Part 9
Phase Shifting Interferometry

• Classical Interferogram Analysis
• Phase Shifting Advantages
• Phase Shifters
• Algorithms
• Removing Phase Ambiguities
• Error Sources

Classical Analysis of Interferograms

Surface Error = \( \frac{\lambda}{2} \frac{\Delta}{S} \)

• Classical Analysis
  • Measure positions of fringe centers.
  • Deviations from straightness and equal spacing gives aberration.
Computer Analysis of Interferograms

Largest Problem
Getting interferogram data into computer

Solutions
• Graphics Tablet
• Scanner
• CCD Camera
• Phase-Shifting Interferometry

Advantages of Phase-Shifting Interferometry
• High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
• Rapid measurement
• Good results with low contrast fringes
• Results independent of intensity variations across pupil
• Phase obtained at fixed grid of points
• Easy to use with large solid-state detector arrays
Phase-Shifting Interferometry

1) MODULATE PHASE
2) RECORD MIN 3 FRAMES
3) CALCULATE OPD

\[ \text{OPD} = \frac{\lambda}{2k} \tan \left[ \frac{C - B}{A - B} \right] \]

Phase Shifting - Moving Mirror

Move \( \lambda/8 \)

\( \pi/2 \) Phase Shift

PZT Pushing Mirror
Phase Shifting - Diffraction Grating

Diffraction Grating +1 Order \( \pi/2 \) Phase Shift
Move 1/4 Period

Phase Shifting - Bragg Cell

Bragg Cell
\( f_0 \)
\( f_0 + f \)
\( f_0 \)
0 Order
Frequency \( f \)
Phase Shifting - Tilted Glass Plate

\[ \frac{\pi}{2} \]

Phase Shift

Phase Shifting - Rotating Half-Wave Plate

Circularly Polarized Light

Half-Wave Plate

45° Rotation

\[ \frac{\pi}{2} \]

Phase Shift
Four Step Method

\[ I(x,y) = I_0 + I_1' \cos[\phi(x,y) + \phi(t)] \]

\[ \begin{align*} 
I_1(x,y) &= I_0 + I_1' \cos \left[ \phi(x,y) \right] \quad \phi(t) = 0 \\
I_2(x,y) &= I_0 - I_1' \sin \left[ \phi(x,y) \right] \quad \phi(t) = \pi/2 \\
I_3(x,y) &= I_0 - I_1' \cos \left[ \phi(x,y) \right] \quad \phi(t) = \pi \\
I_4(x,y) &= I_0 + I_1' \sin \left[ \phi(x,y) \right] \quad \phi(t) = 3\pi/2 
\end{align*} \]

\[ \tan[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \]

Relationship between Phase and Height

\[ \phi(x,y) = \tan^{-1}\left[ \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \right] \]

Height Error \( (x,y) = \frac{\lambda}{4\pi} \phi(x,y) \)
### Phase-Measurement Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three Measurements</td>
<td>$\phi = \tan^{-1} \left[ \frac{I_3 - I_2}{I_1 - I_2} \right]$</td>
</tr>
<tr>
<td>Four Measurements</td>
<td>$\phi = \tan^{-1} \left[ \frac{I_4 - I_3}{I_1 - I_5} \right]$</td>
</tr>
<tr>
<td>Hariharan</td>
<td>$\phi = \tan^{-1} \left[ \frac{2(I_2 - I_4)}{2(I_3 - I_5 - I_1)} \right]$</td>
</tr>
<tr>
<td>Carré Equation</td>
<td>$\phi = \tan^{-1} \left[ \sqrt{\frac{3(I_2 - I_3) - (I_1 - I_4)}{(I_2 + I_3) - (I_1 + I_4)}} \right]$</td>
</tr>
</tbody>
</table>

### Phase-Measurement Algorithm for N Intensity Measurements

For $N$ intensity measurements:

$$\phi = -\tan^{-1} \left[ \frac{\sum_{i=1}^{N} I_i \sin \alpha_i}{\sum_{i=1}^{N} I_i \cos \alpha_i} \right]$$

$$\alpha_i = \frac{2\pi i}{N} \quad \text{for} \ i = 1, \ldots, N$$

Technique is also known as synchronous detection.
Phase-Stepping Phase Measurement

Integrated-Bucket Phase Measurement
Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

\[ I_i = \frac{1}{\Delta} \int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} I_o \left( 1 + \gamma_o \cos[\phi + \alpha_i(t)] \right) d\alpha(t) \]

\[ = I_o \left( 1 + \gamma_o \text{sinc} \left[ \frac{\Delta}{2} \right] \cos[\phi + \alpha_i] \right) \]

Integrating-Bucket \( \Delta=\alpha \)

Phase-Stepping \( \Delta=0 \)

Phase Ambiguities

- If we know sign of Sin and sign of Cosine the Arc Tangent is calculated modulo 2\(\pi\).
- Must correct for 2\(\pi\) ambiguities.
Typical Fringes
For Spherical Surfaces

Fringes

Phase map

Phase Ambiguities
-Before Integration

2 π Phase Steps

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Phase Ambiguities
- After Integration

Phase Steps Removed

Removing Phase Ambiguities

- Arctan Mod $2\pi$ (Mod 1 wave)
- Require adjacent pixels less than $\pi$ difference
  
  \[(1/2 \text{ wave OPD})\]
- Trace path
- When phase jumps by $> \pi$
  - Add or subtract $N2\pi$
  - Adjust so $< \pi$
Error Sources

- Incorrect phase shift between data frames
- Vibrations
- Detector non-linearity
- Stray reflections
- Quantization errors
- Frequency stability
- Intensity fluctuations

Three $\pi/2$ Steps

Phase error due to 5% phase shift calibration error.

Peak-Valley Error (Radians) = 0.0785398

![Graph showing phase error due to 5% phase shift calibration error]
Four $\pi/2$ Steps

Phase error due to 5% phase shift calibration error.

Peak - Valley Error (Radians) = 0.0785302

Phase Error (Radians)

Phase measured

Phase Error Compensating Techniques

Two data sets with $\pi/2$ phase shift.

- Calculate a phase for each set from algorithm, and then average phases.
- Average Numerator and Denominator, and then calculate phase.
Example of Algorithm Derivation

Averaging Technique

4-FRAME (offset = 0)
frames # 1,2,3,4

\[
\frac{l_4 - l_2}{l_1 - l_3} = \frac{N_1}{D_1}
\]

4-FRAME (offset = \(\pi/2\))
frames# 2,3,4,5

\[
\frac{l_4 - l_2}{l_5 - l_3} = \frac{N_2}{D_2}
\]

5-FRAME

\[
\tan \phi = \frac{N_1 + N_2}{D_1 + D_2} = \frac{2(l_4 - l_2)}{l_1 + l_5 - 2l_3}
\]

Schwider-Hariharan Five \(\pi/2\) Step Algorithm

Phase error due to 5\% phase shift calibration error.

Peak - Valley Error (Radians) = 0.0030859

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Error due to vibration

- Probably the most serious impediment to wider use of PSI is its sensitivity to external vibrations.
- Vibrations cause incorrect phase shifts between data frames.
- Error depends upon frequency of vibration present as well as phase of vibration relative to the phase shifting.

Error due to vibration for Schwider-Hariharan algorithm

![P-V phase error due to 0.02 zero to peak waves of vibration](image-url)
Best way to fix vibration problem

- Reduce vibration
- Take data fast
- Take all frames at once
- Measure vibration and introduce vibration 180 degrees out of phase to cancel vibration

Error due to detector nonlinearity

- Generally CCD’s have extremely linear response to irradiance
- Sometimes electronics between detector and digitizing electronics introduce nonlinearity
- Detector nonlinearity not problem in well designed system.
- Schwider-Hariharan algorithm has no error due to 2nd order nonlinearity. Small error due to 3rd order.
Schwider-Hariharan

Phase error due to $\%$ detector nonlinearity of order 3.

<table>
<thead>
<tr>
<th>Phase Error (Radians)</th>
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</thead>
<tbody>
<tr>
<td>-0.002</td>
</tr>
<tr>
<td>-0.001</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.001</td>
</tr>
<tr>
<td>0.002</td>
</tr>
</tbody>
</table>

Peak-Valley Error (Radians) - 0.00480903

Error due to Stray Reflections

- Stray reflections in laser source interferometers introduce extraneous interference fringes.
- Stray reflections add to test beam to give a new beam of some amplitude and phase.
- Difference between this resulting phase and phase of test beam gives the phase error.
Error – Stray Reflections

Phase Error

Measured Beam

Stray Beam

Correct Phase

Test Beam

Phase-Shifting Interferometer

TEST SAMPLE

PZT

PUSHING MIRROR

LASER

BS

IMAGING LENS

DETECTOR ARRAY

DIGITIZER

COMPUTER

PZT CONTROLLER