A Multiple-Beam Interferometer for Use with Spherical Wavefronts

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A multiple-beam interferometer that permits evaluation of autostigmatic systems is described. A small reference sphere is compared with a spherical wavefront in a manner analogous to the comparison of a plane wavefront with the reference flat in a Fizeau interferometer. To prevent walk-off of the fringes, a field lens is used at the center of curvature of the reference sphere. The instrument described has been used for evaluating spherical mirrors, concentric windows, and lenses.

Introduction

Over the last five years the interferometers for measuring relatively fast spherical concave mirrors have undergone several phases of modification in our metrology laboratories. Starting with the conventional Williams interferometer, which uses standard monochromatic sources and a reference surface having a radius of curvature comparable with that of the element tested, we soon adapted the laser source to permit the utilization of a shorter radius of curvature reference surface. This effort met with limited success primarily because the errors introduced by the beam splitter for wide angle, unequal arm interferometers were difficult to assess. In October 1963, it occurred to one of us (J. V.) that the beam splitter could be eliminated and a multiple-beam instrument devised by forming a concentric concave cavity with the reference and tested surfaces, the small reference surface having a highly reflecting, slightly transmitting coating, as in the Fizeau interferometer. Although familiar with Steele's review of compensated interferometers, we overlooked the advantage of compensating this interferometer until Herriott's presentation pointed it out.

Thus, the most recent modification to the interferometers for evaluating spherical wavefronts has resulted in the instrument described here.

The Spherical Wavefront Interferometer, Multiple-Beam (SWIM)

The Fizeau interferometer is a classic example of a multiple-beam, high-finesse interferometer which produces narrow straight line fringes of high contrast and has seen extensive use in evaluating optical surfaces (Fig. 1). Since the fringes are well defined, their positions can be accurately measured. Small fluctuations in the fringes are readily seen and interpreted as small irregularities in the surface being tested. The advent of the laser has enhanced the versatility of the Fizeau interferometer by permitting the interferometer gap to be increased many orders of magnitude. It has also made possible the development of an instrument using the same principle, but comparing a spherical wavefront with a spherical reference surface (Fig. 2). The wavefront can be that produced by reflection from a spherical surface or from an autocollimated optical system. This interferometer provides the same well known advantages over two-beam interferometers as does the Fizeau.

Consider two opposed spherical concentric mirrors that form a cavity. If one of the mirrors is partially transmitting, a spherical wavefront passing through the mirror will pass through its center of curvature and produce multiple reflections in the cavity in a manner analogous to the Fizeau interferometer. Although familiar with Steele's review of compensated interferometers, we overlooked the advantage of compensating this interferometer until Herriott's presentation pointed it out.

The Field Lens

The field lens, then, makes the interferometer capable of producing narrow fringes such as those obtained with
A Fizeau interferometer of narrow gap. However, there are several factors in the design of the lens which must be considered. The purpose of the lens is to image one surface of the cavity onto the other, but both the object and image planes are spherical surfaces curving toward the lens. A multielement lens could be designed to have a Petzval radius that would image one surface on the other, but such a lens would introduce aberrations that would distort the wavefront in the cavity. An appreciable lens thickness would also introduce optical path length differences across the field. Air-glass surfaces reduce the transmittance in the cavity, thus reducing the finesse. In consideration of these difficulties, we decided to make a single element lens with as small a Petzval radius as possible.

Since $1/R = \Sigma 1/nf$ (where $R$ = Petzval radius, $n =$ refractive index, $f =$ focal length) and $f$ is fixed, $R$ varies as $n$. To keep $R$ small, $n$ must be as small as possible; therefore, fused silica with $n = 1.458$ was used. The focal length is determined by the two radii of curvature of the cavity and the need to place the lens at the common center of curvature in order to use as small an area of the lens as possible.

The effect of spherical aberration was considered, but since the working aperture of the field lens is approximately 0.25 mm and its equivalent focal length is about 50 mm, the lens is working at $f/200$. Spherical aberration is not a problem.

The shape factor of the lens was considered with regard to the optical path difference of the rays of the cone passing through the lens. Calculations showed that an entering spherical wave would emerge still spherical within $\lambda/140$, regardless of the shape factor, for a lens 0.2 mm thick. The lenses used in most of the tests were biconvex, 3-mm diam, and 0.2 mm thick. Calculations and measurements also have indicated that the longitudinal position of the field lens is not critical.

In order that the cavity absorb as little light as possible, the field lens received a multilayer dielectric coat that increased its transmittance to over 99.5%. Such a coat has been calculated to cause a nonspherical deformation in the wavefront of less than $\lambda/1000$. Since it is used over an extremely small portion of its surface, the figure and coating thickness uniformity is excellent.

An important additional benefit of the field lens is that it greatly reduces the effect of wavefront aberrations which may originate outside the cavity. Although these are generally neglected for narrow gap interferometers, they cannot be neglected in a large gap interferometer. If a wavefront normally incident on the reference surface has an angle error of $\Delta \theta$ over part of its aperture owing to aberrations in the collimator, the error will introduce fractional fringe displacement $\Delta m \approx (d/\lambda)(\Delta \theta)^2$ (Fig. 4). Therefore, the larger the gap $d$, the more sensitive the interferometer is to these aberrations. However, the field lens, by imaging the test surface on or quite close to the reference surface, makes the large cavity act like a narrow gap interferometer, with $d \to 0$.

The improvement in fringe configuration owing to the reduction of the aberration in the interferometer by the field lens has been demonstrated experimentally. The objective used to collimate the pinhole source was reversed to introduce spherical aberration in the interferometer outside the cavity. Figure 5 shows the effect both without the field lens and with it. The improvement is quite obvious.

The Aplanat

A partially reflecting spherical surface is used as the reference in the interferometer. The element in which this reference surface was incorporated could have taken several forms; it could have been a concentric meniscus.
LENS A

Fig. 4. Fractional fringe displacement. \( m\lambda = 2d \cos \theta, \Delta m/m = -[\tan(\Delta \theta/2)] \Delta \theta = (\Delta \theta)^2/2 \text{ or } \Delta m = d(\Delta \theta)^2/2, \) where \( m = \) order of interference, \( \lambda = \) wavelength, \( d = \) interferometer gap, and \( \Delta \theta = \) angle error.

LENS B

Fig. 5. Spherical aberration introduced in interferometer: (a) no field lens; (b) with field lens.

A lens, used such that its center of curvature was at the focus of a positive lens with the cone from the positive lens passing through the meniscus without refraction. Fringes between the two concentric surfaces may have caused difficulty, however. The sphere might also have been the last surface of a positive lens, again having the focus at the center of curvature. In order to gain an increase in the cone angle from the positive lens, we decided to use a positive single element aplanat. Since the light used is monochromatic, no color correction is required. In addition, an aplanat is free of spherical aberration and coma when used at the conjugates for which it is designed. Since the speed of the cone from a lens is increased by a factor equal to the refractive index of the aplanat, a relatively slow lens may be used behind the aplanat. For example, if it is desired to produce an \( f/2 \) cone from a positive aplanat of index 1.5, it is necessary to use only an \( f/3 \) objective behind it.

The reflectance of the reference surface of the aplanat (i.e., the concave side) should match that of the test object if maximum contrast in the fringe pattern is to be obtained. The primary test object used in the experiments was a 71-cm radius of curvature, 45.7-cm diam sphere with a 5-cm perforation in its center. The piece is aluminized and has a reflectance of about 88%. The reference surface received a multilayer dielectric coat of extremely low absorption and a transmittance of 10–15%. Its reflectance, therefore, was approximately that of the test sphere. Other test objects have been evaluated in autocollimation, such as photographic objectives with as many as twelve air–glass surfaces, and the fringes were not as narrow or of high contrast, but they were still acceptable. Had the reference surface coating been changed to match the reflected beam, the fringes would have improved in contrast.

The Interferometer

An optical schematic of the instrument is shown in Fig. 6. Light from a laser is brought to focus by a microscope objective; this focus is at the long conjugate of another microscope objective. A 12-\( \mu \)m pinhole is mounted at its short conjugate. The purpose of the pinhole is to remove artifacts produced by the laser and objectives and to block modes not parallel to the laser axis.

The pinhole is at the back focus of a 100-mm, \( f/3 \) doublet (A), which collimates the light. The light is reflected by a beam splitter and passes through another 100-mm, \( f/3 \) doublet (B) where it is focused at one conjugate of the aplanat. The other conjugate is at the center of curvature of the concave surface, where the pinhole is imaged. The field lens is placed so that its first nodal point is approximately there, with the center of curvature of the sphere under test placed at the second nodal point. Such an arrangement uses only a small portion of the lens, about 0.3 mm in diameter. The aplanat's convex surface and the doublets have low reflectance coatings, as does the back side of the beam splitter. This was done to reduce internal reflections to a minimum.

The light reflected from the reference surface and the test piece passes back through doublet B, the beam splitter, doublet C, and onto a 10 cm \( \times \) 12.5 cm film holder. A guillotine type shutter is placed at the focus of lens \( C \). When the shutter is dropped by deactivating a solenoid, a fold mirror in front of the shutter is simultaneously swung out of the way. This mirror and a low power telescope are used to view the fringe pattern until the exposure is made, as with a reflex camera. A low power microscope is interchangeable with the telescope and is used to align the interferometer with the test piece by superimposing the series of bright pinhole images that result from multiple reflections within the cavity. The aplanat and lens \( B \) are mounted in a

![Fig. 6. Optical schematic of the instrument.](image-url)
housing which can be moved transversely to achieve final alignment, while the entire interferometer can be moved longitudinally. Figure 7 shows the completed instrument.

A single mode, continuous wave gas laser is essential for efficient operation of the interferometer. We originally used a standard 300-mm tube length laser for some early work on an experimental model, but the moding of the laser often caused two sets of fringes to appear. Since our interferometer gap was approximately 2 the laser gap, the second set of fringes was displaced about half a fringe spacing, making it appear as the pattern of a single wavelength. Each set was of lower contrast than when the laser was operating in a single mode. If the laser happened to be working in two modes when an interferogram was made, a twofold error would result in the data reduction if it were not known that two sets of fringes were present.

Alignment of the optics is extremely critical—the pinhole must fall precisely at the center of curvature of the aplanat’s reference surface. For best results, the aperture must be evenly illuminated, which means that the pinhole must be evenly illuminated. The even pinhole illumination is achieved by normal means of adjusting the two microscope objectives with respect to the laser beam and the pinhole.

An assessment of the figure of the reference sphere was made by using a large spherical mirror as a test piece and making two overlapping interferograms, where a given portion of the test mirror is viewed by two parts of the reference sphere (Fig. 8). One set of fringes was normalized to the other, and tilt and power were corrected. The extent to which the fringes in the overlapping area do not coincide is a measure of the error in the reference sphere. It is advisable that the fringes not be in an area where their shape changes rapidly, for the normalization will not be accurate in this case.

**Working Distance**

During the course of the development of the spherical wavefront interferometer, the question arose as to the practical limit of the working distance of the instru-

Air gaps, up to about 1 m, have been realized in practice, but an experiment to measure long separations requires an elaborate apparatus to nullify vibration and air turbulence. For this reason it was decided to compute the maximum path length that could be used.

The modulation which can be expected for the cavity is

$$M = \frac{\sin(\pi \chi \Delta \nu / c)}{\pi \chi \Delta \nu / c}$$

where \( \chi \) = optical path length = 2 \( \times \) cavity length, \( \Delta \nu \) = spectral linewidth of laser source, and \( c \) = velocity of light. If we assume a modulation of 0.8 and \( \Delta \nu \) is in the range of 100 KHz to 10 KHz (which is a realistic approximation for the single mode laser used), \( \chi/2 = \) cavity length \( \approx \) 500-5000 m at a finesse of 2. If a finesse of 20 is required, the maximum cavity length would be 50-500 m.

Vibration and air turbulence are two factors that must be considered when a long cavity is contemplated. Air turbulence can be eliminated by evacuating the working space; pneumatic mounts have shown their worth in drastically reducing vibration. Using such mounts under a massive lens bench, we have obtained high contrast interferograms at the center of curvature of a 3.2-m radius spherical mirror.

![Fig. 7. Completed instrument.](image)

![Fig. 8. Tilt and power corrected.](image)

![Fig. 9. Uses of spherical wavefront interferometer.](image)
Applications

There are numerous uses to which the instrument can be put, in addition to testing spheres. Figure 9 shows four other possibilities. Any autostigmatic system can be evaluated if its absorption is not too high or if it is not so poor in quality that the wavefront in double transmission is deformed by a factor greater than two.

One interesting and useful feature of this interferometer is that a lens (or other system) can be tested in autocollimation and the lens under test can double as a field lens. As explained previously, the purpose of the field lens is to image the test surface onto the reference surface. If now a lens is being tested in autocollimation, the field lens at the center of curvature may be eliminated if the autocollimating surface is imaged by the test lens onto the reference sphere. If this is impractical because of space limitations, a field lens can be used which, in combination with the test lens, images the autocollimating surface onto the reference surface. It should be pointed out that fairly good interferograms can be obtained even without a field lens if the lens under test images the autocollimating flat not too far from the reference surface.

Figure 10 shows examples of using a test lens as a field lens. Figure 10(a) shows fringes obtained when the autocollimating flat is positioned so it is imaged by the 75-mm objective onto the reference sphere. Figure 10(b) is the same setup, but with a field lens inserted at the center of curvature. Note that the fringes are less symmetrical since now the field lens does not image the flat onto the aplanat. Figure 10(c) shows the effect of moving the collimating flat close to the test lens so that the test lens images the flat about 76 cm out from the reference surface. Figure 10(d) is under conditions the same as Fig. 10(c), but with the field lens in place. Here the test and field lenses combine to image the flat correctly, and the fringes improve.

Background Interference Fringes Owing to Misalignment

At one time during the course of the investigations of the interferometer, an array of background interference fringes was observed. It was particularly obvious when a flat or convex sphere was placed at or near the center of curvature of the aplanat's reference surface. The fringes were very pronounced only under certain conditions, but they badly distorted the main fringe pattern. When a reflecting surface was placed very close to the reference surface center of curvature, fringes were most apparent; these were due to the reflected image of the reference surface acting like a test sphere. The fringes were interrupted by closely spaced background fringes which were more apparent in the vicinity of a fringe, although closer examination revealed their presence throughout the interferogram [Fig. 11(a)]. It is interesting to note that when the interferometer was used for evaluation of a sphere, the background fringes were quite faint and did not appreciably alter the normal fringes.

The explanation for the additional fringes was arrived at* by considering that the pinhole image from the collimating system was laterally displaced from the center of curvature of the reference surface. If a mirror were placed at (or near) the center of curvature, the pinhole image would be reimaged on the other side of the center of curvature. Such a situation results in

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* We are indebted to J. M. Burch, National Physical Laboratory, Teddington, U. K., for this analysis.
interference between two slightly separated pinholes, giving straight two-beam fringes, each corresponding to either an even or odd number of traversals of the cavity.

When an autocollimating microscope was placed at the reference surface center of curvature, it was observed that the pinhole image had indeed become laterally displaced. When the shift was remedied, the background fringes were eliminated.

Figure 11(b) shows that when a test sphere (convex) was placed concentric with the reference sphere, the background fringes were much less evident and the primary fringes were not affected. In every case, when a mirror is placed at the reference surface center of curvature, the background fringes are obvious.

The interferometer described was developed from a concept to an experimental model to a working instrument.

References


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4. W. E. Williams, Application of Interferometry (John Wiley & Sons, Inc., New York, 1966). A new graduate-level textbook, including authoritative reviews covering selected topics in laser technology (ruby lasers; solid-state lasers; organic lasers; Q switching; and optical resonator modes) by noted experts in the field.