Pupil aberrations and effects

Lens design OPTI 517

Prof. Jose Sasian
OPTI 518

College of Optical Sciences
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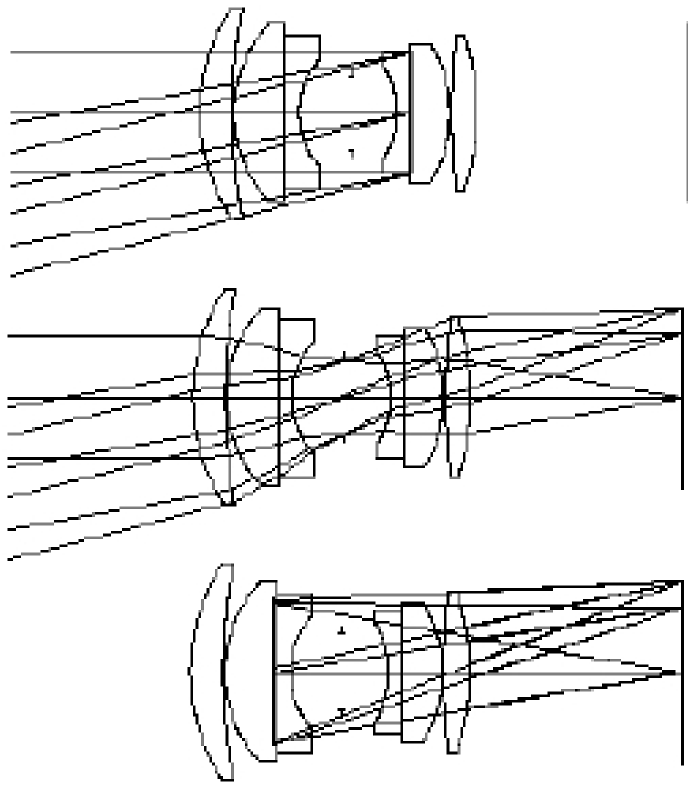
Pupils



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Some references

- T. Smith, “The changes in aberrations when the object and stop are moved,” Trans. Opt. Soc. 23, 139-153, 1921/1922
- C. C. Wynne, “Primary aberrations and conjugate change,” Proc. Phys. Soc. Lond. 65 b, 429-437 (1952)
- J. Sasian, Interpretation of pupil aberrations in imaging systems, SPIE V. 6342-634206 (2006)

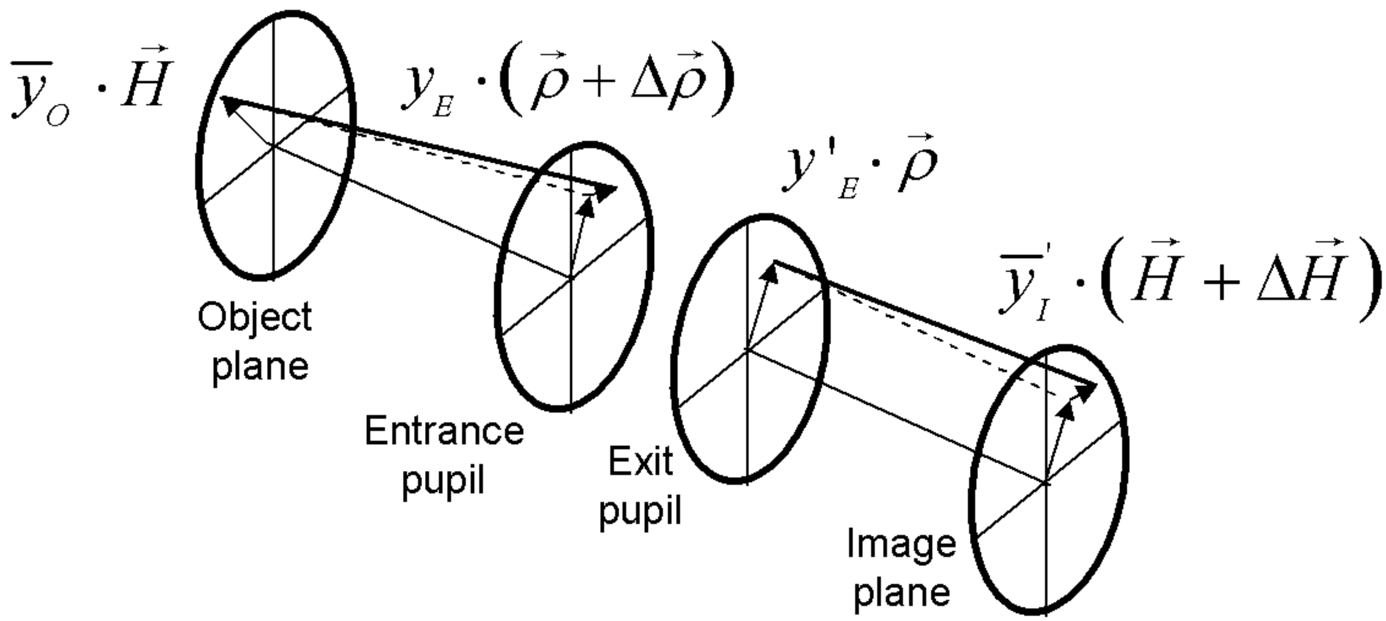
Pupil aberration function

$$W\left(\vec{H},\vec{\rho}\right)=W_{000}+W_{200}\left(\vec{H}\cdot\vec{H}\right)+W_{111}\left(\vec{H}\cdot\vec{\rho}\right)+W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)$$
$$+W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^{2}+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^{2}$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{400}\left(\vec{H}\cdot\vec{H}\right)^{2}$$

$$\overline{W}\left(\vec{H},\vec{\rho}\right)=\overline{W}_{000}+\overline{W}_{200}\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{111}\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{020}\left(\vec{H}\cdot\vec{H}\right)$$
$$+\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^{2}+\overline{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^{2}$$
$$+\overline{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{400}\left(\vec{\rho}\cdot\vec{\rho}\right)^{2}$$

Pupil aberrations

- Object-image interchange role with entrance-and exit pupils
- The chief ray becomes the marginal ray, and the marginal ray becomes the chief ray. Lagrange invariant changes sign
- Image and pupil aberrations are connected



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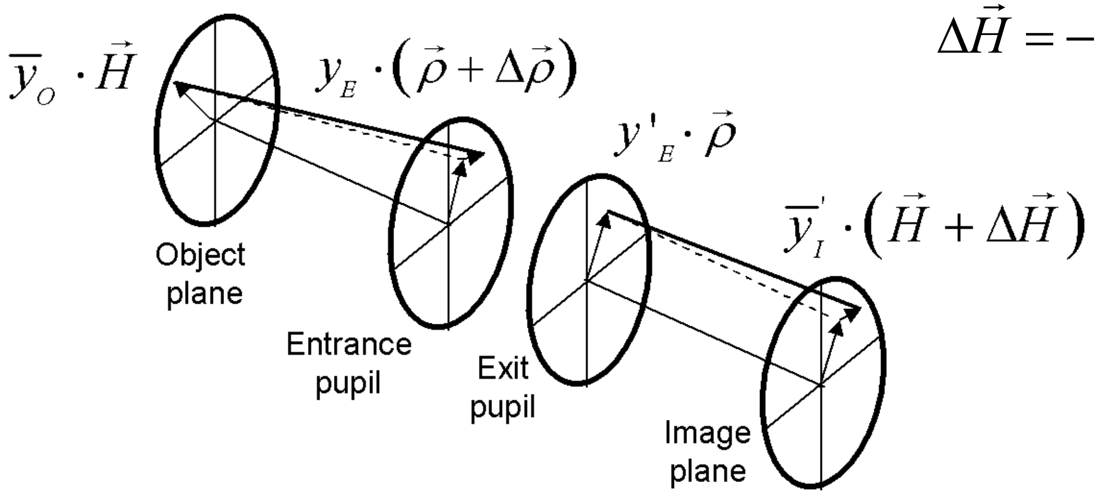
Identity between pupil and image aberration coefficients	
$\bar{W}_{040} = W_{400}$	
$\bar{W}_{131} = W_{311} + \frac{1}{2}\mathcal{K} \cdot \Delta\left\{\bar{u}^2\right\}$	
$\bar{W}_{222} = W_{222} + \frac{1}{2}\mathcal{K} \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{220} = W_{220} + \frac{1}{4}\mathcal{K} \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{311} = W_{131} + \frac{1}{2}\mathcal{K} \cdot \Delta\left\{u^2\right\}$	
$\bar{W}_{400} = W_{040}$	

The displacement vector at the entrance pupil

$$\Delta\vec{H} = -\frac{1}{Ж}\nabla_{\rho}W\left(\vec{H},\vec{\rho}\right) + O^{(5)}$$



$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_{H}\overline{W}\left(\vec{H},\vec{\rho}\right) = -\frac{1}{Ж}\cdot\left\{\begin{array}{l} 4\cdot\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \overline{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\overline{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{array}\right\}$$

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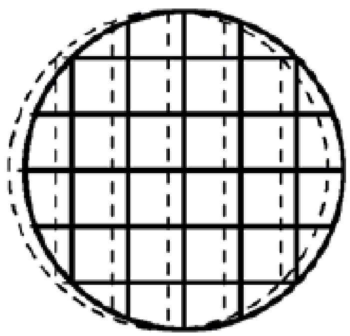
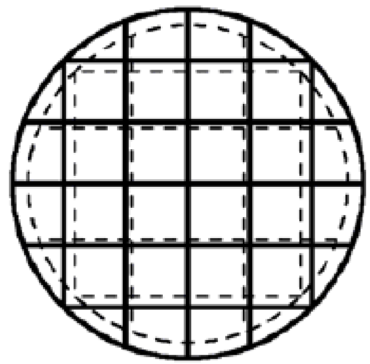
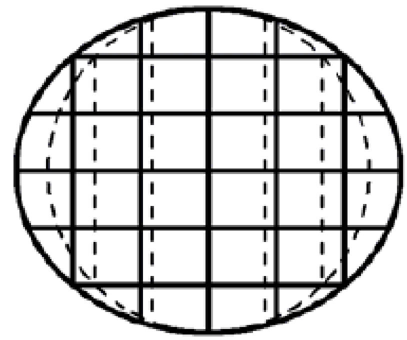
Beam deformation at pupil

$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_H \overline{W}\left(\vec{H},\vec{\rho}\right) = -\frac{1}{Ж}\cdot\begin{Bmatrix} 4\cdot\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \overline{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\overline{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{Bmatrix}$$

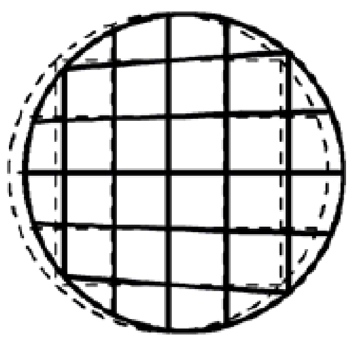


$\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^2$

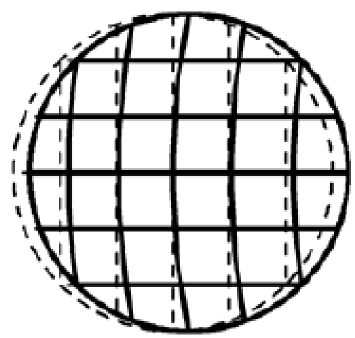




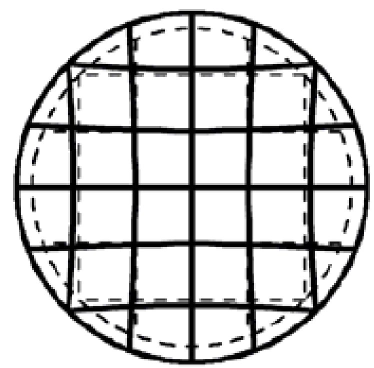
$\overline{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)$



$\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$

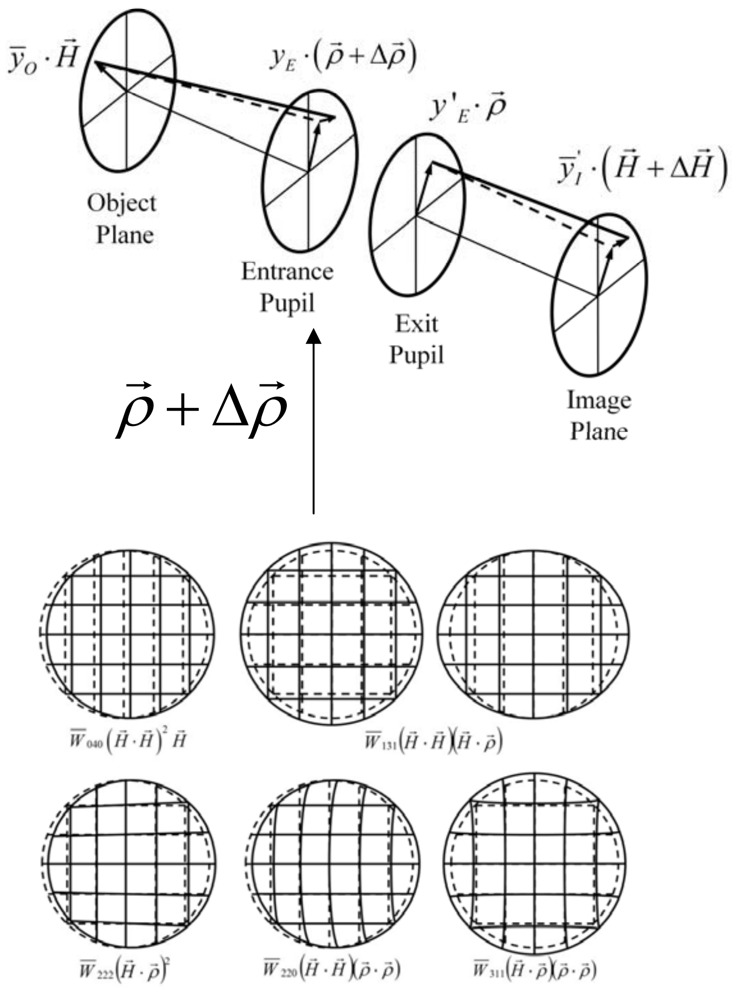


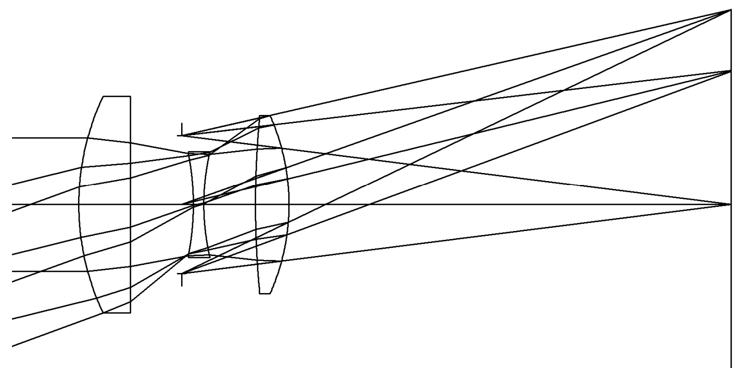
$\overline{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$



$\overline{W}_{311}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$

Distortion at entrance pupil represents a cross-section deformation





$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_H \overline{W}\left(\vec{H},\vec{\rho}\right)$$

$$= -\frac{1}{Ж}\cdot\begin{Bmatrix} 4\cdot\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \overline{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\overline{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{Bmatrix}$$

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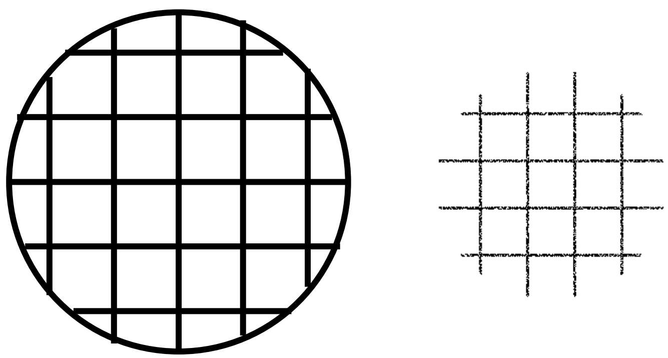
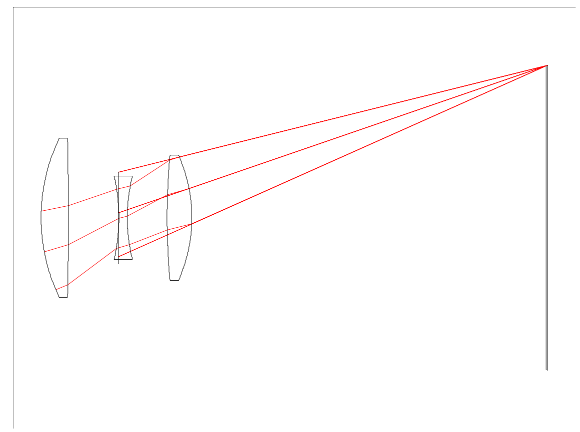
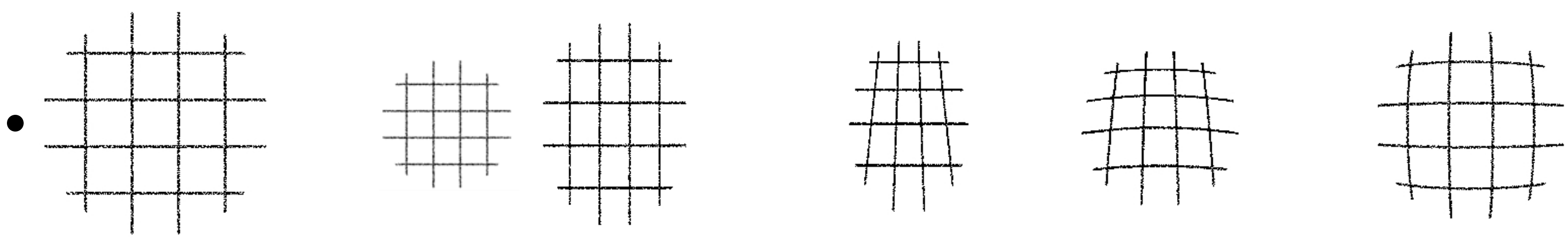
Pupil aberration interpretation (axially symmetric systems)



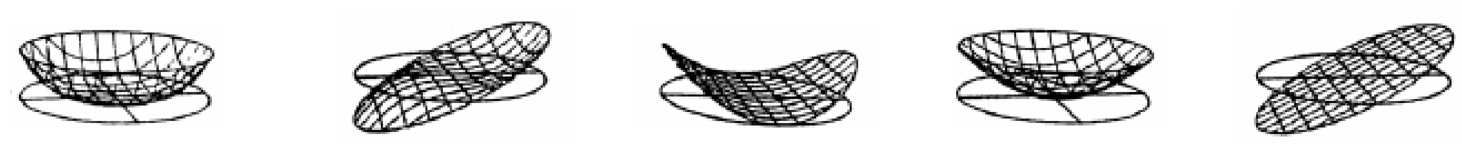




$\overline{W}_{040}$ $\overline{W}_{131}$ $\overline{W}_{222}$ $\overline{W}_{220}$ $\overline{W}_{311}$

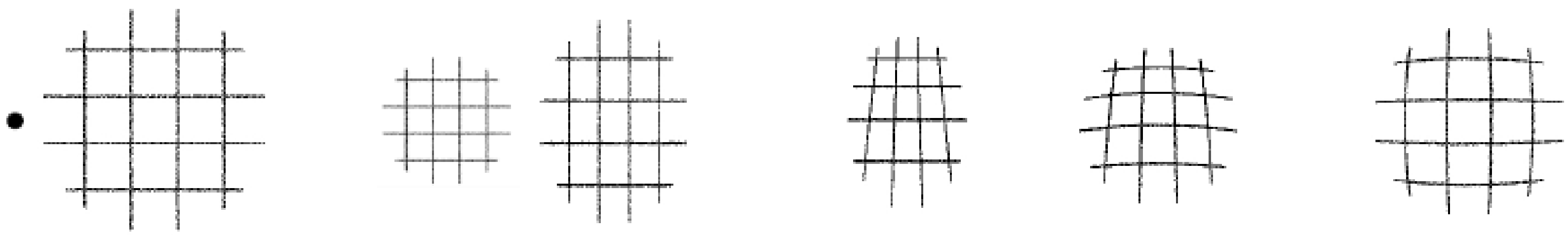
Image vs. Pupil aberrations

Basic wavefront deformation shapes



W_{040} W_{131} W_{222} W_{220} W_{311}

Basic cross-section deformation shapes



$\overline{W}_{040}$ $\overline{W}_{131}$ $\overline{W}_{222}$ $\overline{W}_{220}$ $\overline{W}_{311}$

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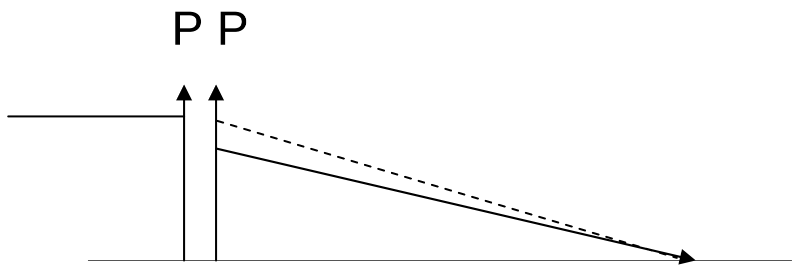
Pupil aberrations consequences

- Can now determine aberration change upon object shift
- Spherical aberration of the pupil
- Coma of the pupil
- Astigmatism and field curvature of the pupil
- Extrinsic sixth-order aberrations
- Bow-Sutton conditions

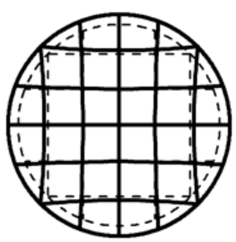
Effects from pupil aberrations

- F/# change
- Kidney bean effect. This is a partial obscuration in the form of a kidney bean caused by pupil spherical aberration
- Loss of telecentricity in relay systems. The chief slope varies as a function of the field of view.
- Vignetting. Spherical aberration of the pupil can lead to light vignetting.
- Pupil walking. Notably in fish eye lenses.
- Slyusarev effects. Due to pupil coma, the exit pupil changes size impacting the relative illumination
- Pupil Apodization

f/# change



$$\Delta y = \frac{1}{\bar{u}} \overline{W}_{311}$$



Δy

$$f/\# = \frac{f}{d} = \frac{f}{d - 2\Delta y}$$

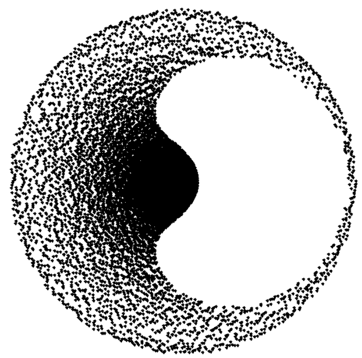
(fourth-order contribution)

System corrected for all fourth-order aberrations over entire object and image spaces (Described by D. Shafer)

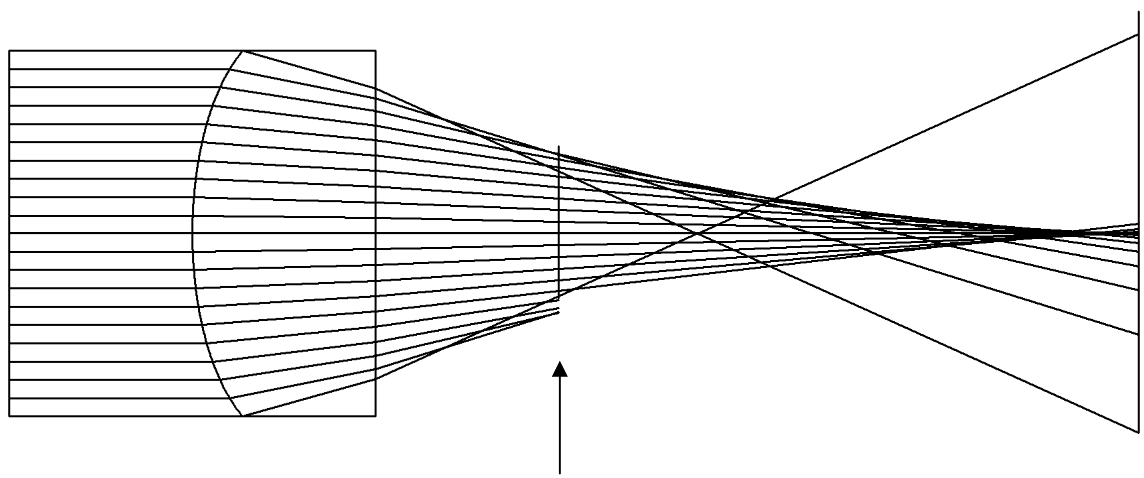
- Two mirrors
- Six reflections
- Afocal m=+/- 1
- Spacing R/2 or 0.707 R
- R1=R2

Kidney bean effect

From pupil spherical aberration



Chief rays



Off-axis clipping aperture (eye iris)

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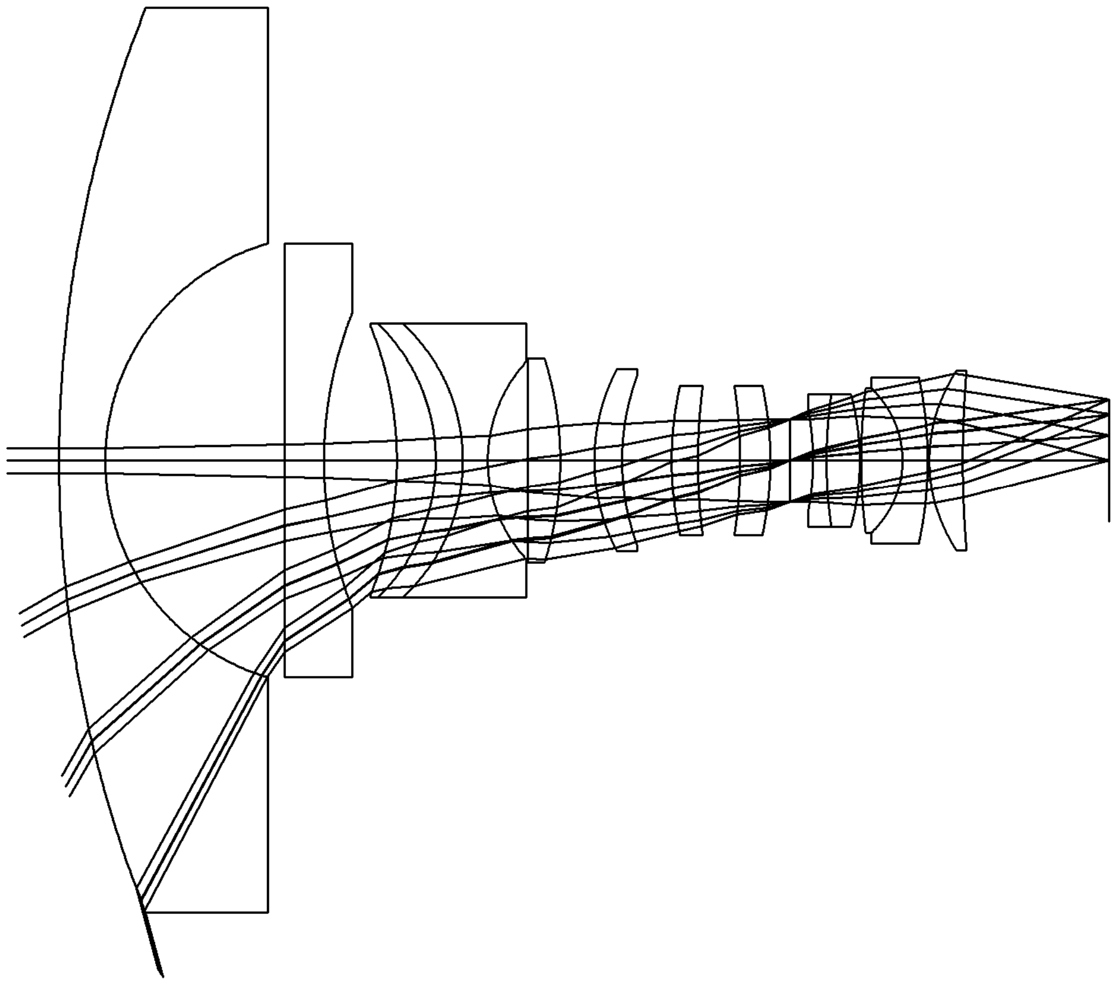
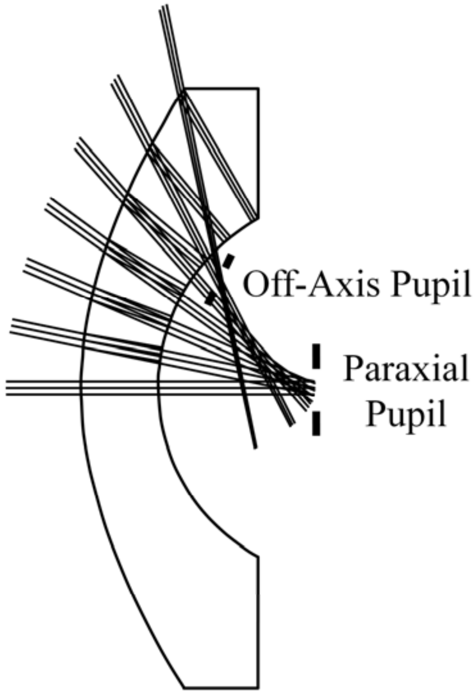
Entrance pupil ‘walking’





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Bow-Sutton conditions

- If the lens is symmetrical but the conjugates are not equal, then distortion will be corrected only if the entrance and exit pupils are free from pupil spherical aberration.
- Similarly, lateral color will be absent if the entrance and exit pupils are free from axial chromatic aberration.

See Kingslake's lens design fundamentals
New edition with Barry Johnson

Bow-Sutton Conditions

- Since the lens is symmetrical ubar’=ubar
- Thus Coma pupil = image distortion
- A unit magnification the system is fully symmetrical for the stop and for the image systems and so, Coma pupil = image distortion=0
- When the object moves at other conjugate coma of the pupil according to stop shift equations is,
- New pupil coma=old pupil coma + pupil spherical aberration * y/ybar.
- The old coma is zero, thus,
- New pupil coma=pupil spherical aberration * y/ybar.
- Therefore if pupil spherical aberration is zero then the new pupil coma is zero.
- And therefore image distortion is zero.
- ubar’=ubar, still holds for the second conjugate

$$\bar{W}_{131} = W_{311} + \frac{1}{2} Ж \cdot \Delta\left\{\bar{u}^{2}\right\} \qquad \bar{W}^{*}_{131} = \bar{W}_{131} + \frac{\Delta y}{\bar{y}} \bar{W}_{040}$$

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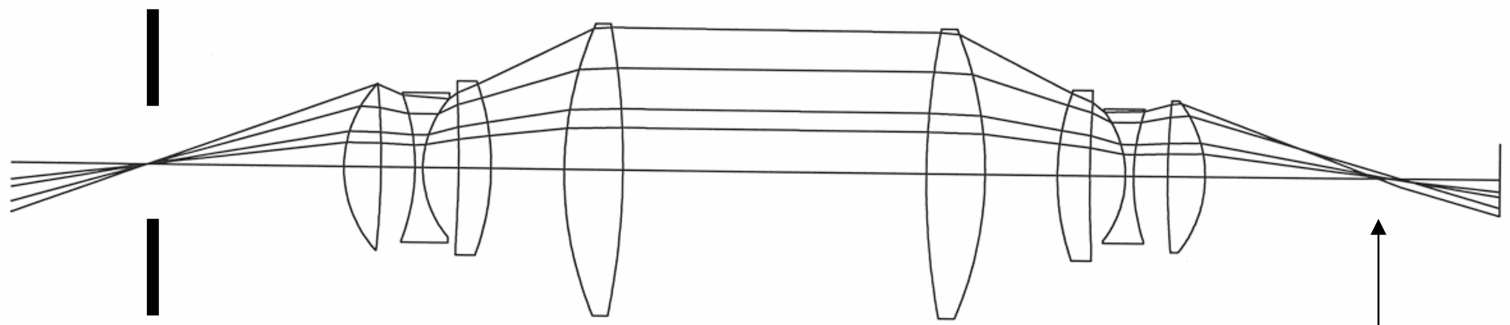
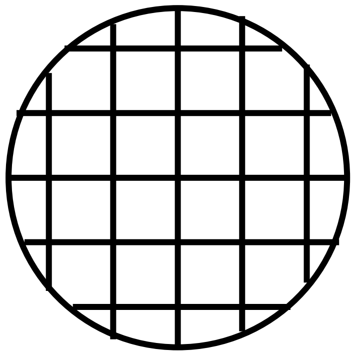
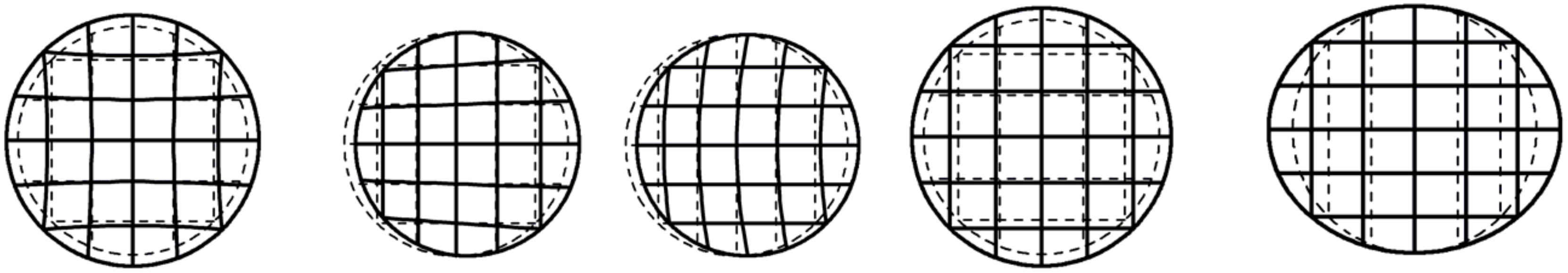
Loss of telecentricity



Change in relative illumination

- Vignetting
- Image distortion
- Cosine to the fourth law
- Pupil distortion





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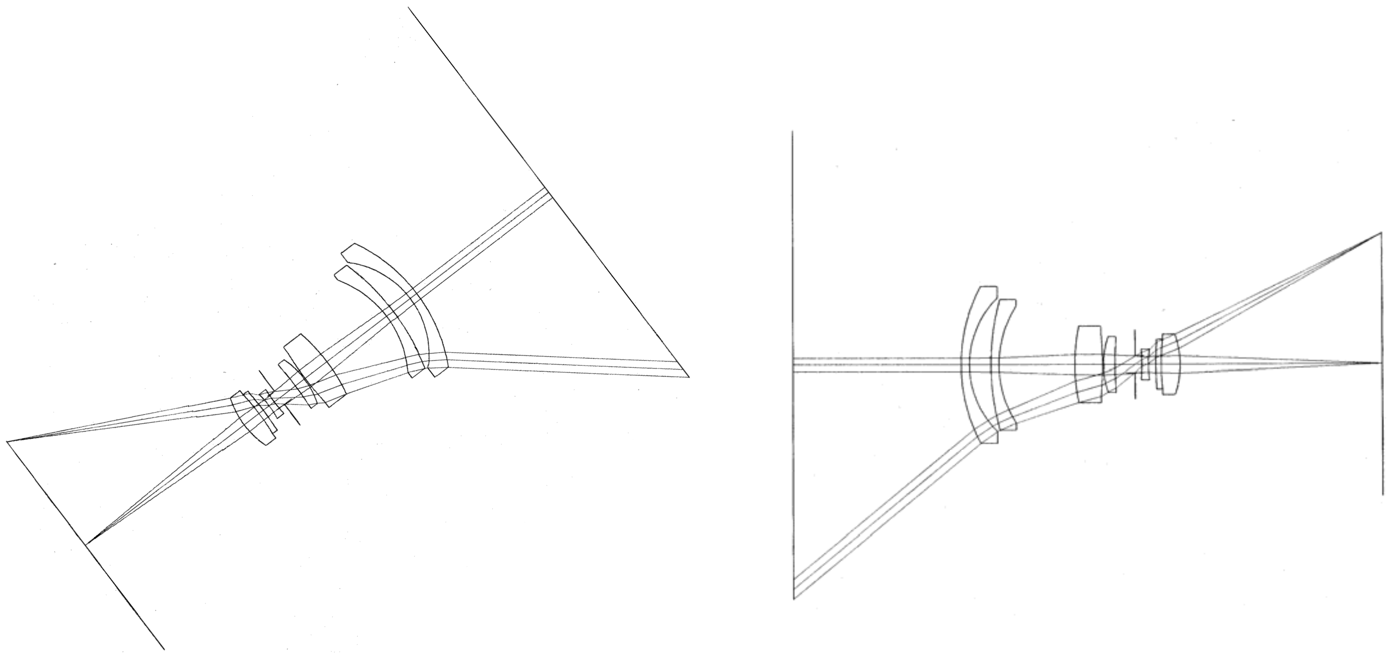
Slyusarev effect



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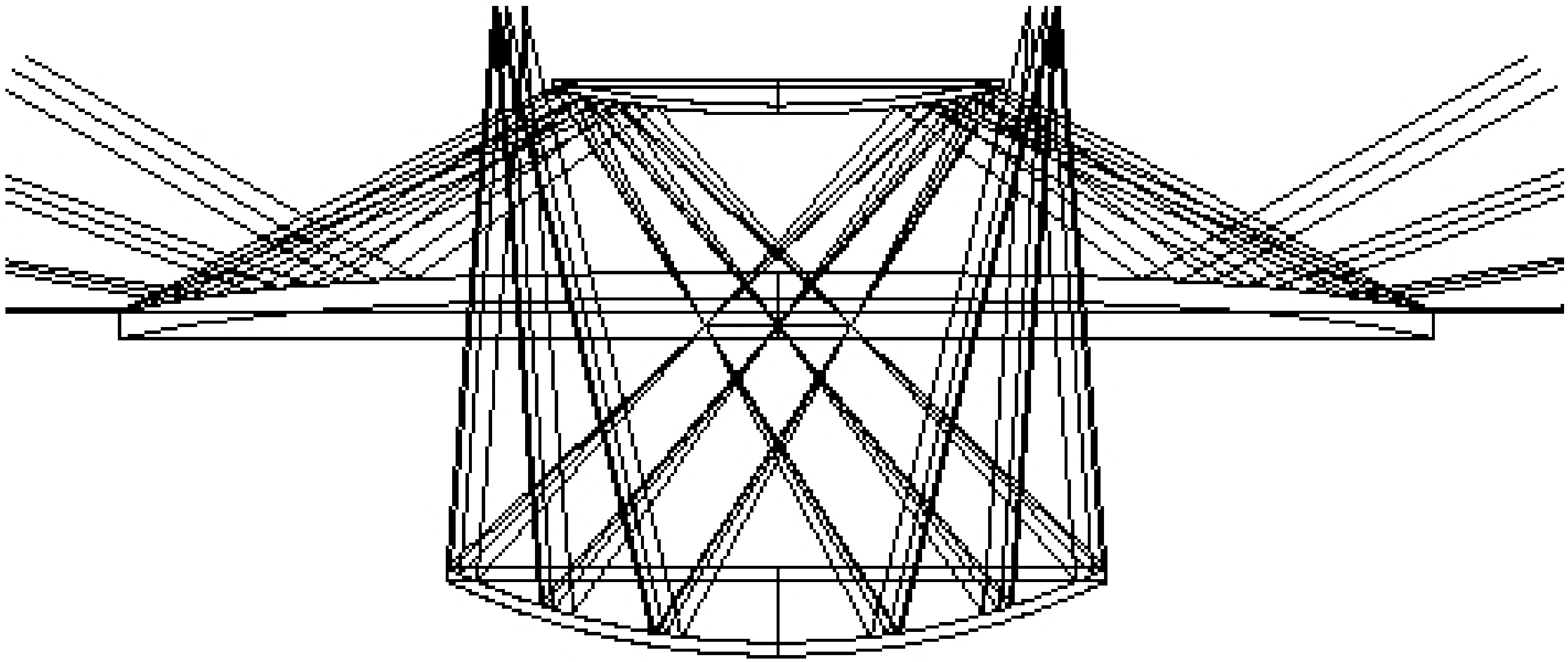
Effects from pupil coma



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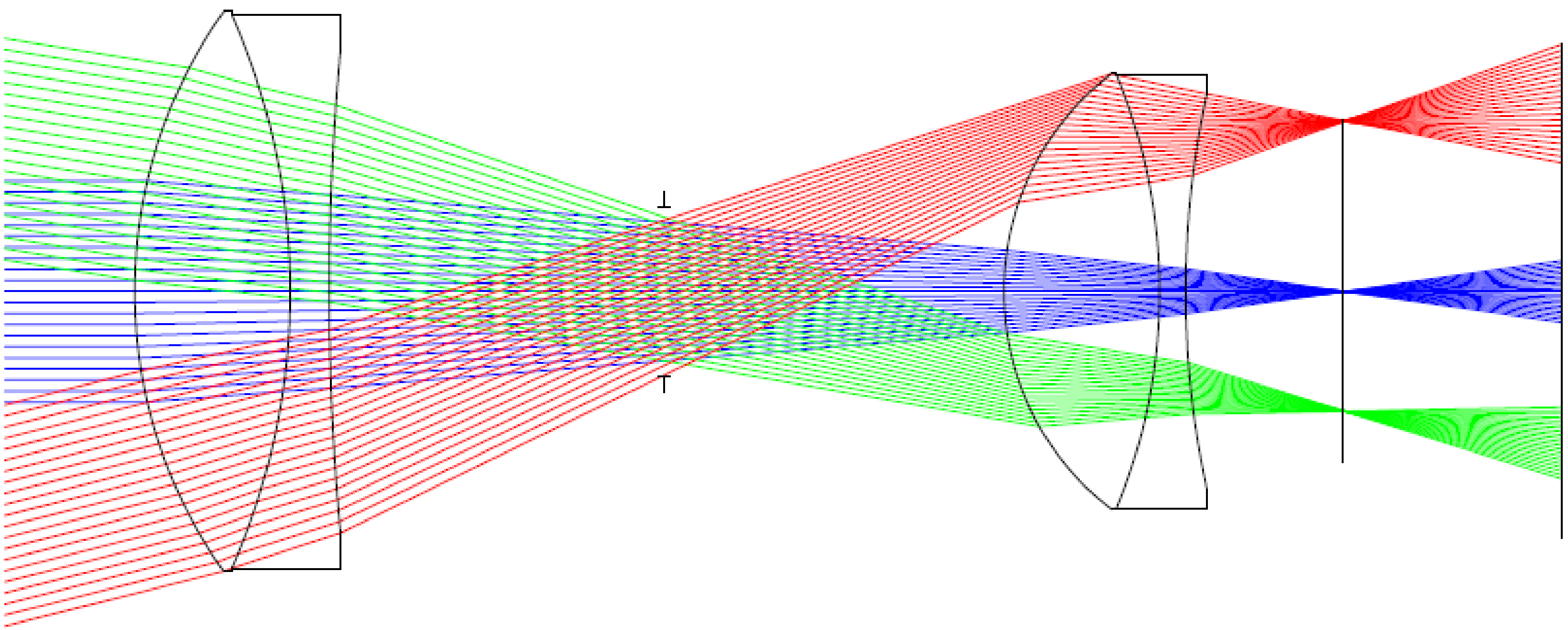
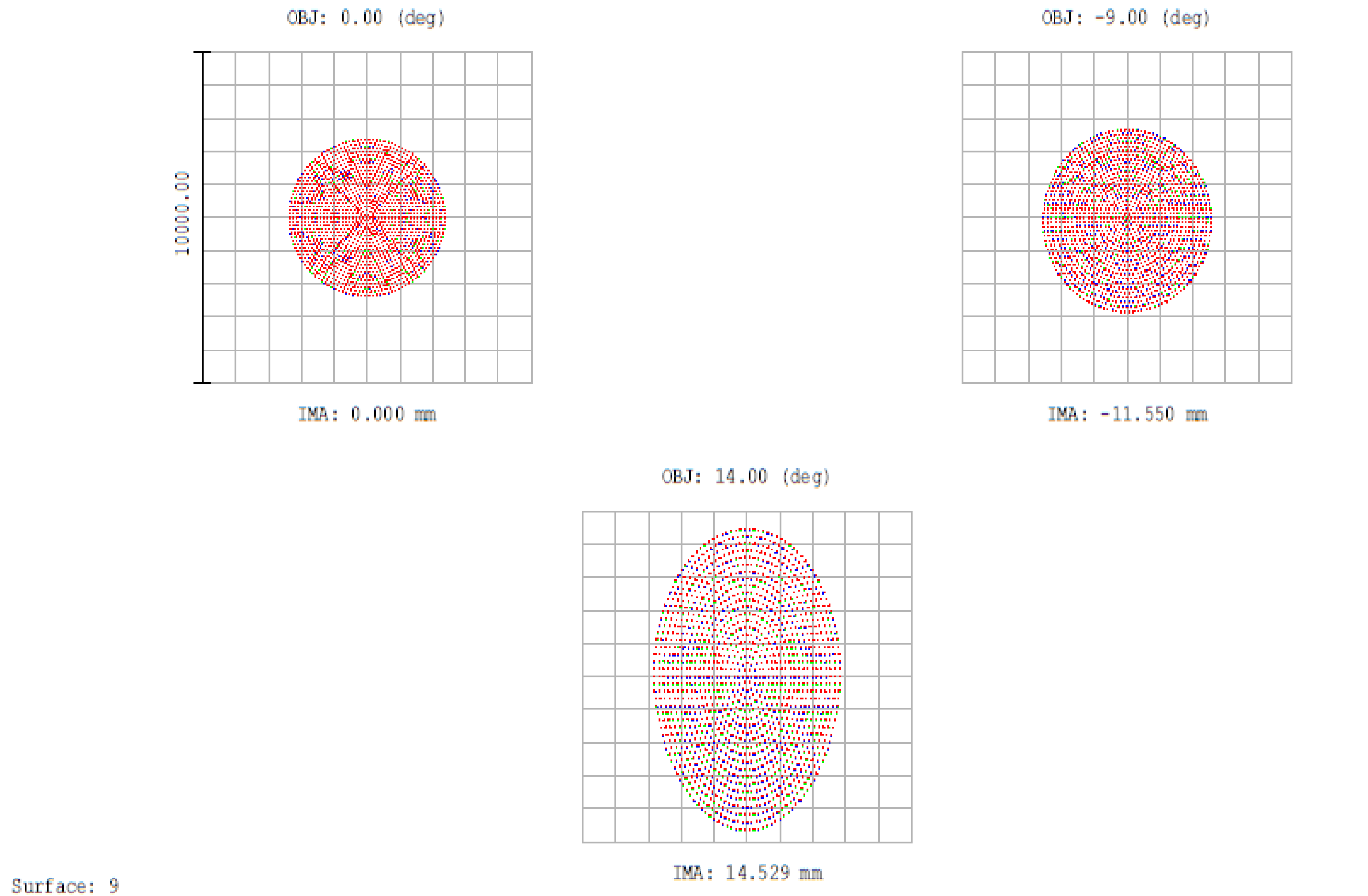
Petzval type lens example

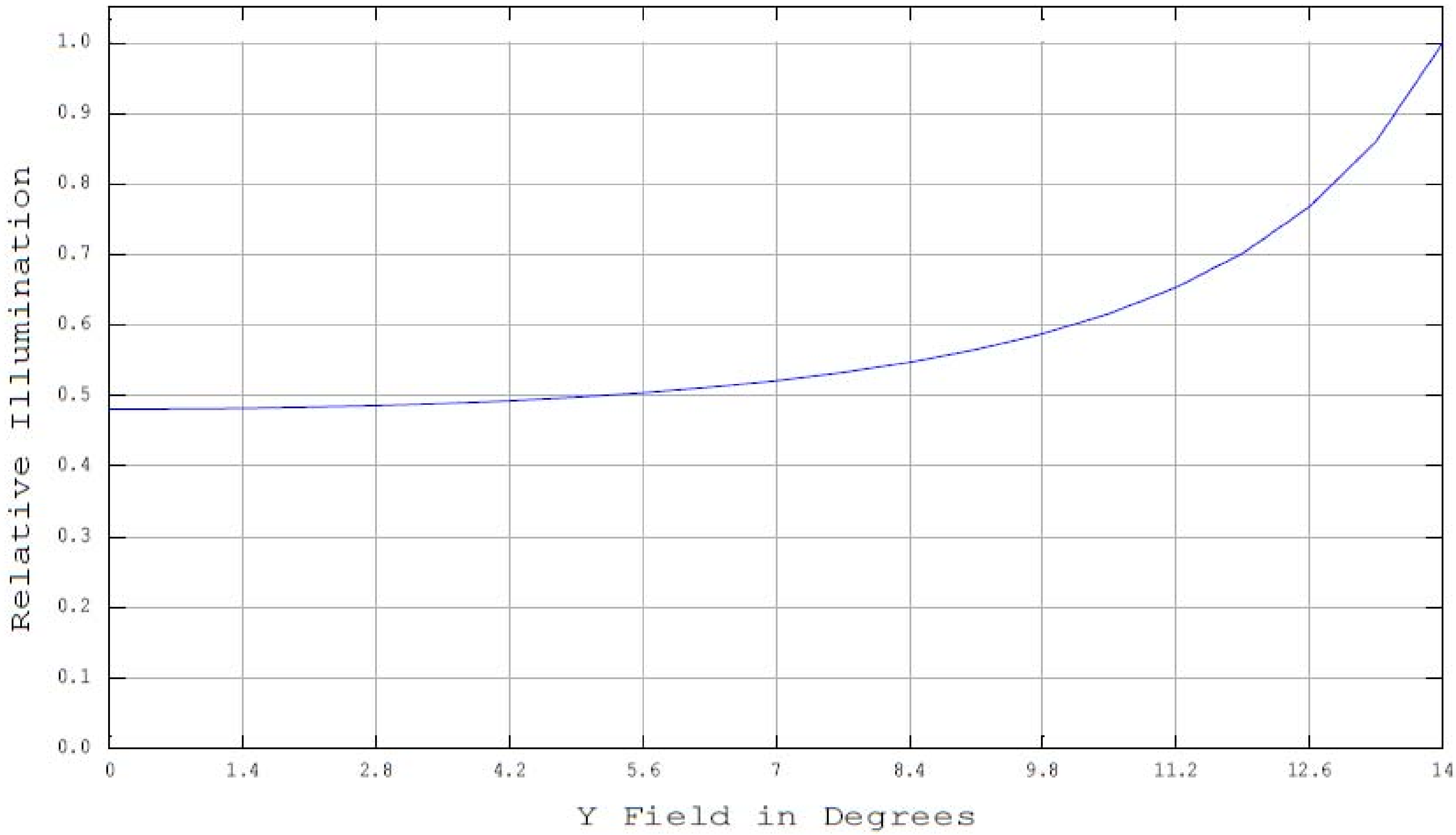


Note nearly telecentricity
Issues with MTF calculation

Beam foot print at exit pupil



Relative illumination



Summary

- Pupil aberrations
- Effects from pupil aberrations