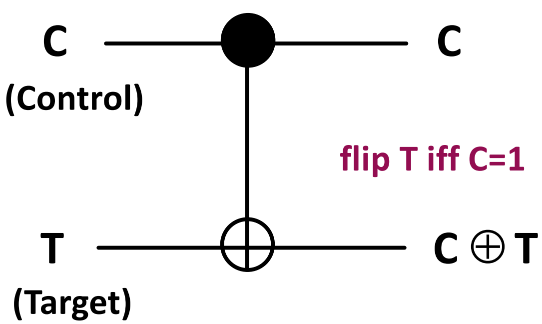
Introduction and Overview (Preskills Notes)

9-03-2024

Controlled-NOT (CNOT)



Truth Table

C	T	$C \oplus T$
0	0	0
0	1	1
1	0	1
1	1	0

Circuit maps logical basis states as

C	T
0	0
1	1
0	1
1	0

→ CNOT →

C	T
0	0
1	0
0	1
1	1

→ meas. T →

C	T
0	0
1	0
0	1
1	1

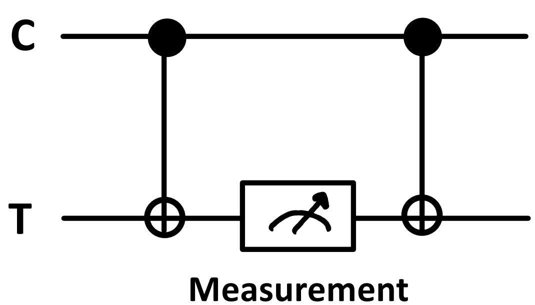
→ CNOT →

C	T
0	0
1	1
0	1
1	0

$T=0$ iff same

$T=1$ iff different

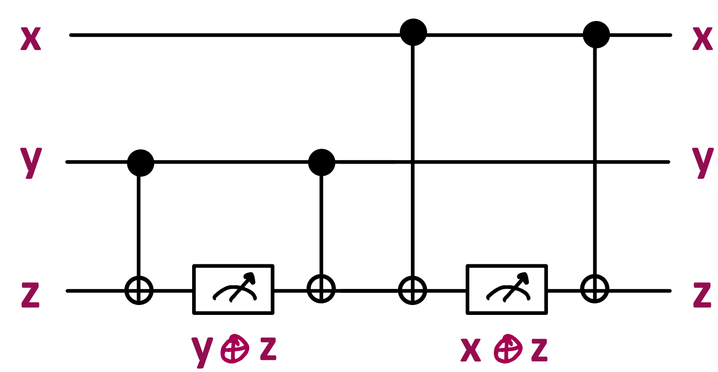
Quantum Circuit for joint measurement



Measurement in $\{|0\rangle, |1\rangle\}$ basis

yields $C \oplus T$

Full circuit to obtain Error Syndrome



* iff qubit flip, binary address = $(y \oplus z, x \oplus z)$

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Circuit maps logical basis states as

CNOT → meas. T → CNOT

C	T
0	0
1	1
0	1
1	0

C	T
0	0
1	0
0	1
1	1

C	T
0	0
1	0
0	1
1	1

C	T
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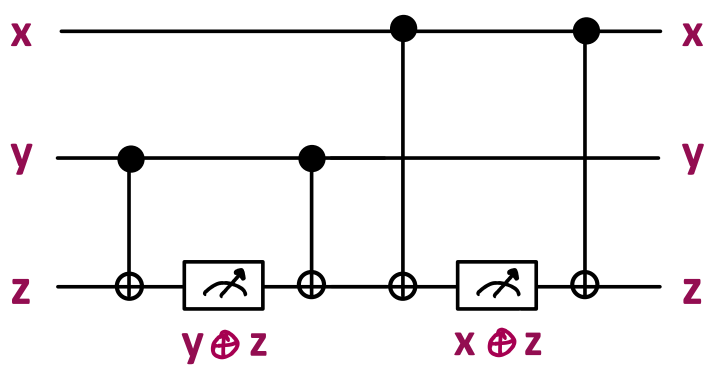
Full circuit to obtain Error Syndrome



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Quantum Phase Error

$|0\rangle \to |0\rangle$

$|1\rangle \to -|1\rangle$

Encoding

$$|0\rangle \to |\bar{0}\rangle = \frac{1}{2^{3/2}} \underbrace{(|0\rangle+|1\rangle)}_{x'}\underbrace{(|0\rangle+|1\rangle)}_{y'}\underbrace{(|0\rangle+|1\rangle)}_{z'}$$

$$|1\rangle \to |\bar{1}\rangle = \frac{1}{2^{3/2}} (|0\rangle-|1\rangle)(|0\rangle-|1\rangle)(|0\rangle-|1\rangle)$$

Relabel

$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) = |0'\rangle$$

$$\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) = |1'\rangle$$

Measure in basis

$$\{|0'\rangle, |1'\rangle\} \to y' \oplus z', \; x' \oplus z'$$

Error Syndrome

* Iff phase error, binary address $= (y' \oplus z', x' \oplus z')$
* Analogous to bit-flip code, just in different basis

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Quantum Phase Error

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- Iff phase error, binary address $= (y' \oplus z', x' \oplus z')$
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Shor's 9-bit code

- Combines flip/phase error correction
- Corrects one flip or phase error

General principle of error correction

- Encode *p* logical qubits in *n* physical qubits.
- Valid Logical States form 2^p–dimensional subspace $\mathcal{E}_p$ (code space) in *n*-qubit (2^n–dimensional) Hilbert space $\mathcal{E}_N$
- Errors displace system into orthogonal (distinguishable) subspaces.

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Shor's 9-bit code

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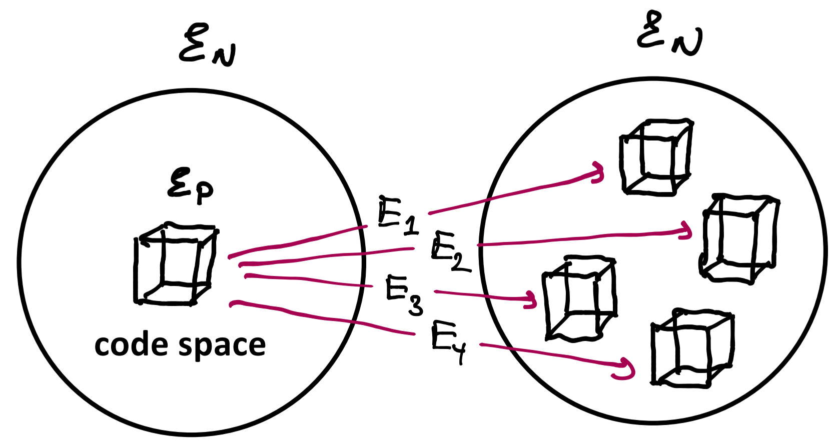
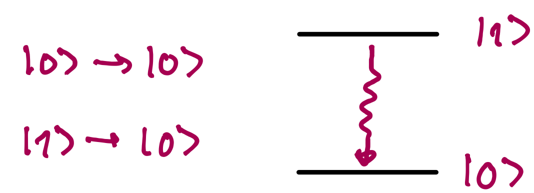
Geometric illustration



What about non-Unitary errors?

e. g., decay



Problem: Errors not displaced into orthogonal subspaces

Solution: "Quantum jump codes", monitors the environment

Other kinds of errors?

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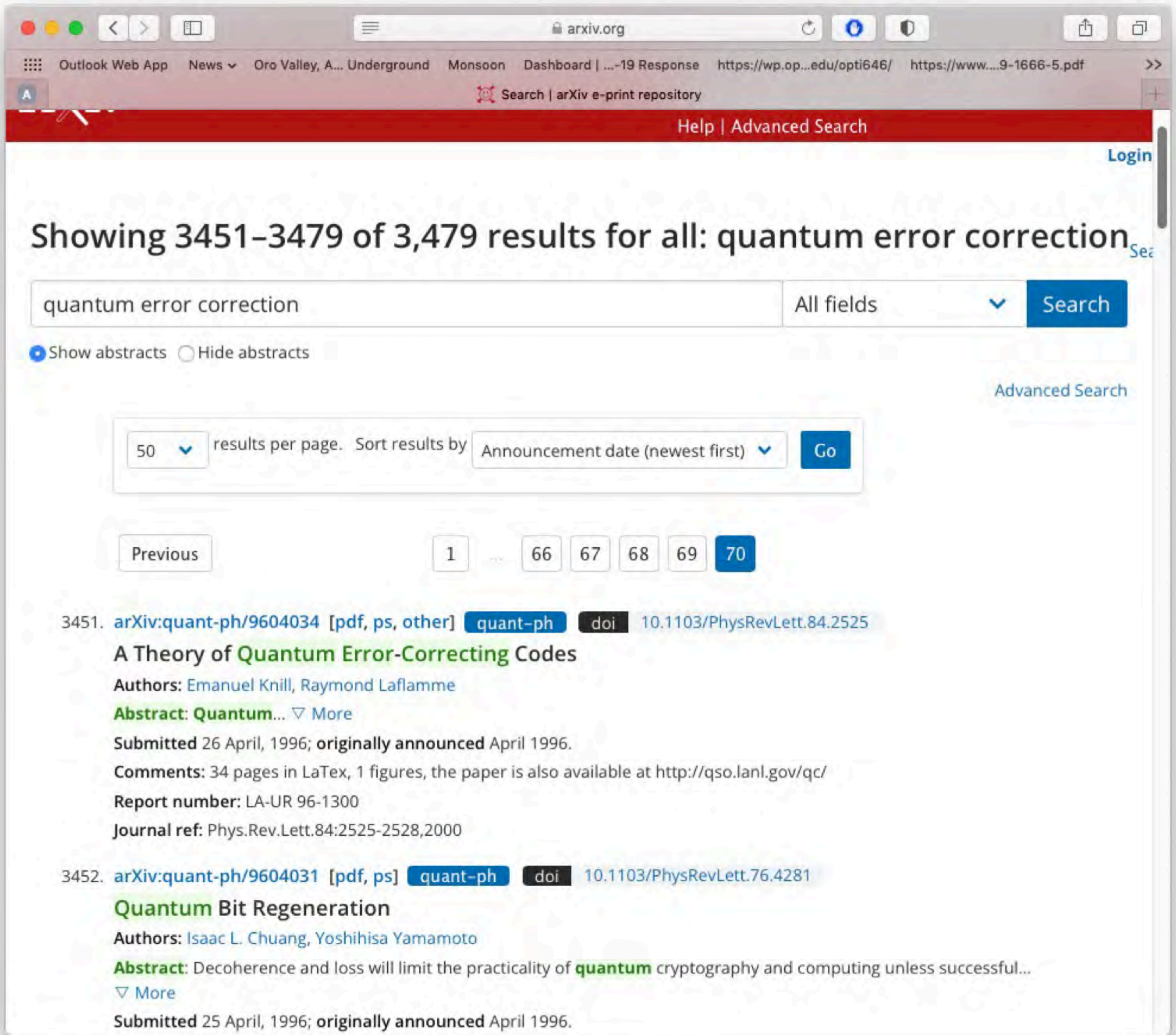
Catnip for Theoretical Physicists & Computer Scientists



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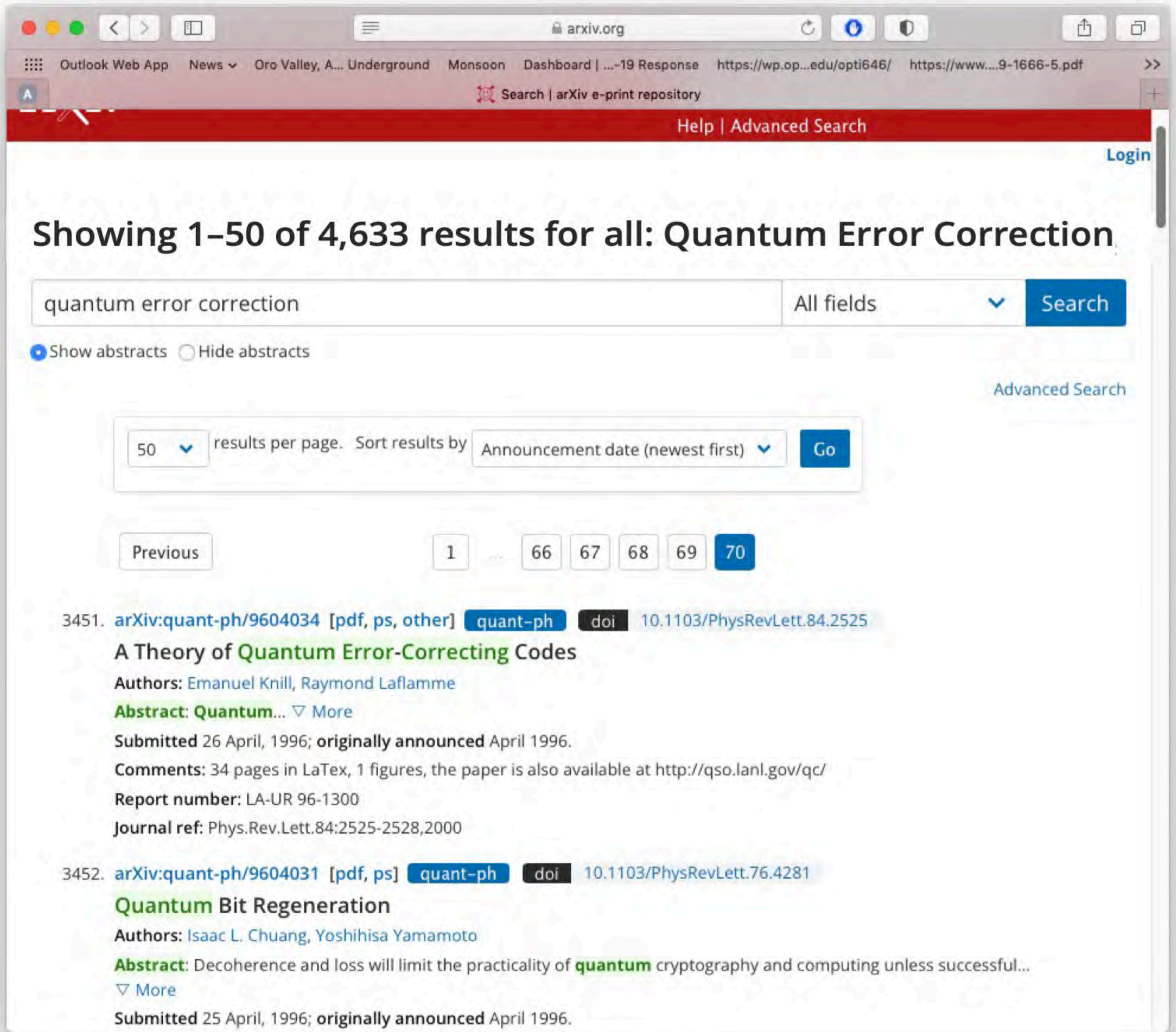

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Quantum Hardware

Physical Implementation is Extremely demanding !

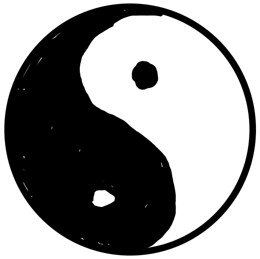
Requirements

1. Storage: Quantum memory.
2. Gates: We put computation U_f together from 1 and 2-qubit operations.
3. Readout: Method to measure qubits.
4. Isolation: No coupling to environment to avoid decoherence & errors
5. Precision: Gates, readouts must be highly accurate

Inherent Contradictions

2. Gates vs **4. Isolation**

coupling between qubits — no coupling to environment



To build a Quantum Computer:

Choose, find or invent a system with acceptable tradeoffs.

6. Error Correction must not create more errors than it corrects.

7. Thresholds for Error Correction and Fault Tolerance

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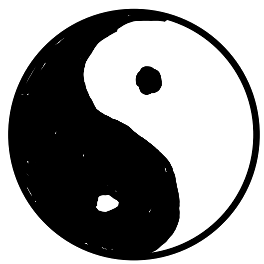
Inherent Contradictions

2. Gates vs **4. Isolation**

coupling between qubits — no coupling to environment



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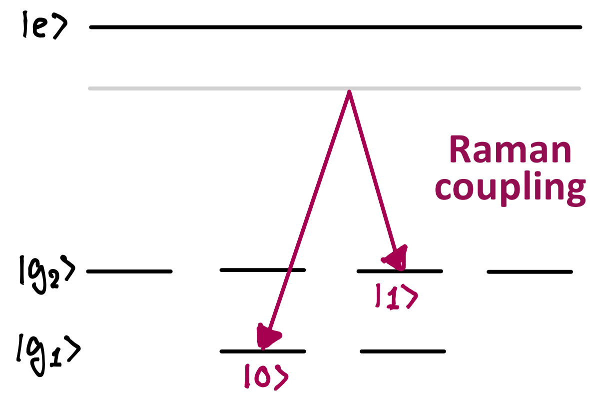
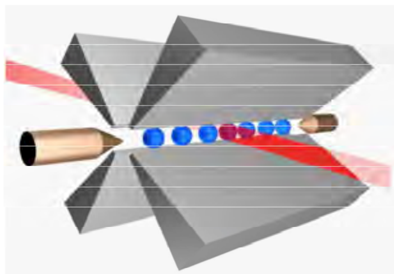
Ion Trap Quantum Computing

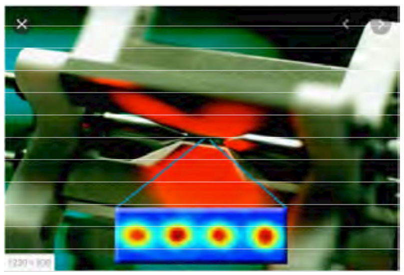
First to demonstrate a Quantum Gate

* Qubit is encoded in the electronic ground state of an atomic ion



* Early design with a few ions in large trap





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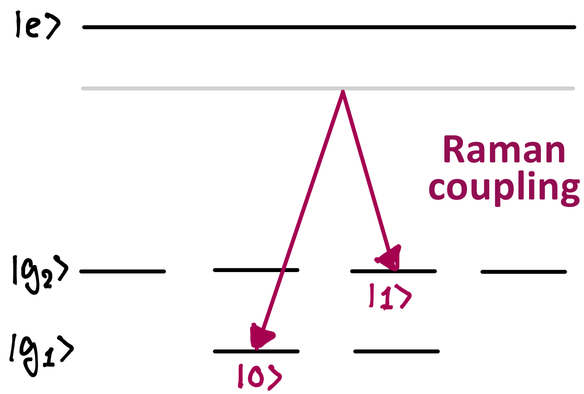

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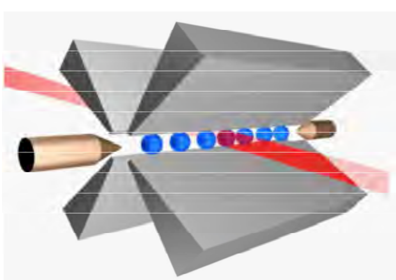
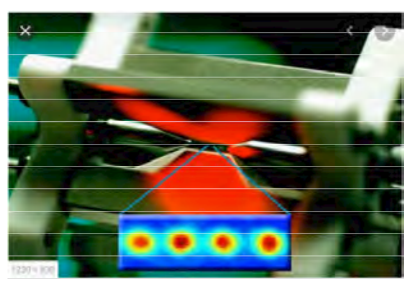
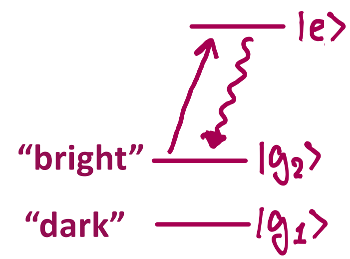




Requirements

1. Storage: 10s-100s coherence time
2. Gates: Use collective vibrations as "quantum bus"
3. Readout: Fluorescence



Cirac & Zoller: 5 laser pulses

CNOT gate between any 2 ions in linear array

Wineland: 3 laser pulses enough for CNOT

Use this example serves as conceptual template

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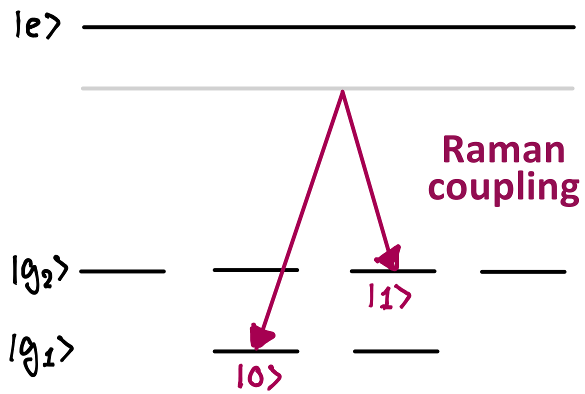

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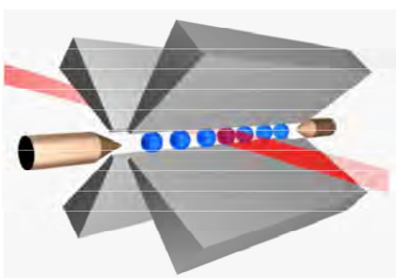
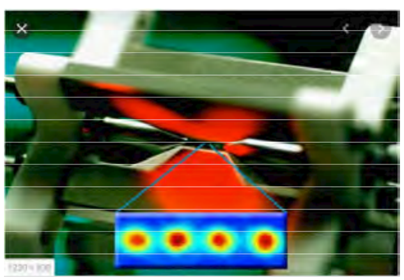
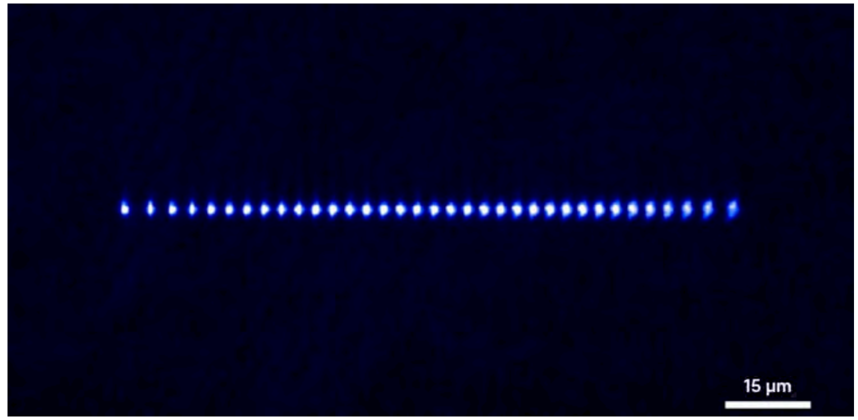




* Scaling up in Linear Ion Traps



* Limitations