Begin
8-27-2024

Introduction and Overview (Preskills Notes)

Physics of Information: John von Neumann
Alan Turing

Notions: What is a computation ?
What is computable

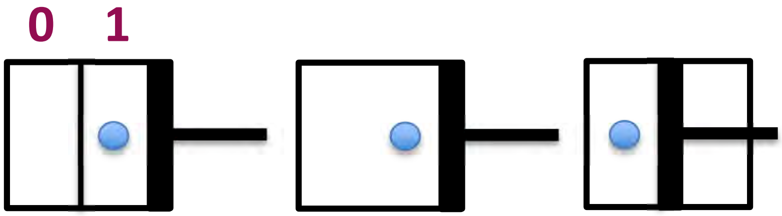
Formulation of Computer Science that is Device Independent

1937 Turing Machine:

https://www.youtube.com/watch?v=E3keLeMwfHY

Landaur: Information is Physical !

Example: Erasure = Dissipation



Entropy: $\Delta S_{gas} = -k \ln 2$

Work: $W = kT \ln 2 = 0.96 \times 10^{-23} \frac{J}{K} \cdot 300K$

$\sim 3 \times 10^{-21} J \sim 0.02 eV$

Is there a way around it ?

Reversible Computation !

But we need a different gate set !

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John von Neumann
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Wikipedia:

A Turing Machine (TM) is a mathematical model of computation describing an abstract machine that manipulates symbols on a strip of paper according to a table of rules.

The TM operates on an infinite tape divided into cells, each of which can hold a symbol drawn from a finite set.

At each step the head reads the symbol in the cell. Then, based on the symbol and the TM's present state, the machine writes a symbol in the cell, and moves the head one step to the left or the right, or halts the computation.

Church – Turing Thesis:

Everything that is computable can be computed on a Turing Machine with at most polynomial overhead.

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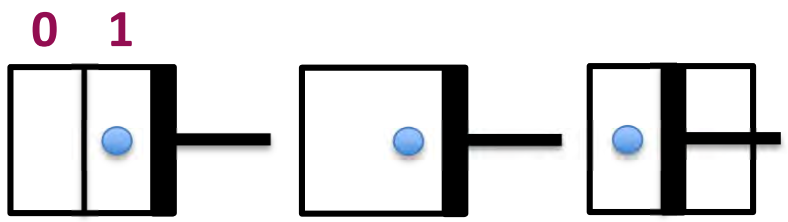

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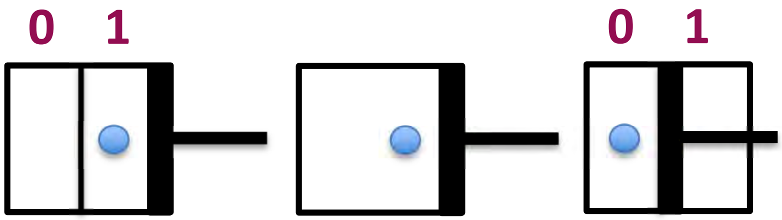

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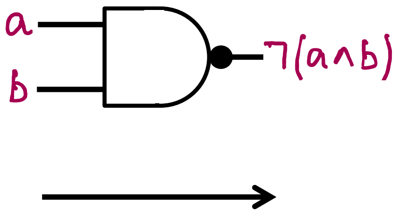
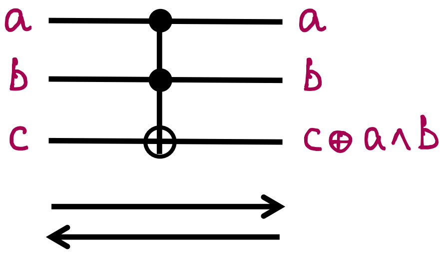


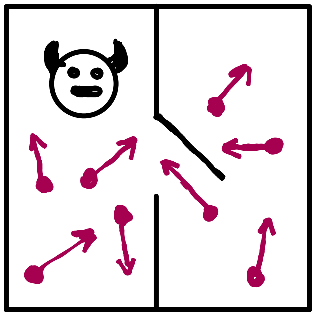
irreversible

Toffoli Gate:



reversible

Maxwells Demon:



Information is Physical !

Quantum Information

Carl Caves: Quantum States are states of knowledge

Physics is Information !

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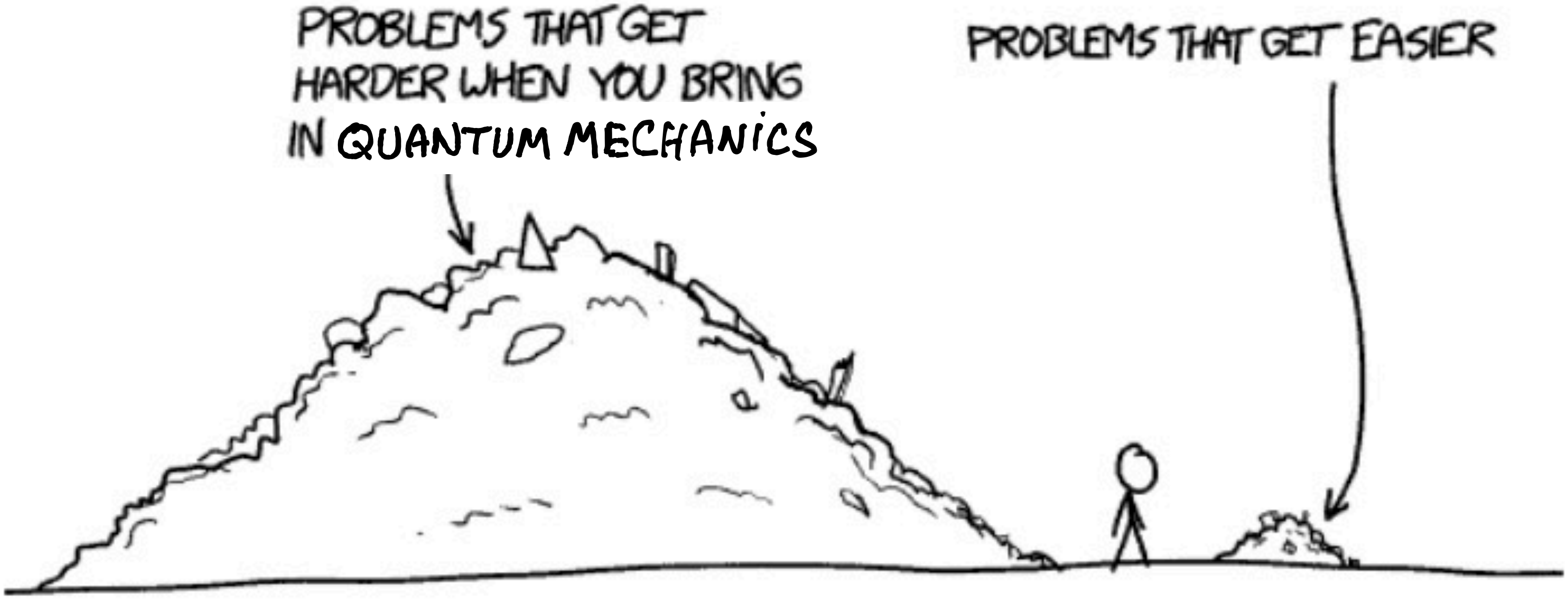

Source: xkcd.com

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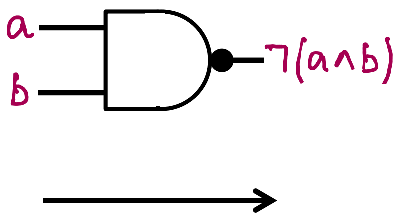

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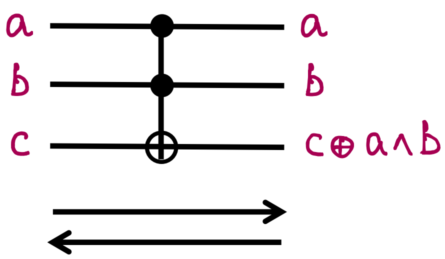

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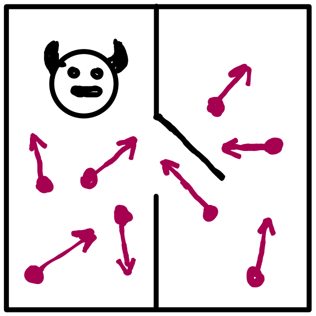

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New properties of QM

Measurement:

$$[A,B] \neq 0 \Rightarrow \Delta A \Delta B \geq \frac{\hbar}{2} |\langle [A,B] \rangle|$$

Acquire Info ⇨ Disturb system

Randomness:

Outcome fundamentally unpredictable

"Collapse" of wavefunction

Cannot determine state of a quantum system if initially unknown } Cannot Copy
No cloning theorem

Entanglement:

Non-local correlations

pure state, entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$\rho = \frac{1}{2} (|00\rangle\langle 00| + |11\rangle\langle 11|)$$

mixed state,
not entangled

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Quantum Computing

Does QM impact Computation?

Peter Shor (1994): YES ! → **Quantum Fourier Transform**

↓

Factoring !

DFT on N bits	$\mathcal{O}[(2^N)^2]$	**steps**
FFT on "	$\mathcal{O}[N2^N]$	**"**
QFT on "	$\mathcal{O}[N \operatorname{Log} N]$	**"**

Efficient Factoring

Factoring $n = p \times q$ → **RSA encryption**

large integer ↑ ↑ large prime numbers

- easy to verify solution	$\operatorname{Log} n$	**steps**
- very hard to solve	$\mathcal{O}[n]$	**steps**

↓

Exponential in # of digits

QFT → Solvable

Preskill Ch. 1, p. 5-6 $T \propto e^{1.9(\operatorname{Log} n)^{1/3}} e^{(\operatorname{Log}\operatorname{Log} n)^{2/3}}$

Best Classical Algorithm

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(1998) 130 digits in 1 month

⬇

400 digits in 10^{10} years

(2022) 24 yrs = 16 Moores Law doublings

2^{16} = 65,536 ➡ **400 digits ~ 150kYrs**

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Shors algorithm: $\mathcal{O}[(\mathrm{Log}\, n)^3]$ ← Polynomial in # of digits

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Efficient Factoring

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large integer → n; p, q ← large prime numbers

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