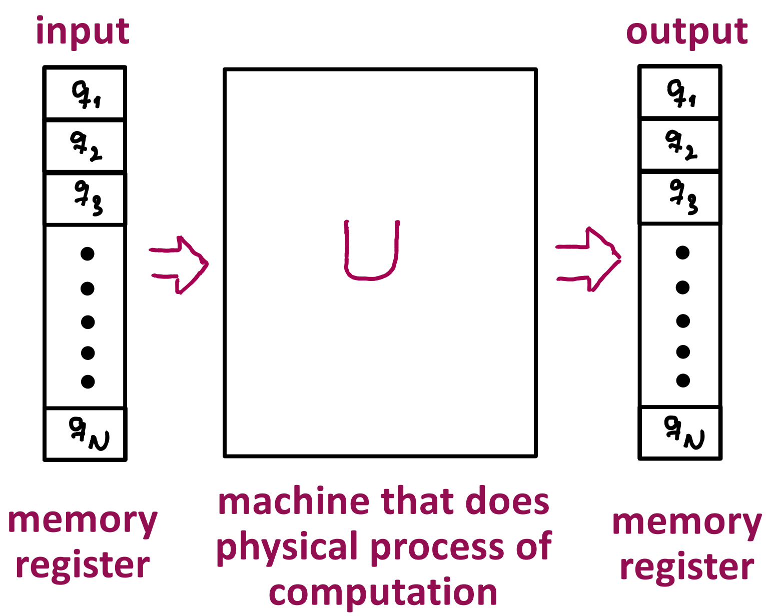
Introduction and Overview (Preskills Notes)

OK – Plausible QM can do more

Where does the QC's power come from?

Visualization of Computation



Classical: Register is in one of the logical states $\underbrace{x = q_1 q_2 q_3 \dots q_N}_{\text{binary \#}}$

Reversible transformation $U: x \to y$

Quantum: Register can be in any coherent superposition of logical states $|x\rangle$

Unitary transformation $U: |x\rangle \to |y\rangle$

Maps basis to basis $U: \{|x\rangle\} \to \{|y\rangle\}$

Quantum Parallelism

$$|\psi_{in}\rangle \to \sum_x a_x |x\rangle \to$$

$$\to |\psi_{out}\rangle = U|\psi_{in}\rangle = \sum_x a_x |y\rangle = \sum_x b_x |x\rangle$$

Machine processes 2^N inputs "in parallel" !

Beware: measurement collapses Q. Register into a single basis state at random

$$\boxed{\text{We get one random result out of } 2^N}$$

Introduction and Overview (Preskills Notes)

8-29-2024

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Quantum Sampling Problem

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Quantum Algorithms look for global properties of functions – symmetry, periodicity, etc.

* Classical -> requires many function evaluations

* Quantum -> design U so measurement gives answer with high probability

* ∃ classes of problems (sampling problems) which are classically hard but quantum "easy"

Google "Quantum Supremacy"


Expert insight into current research

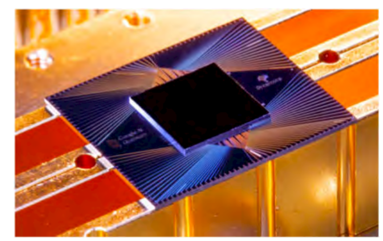
News & views

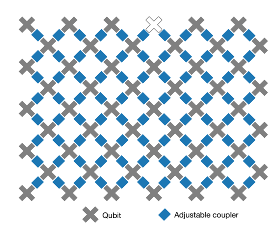
Quantum information

Quantum computing takes flight

William D. Oliver

A programmable quantum computer has been reported to outperform the most powerful conventional computers in a specific task – a milestone in computing comparable in importance to the Wright brothers' first flights. See p.505





Introduction and Overview (Preskills Notes)

9-03-2024

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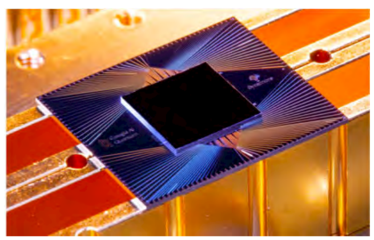
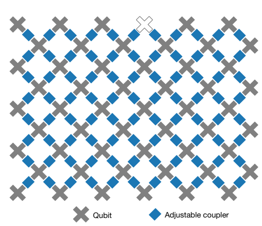
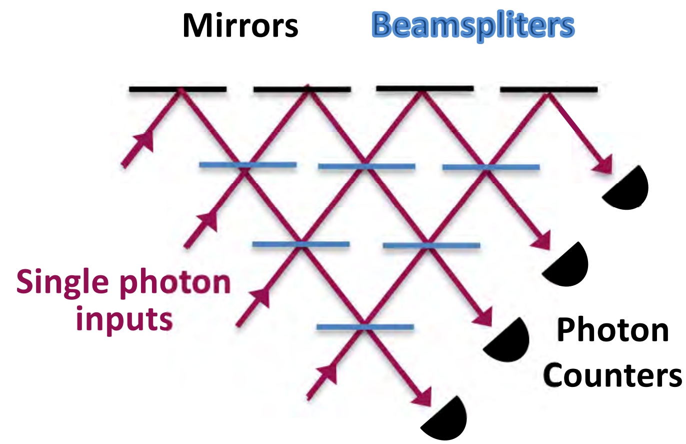




Boson Sampling

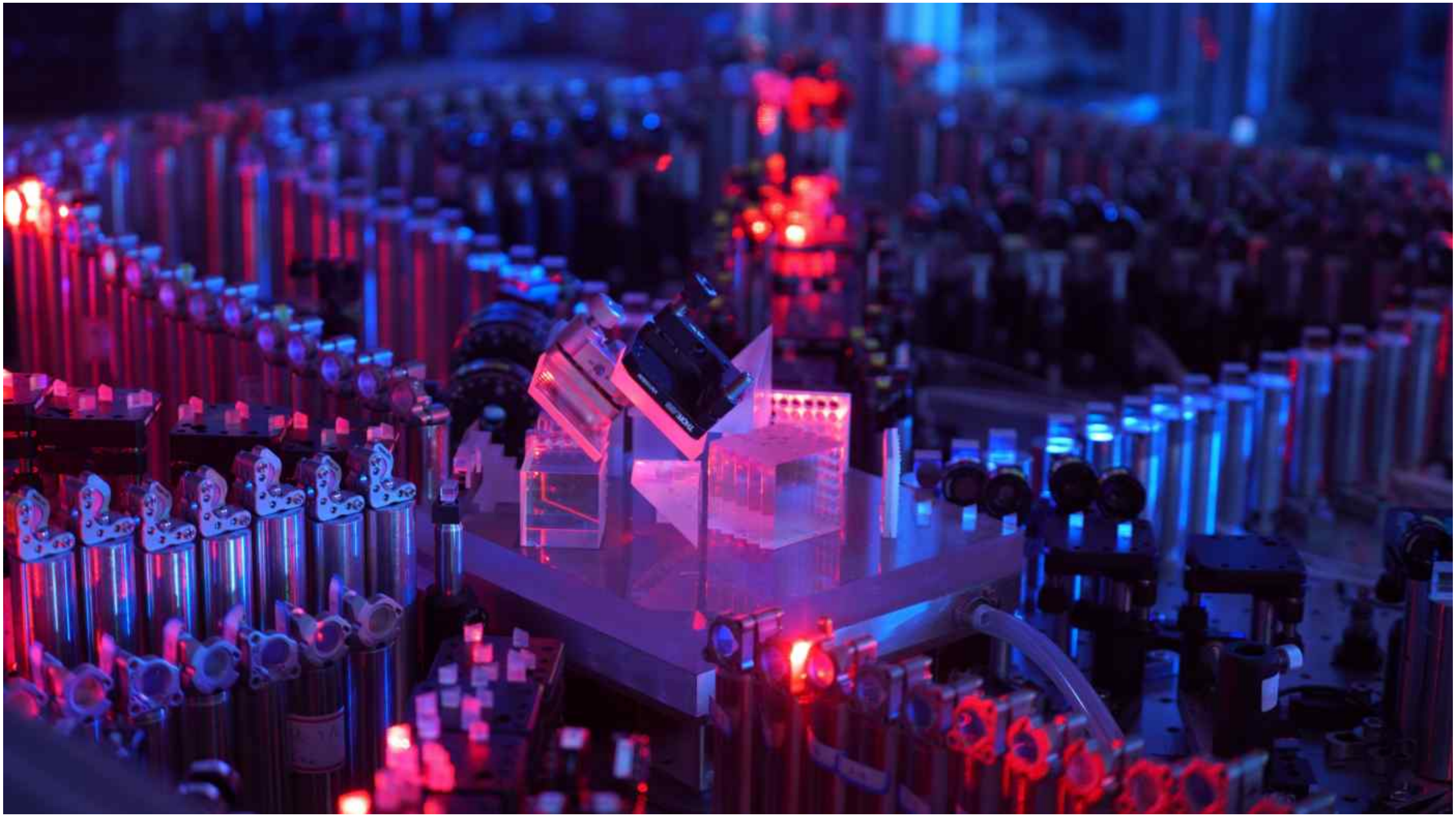
An example from Optics/Photonics

Setup



An optical quantum computer developed by a team of Chinese researchers including those from the University of Science and Technology of China. (courtesy of Han-Sen Zhong of the research group)

Introduction and Overview (Preskills Notes)



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Introduction and Overview (Preskills Notes)

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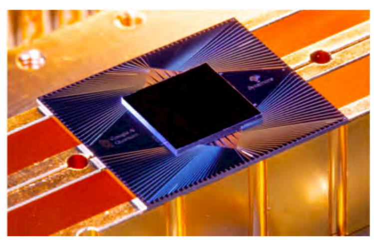

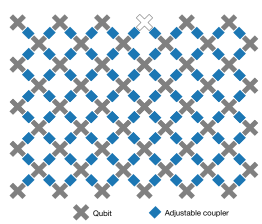

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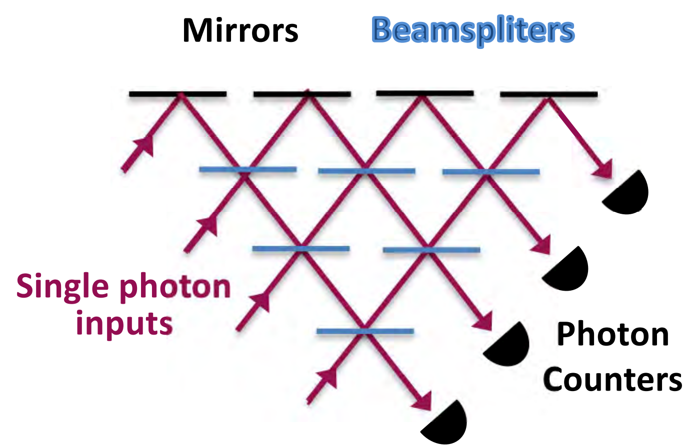
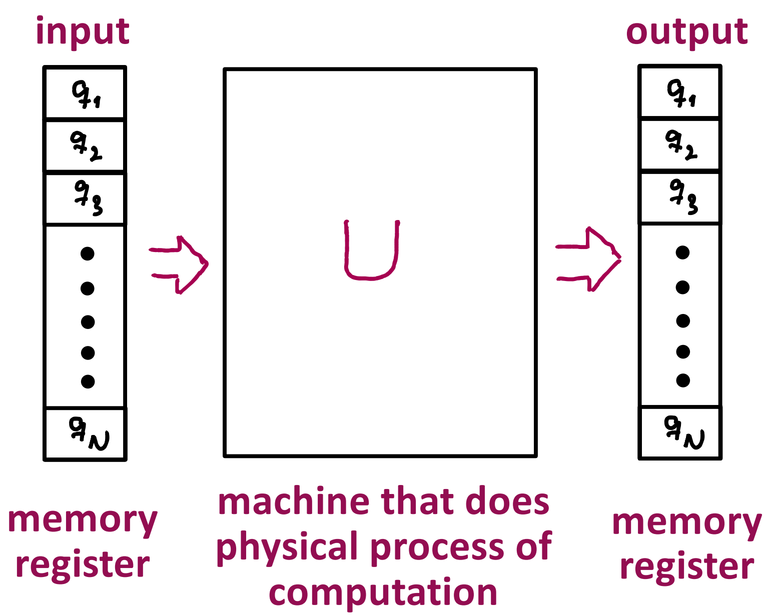


Imagine aligning that thing…!

Introduction and Overview (Preskills Notes)

Back to Universal Computation

Visualization of Computation

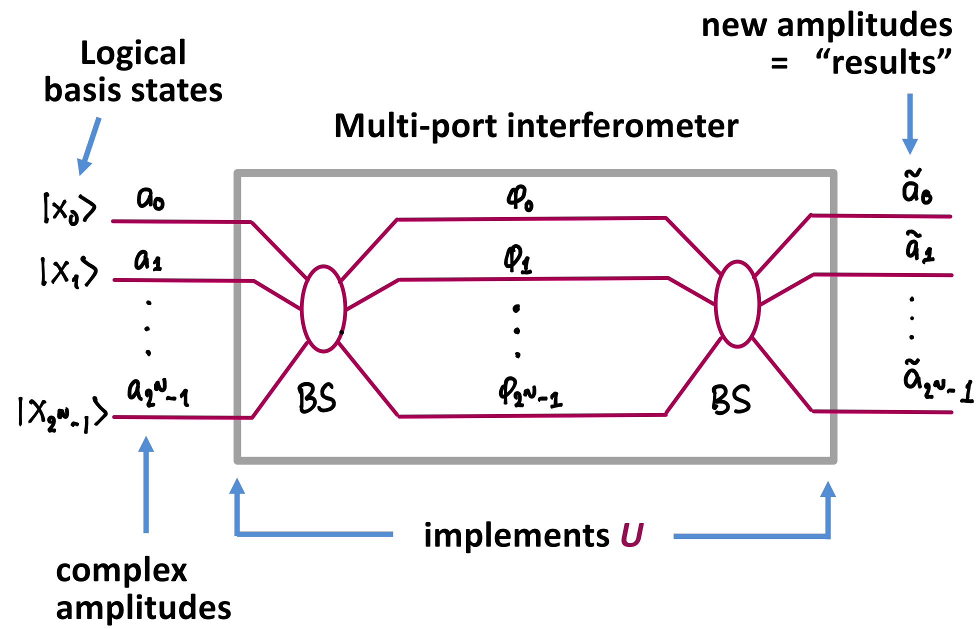


Classical: Register is in one of the logical states $\underbrace{x = q_1 q_2 q_3 \ldots q_N}_{\text{binary \#}}$

Reversible transformation $U: x \to y$

What might be inside the machine ?

Wave interference w/classical fields ?



Note: N - qubit register ➡ 2^N "paths"

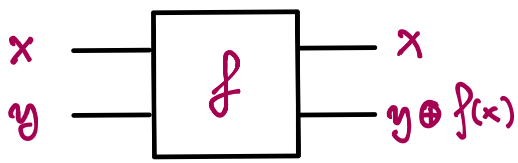
Beware of Resource Scaling !

Introduction and Overview (Preskills Notes)

Quantum Advantage

David Deutsch: Toy problem that shows Quantum Advantage

Oracle



(Black Box)

Is the function

constant $f(0) = f(1)$

balanced $f(0) \neq f(1)$

Classical Box: Need 2 queries $f(0)$ & $f(1)$

Introduction and Overview (Preskills Notes)

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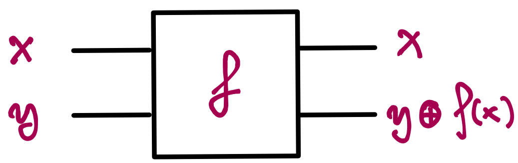

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Quantum Box: In 3 steps can show that

(1) $U_f: |x\rangle|y\rangle \to |x\rangle|y \oplus f(x)\rangle$

(2) $U_f: |x\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \to |x\rangle\frac{1}{\sqrt{2}}(|f(x)\rangle - |1 \oplus f(x)\rangle)$

$$= |x\rangle\frac{1}{\sqrt{2}}(-1)^{f(x)}(|0\rangle - |1\rangle)$$

(3) $U_f: \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$\to \frac{1}{2}\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)(|0\rangle - |1\rangle)$$

Quantum Advantage

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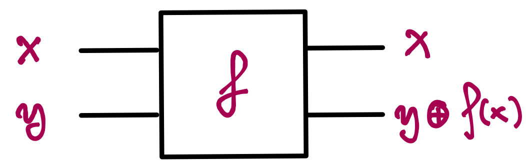

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Quantum Computation:

Input $|x\rangle|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$\xrightarrow{U_f} \left[(-1)^{f(1)}|0\rangle + (-1)^{f(0)}|1\rangle\right]\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Measure 1st qubit in basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$\longrightarrow$ $|+\rangle$ if constant, $|-\rangle$ if balanced

Quantum Speedup: can solve w/1 query

Introduction and Overview (Preskills Notes)

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Quantum Advantage

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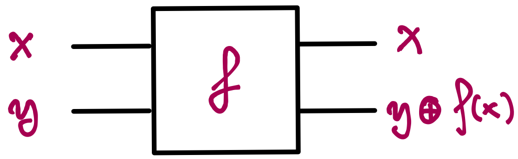

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$\boxed{\text{Quantum Speedup: can solve w/1 query}}$

Key aspect of Deutsch's algorithm:

We are looking for a global property of the function f

N bit binary number

Generally: $U_f : |x\rangle|0\rangle \rightarrow |x\rangle|f(x)\rangle$

Input $|\psi_{in}\rangle = \left[\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right]^{\otimes N}|0\rangle$

$$= \frac{1}{2^{N/2}}\sum_{x=0}^{2^N-1}|x\rangle|0\rangle$$

compute once

Output $|\psi_{out}\rangle = \frac{1}{2^{N/2}}\sum_{x=0}^{2^N-1}|x\rangle|f(x)\rangle$

$\boxed{\text{Global properties encoded in state, trick is to extract desired information}}$

Introduction and Overview (Preskills Notes)

Quantum Advantage

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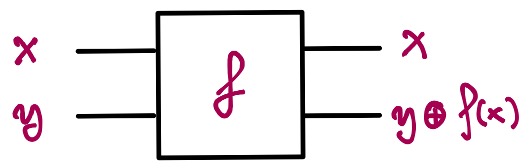

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Peter Shor: Period finding, QFT, Factoring

Introduction and Overview (Preskills Notes)

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Peter Shor:	**Period finding, QFT, Factoring**

Next: Will this work with real-world Quantum Hardware ?

Faulty gates, decoherence !

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Introduction and Overview (Preskills Notes)

Quantum Computation ➡

* Cannot tolerate dissipation
* Destroys superposition and entanglement

What to do?	Error Correction!

Classical Error Correction:

Simple example: Redundancy protects against bit flips

Encode: $0 \to (000)$
$1 \to (111)$

Errors: $(000) \to (100)$
$(111) \to (011)$

correct by majority vote

Von Neumann:

* A classical computer w/faulty components can work, given enough redundancy
* Classical error correction is well developed and highly sophisticated...

* Quantum Errors

1) Bit Flip $|0\rangle \to |1\rangle$, $|1\rangle \to |0\rangle$, phase flip $|0\rangle \to |0\rangle$, $|1\rangle \to -|1\rangle$
2) Small errors $a|0\rangle + b|1\rangle$ a, b can change by ε errors accumulate
3) Measurement disturbs ➡ collapse of quantum states
4) No cloning ➡ Cannot protect by making copies

Introduction and Overview (Preskills Notes)

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Example: Peter Shor's code for bit flip error when $P(\text{error}) \ll 1$

Encode: $|0\rangle \to |\bar{0}\rangle \equiv |000\rangle$, $|1\rangle \to |\bar{1}\rangle \equiv |111\rangle$ (3 bit code)

$$a|0\rangle + b|1\rangle \to a|000\rangle + b|111\rangle$$

Single-qubit measurement ⇨ collapse of state, destroys info, no majority voting !

Collective 2-qubit measurement:

- for $|x, y, z\rangle$ measure $y \oplus z$, $x \oplus z$ (never measure individual bits)

- if $|000\rangle$, $|111\rangle$ these observables $= 0$

- if one bit-flip, at least one observable $= 1$

- easy to check that $(y \oplus z, x \oplus z) =$ binary address of qubit flip

$|000\rangle \to |010\rangle$ $(1, 0) =$ 2nd bit

Introduction and Overview (Preskills Notes)

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Example: Peter Shor's code for bit flip error when P(error) << 1

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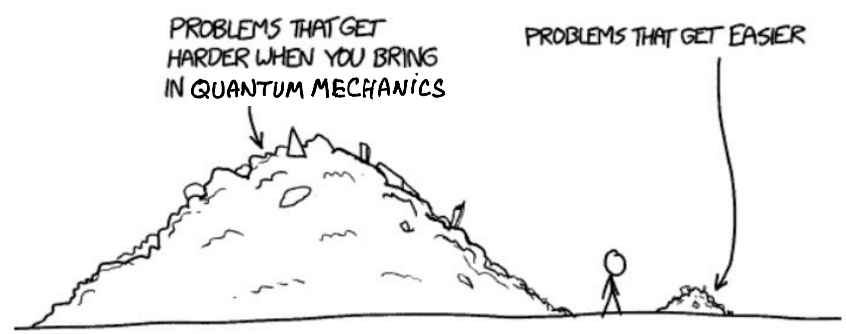
Small errors: $|000\rangle \to |000\rangle + \varepsilon|001\rangle$
$|111\rangle \to |111\rangle + \varepsilon|110\rangle$

Quantum mechanics to the rescue !

- mostly no error detected
  - → collapse into $|000\rangle$ resp. $|111\rangle$
- sometime error detected
  - → collapse into $|001\rangle$ resp. $|110\rangle$
  - → full bit flip, correct as such



Source: xkcd.com

Introduction and Overview (Preskills Notes)

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How to implement ?

Quantum circuit + single qubit measurement

Quantum Gates – work on superpositions, and entangled states

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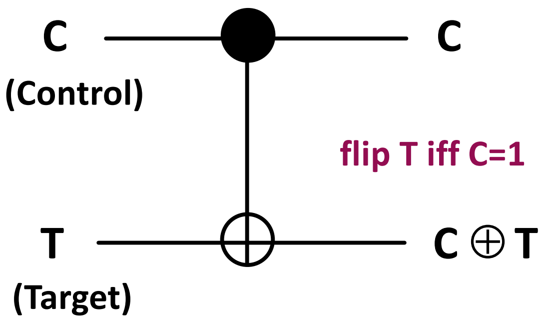
Introduction and Overview (Preskills Notes)

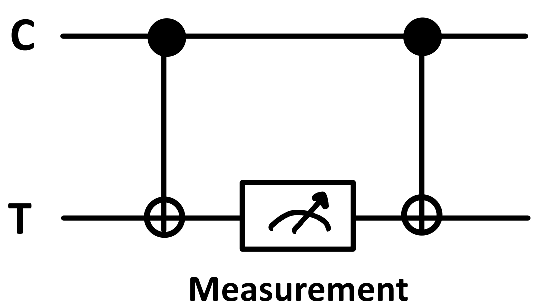
Controlled-NOT (CNOT)



Truth Table

C	T	C ⊕ T
0	0	0
0	1	1
1	0	1
1	1	0

Quantum Circuit for joint measurement



Measurement in $\{|0\rangle, |1\rangle\}$ basis

yields $C \oplus T$

Circuit maps logical basis states as

C	T
0	0
1	1
0	1
1	0

CNOT →

C	T
0	0
1	0
0	1
1	1

meas. T →

C	T
0	0
1	0
0	1
1	1

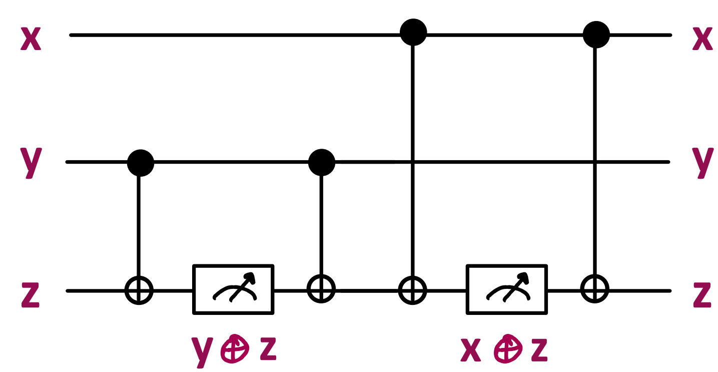
CNOT →

C	T
0	0
1	1
0	1
1	0

T = 0 iff same

T = 1 iff different

Full circuit to obtain Error Syndrome



* iff qubit flip, binary address = $(y \oplus z, x \oplus z)$