

# Optical Insights into Renaissance Art

By David Hockney and Charles M. Falco

*The Hermitage, St. Petersburg*

*In this feature, world-renowned artist David Hockney and University of Arizona optical sciences professor Charles Falco explain how Hockney's observation that certain Renaissance paintings seemed almost "photographic" in nature led them to launch an inquiry into the possibility of finding scientific evidence that some of the Old Masters relied on optical aids. Hockney's visual observations received scientific validation when application of basic optics principles to a number of Renaissance paintings began generating remarkably consistent results.*

**A**n article published earlier this year in *OPN* stated “The *camera obscura* has enjoyed two lives, one that has been fully documented by art historians, and a second, comparatively unknown, as an object of scientific speculation.”<sup>1</sup> The author of that piece may have given a bit too much credit to art historians since, as we describe below, very recent work now shows that the use of projected images in art goes back at least 150 years further than previously thought. Since portraits painted by Renaissance masters now provide important *scientific* documentation of the early use of optical instruments, this discovery has significant implications for the history of science as well as the history of art. In an era in which the divide between artists and scientists was not as large as it is today, and with a dearth of contemporary written accounts, the paintings themselves have become primary source documents.

How did we decide to undertake this research? At a recent exhibition, David Hockney observed a certain quality in the 19th century portraits of Jean-Auguste-Dominique Ingres that suggested that the artist had used some sort of optical device as an aid.<sup>2</sup> This led to a detailed examination of a large number of European paintings. The outcome? The “photographic quality” observed in the Ingres portrait was traced back to as early a work as that of Robert Campin (circa 1430). For a complete account of this extensive visual investigation, including its significance within the context of our current understanding of Renaissance as well as modern art, please see the list of references.<sup>3</sup>

This article describes the variety of scientific evidence we discovered to support and extend our investigation. We begin with a brief review of the relevant properties of imaging optics. The discussion employs the language of photography since, as will be shown, a number of Renaissance paintings share the same optical basis as these modern photographic instruments.

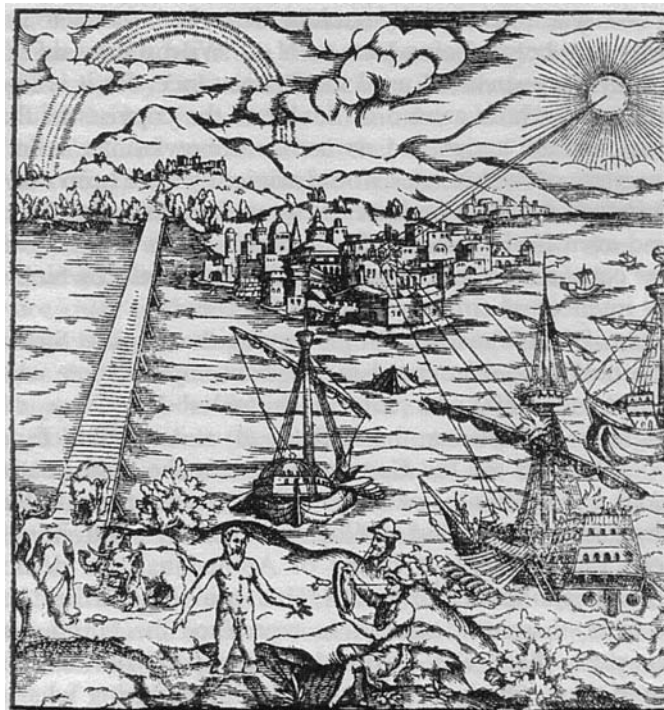
### Focal length and geometry

The “normal” lens for a given film format is one with a focal length roughly the diagonal of the negative, or 43 mm in the case of 35 mm film.<sup>4</sup> Lenses of focal length significantly shorter or longer than the film diagonal result in perspectives that are termed, respectively, wide-angle or telephoto. Although the reader will find this obvious, it’s worth restating in the context of our work that if a larger piece of film is used, since its diagonal will be longer (325 mm in the case of an 8" × 10" piece of film), it will be necessary to use a lens of equivalently longer focal length to obtain the same perspective.

Some photographs contain enough information to allow us to make an estimate of the focal length of the lens used. In the case of Figure 1, starting from the height of the people ( $\approx 1.8$  m) we can compute the horizontal distance across the scene at the location where the most distant two are standing as approximately 2.2 m. If we estimate the length of the console



**Figure 1.** Photograph taken with a 35 mm camera (negative size 24 mm × 36 mm). As described in the text, this photograph contains enough information to allow us to estimate the focal length of the lens used.



**Figure 2.** Frontispiece to *Opticae Thesaurus, with Vitellionis Thurinopoloni Opticae Libri Decem*, edited by Federico Risner (Basle, 1572). A variety of optical phenomena are illustrated in this engraving. To the right we see Archimedes use of “burning mirrors” to set fire to the Roman fleet. In the foreground we see an image projected by a mirror (albeit, not drawn inverted, as would have to be the case for a concave mirror).

of a molecular beam epitaxy (MBE) machine to be 2 m, the horizontal included angle  $\theta$  is given by:

$$\tan(\theta/2) = [1/2 \times \text{horizontal distance across scene}] / [\text{length of MBE console}]$$

or

$$\theta = 2 \tan^{-1} [2.2 \text{ m} / 2 \times 2 \text{ m}] \quad (\text{Eq. 1})$$

$$\cong 58^\circ$$

which corresponds to a 22-mm focal-length lens (the lens used was actually 24 mm).

### Effect of aperture on depth of field

Although a lens can be focused perfectly at only one specific distance at a time, a field on either side of that distance will be acceptably sharp. How far that depth of field (DOF) extends depends on three factors: the permissible circle of confusion  $C$ , the relative aperture of the lens  $f\#$ , and the image magnification  $M$  relative to the original scene, as given by:<sup>4</sup>

$$\text{Total DOF} = 2 C \times f\# (M + 1) / M^2 \quad (\text{Eq. 2})$$

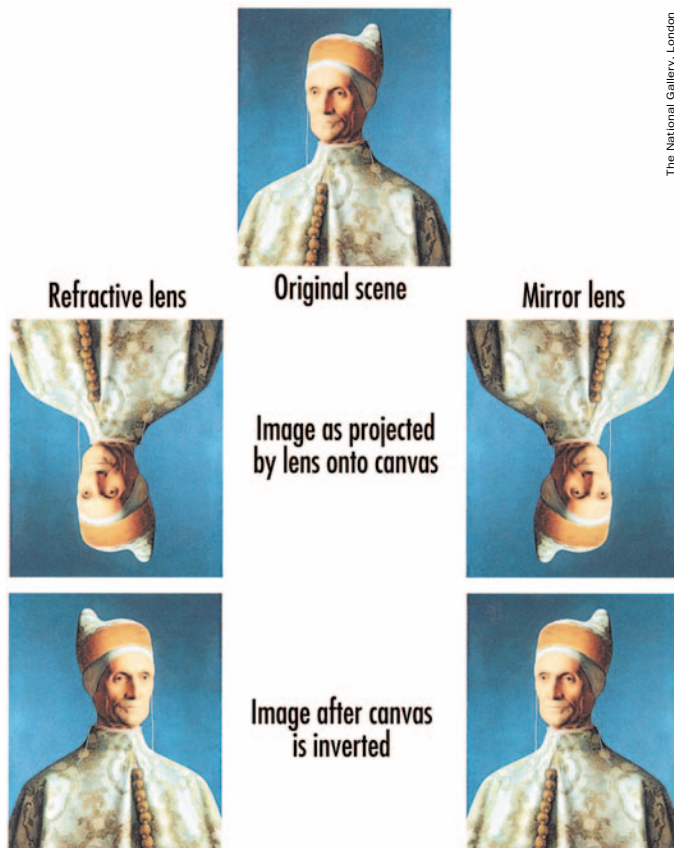
The circle of confusion  $C$  is the smallest feature on an image that a viewer can distinguish from a point. For purposes of photography, it is commonly accepted that on a high-quality print, an image area smaller in diameter than  $\approx V/1000$ , where  $V$  is the viewing distance, will be indistinguishable from a point. As an example, for a large photograph produced with modern camera and enlarger lenses, and examined at a distance of 50 cm, features smaller than about 0.5 mm would appear to be points.

Next is the relative aperture  $f\#$ , which is the ratio of the focal length of the lens  $f$  to its effective diameter  $D$ :

$$f\# = f / D \quad (\text{Eq. 3})$$

From Equations 2 and 3 it can be seen that for a given focal length lens, the DOF depends inversely on the effective diameter  $D$ , and thus can be increased by masking off ("stopping down") the outer area of the lens.

Finally, there is the magnification  $M$ . This is the ratio of the image size to the subject size, which is also the ratio of the lens-image distance to the lens-subject distance:



**Figure 3.** *The Doge Leonardo Loredan*, Giovanni Bellini, 1501–05. This figure shows how a real image would be projected onto film, or a canvas, by a refractive lens [left] and by a concave mirror ("mirror lens") [right]. The fact that, after subsequently inverting the image, a mirror lens has left the original symmetry of the scene unchanged, has significant advantages for an artist using such a lens as an aid.



**Figure 4.** *The Marriage of Giovanni Arnolfini*, Jan van Eyck, 1434. 59.7 cm  $\times$  81.8 cm. The distorted image of the window reflected by the mirror on the back wall shows that it was convex. If the back side of this convex mirror had been silvered, the resulting concave mirror could have been used to form images.

$$M = \text{image size/subject size} \\ = \text{lens-image distance/lens-subject distance} \quad (\text{Eq. 4})$$

Returning to Equation 2, if we are able to estimate the *DOF* in a given photograph, and also to extract the geometrical factors necessary to estimate the magnification and focal length of the lens used by the photographer, we can then use Equation 3 to calculate the effective diameter of the lens.

### Effect of focus on magnification and on vanishing points

The Gaussian lens formula relates the focal length of a lens  $f$  to the distance between it, the subject and the image:

$$1/f = 1/d_{\text{lens-subject}} + 1/d_{\text{lens-image}} \quad (\text{Eq. 5})$$

Since as Equation 4 shows, the magnification depends on the relative distances, a consequence of Equation 5 is that if a lens is moved for purposes of refocusing, the relative distances change, as does the magnification. However, this effect is seldom noticed since the “typical” photograph involves a small magnification. In a common snapshot, for instance, the image of a 180 cm tall person ends up less than 24 mm high on the film, resulting in a magnification of only  $\sim 10^{-3}$ .<sup>3</sup> As  $M$  increases, however, changes in  $M$  upon alteration of the focus can become noticeable.

Another principle of photography important to the research at hand is that parallel lines converge to a vanishing point related to the focal length and position of the lens. Thus, refocusing a lens, or indeed, moving it for any reason, will change not only the magnification of the image but also the vanishing point.

### Optical aberrations

Various aberrations (astigmatic, coma, chromatic, etc.), all of which degrade the quality of the image, are exhibited by lenses. For example, in the case of spherical aberration, rays from the outer edges of a lens are brought to focus closer to the lens than are the central rays. The only way to reduce this effect in a single-element lens is to decrease its effective diameter. In the range of lens sizes that concern us, other aberrations are also reduced as the lens diameter is decreased. Unfortunately, stopping a lens down reduces the brightness of the image, resulting in a tradeoff between sharpness and brightness.

### The concave mirror as lens

The optical properties of concave mirrors have been studied since the time of Euclid. Figure 2, for instance, is an engraving from a 1572 book showing what appears to be Archimedes’ use of focused light from several concave mirrors to defend Syracuse from a Roman fleet in 212 BC. We discovered in the course of our research, however, that outside the scientific community, there is scant awareness of the fact that an image can be formed with a concave mirror. For this reason, we will use the term “mirror lens” in this article to make explicit the

imaging properties of the concave mirror. Although both refractive lenses and mirror lenses can form images, images formed by mirror lenses have a particularly significant characteristic. In both cases the image is inverted but, since a mirror reverses left to right, the result is that the symmetry of the final image created by a mirror lens is identical to that of the subject. This is illustrated in Figure 3. The importance of this from the point of view of an artist fashioning a painting is discussed elsewhere.<sup>3</sup>

We will emphasize mirror lenses in our discussion since we have uncovered a variety of circumstantial evidence pointing to their possible use. However, it should be noted that we have as yet found no scientific evidence that might distinguish between portraits made with the aid of concave mirrors rather than refractive lenses. What evidence is there that the fabrication technology to produce such an optical element existed in the early Renaissance? Jan van Eyck gives us one answer in the 1434 portrait shown in Figure 4. Until an opaque

## Although both refractive lenses and mirror lenses can form images, images formed by mirror lenses have a particularly significant characteristic.

protective coating had been applied to the back side of the convex mirror on the wall, its obverse side would have been a mirror lens. In addition, concave mirrors of polished bronze and speculum metal did exist in Medieval times and in antiquity.

### Summary of relevant lens effects

To summarize the specific aspects of lenses we have drawn upon for our analysis of Renaissance paintings:

- If we know the geometry of the original scene, and the size of the canvas (film), we can calculate the focal length of the lens used;
- From the focal length and depth of field we can calculate the diameter of the lens;
- If a lens is moved to alter the focus, the magnification of the image will change;
- If a lens is moved between two exposures, a second vanishing point will be created;

- Concave mirrors (“mirror lenses”) can be used to create images;
- Mirror lenses leave the symmetry of the scene unchanged;
- The image formed by any simple lens—mirror or refractive—will exhibit significant aberration that can be reduced only by decreasing the diameter of the lens;
- Spherical aberration, astigmatism, and coma limit the useful area of an image, even if the diameter of the lens is reduced.

### **Husband and Wife, Lorenzo Lotto, circa 1523-4**

Figure 5 has turned out to be a remarkably rich painting in the context of our work, so we will discuss our analysis of it in some detail. Figure 6 shows the central geometric pattern on the tablecloth in this painting. What is immediately striking is that Lotto painted a pattern that appears to go out of focus as it recedes into the scene, just as happens in a photograph when the DOF of the lens is exceeded. Since the human eye automatically refocuses as it traverses different depths of a scene, such an out of focus feature would not have been visualized by Lotto’s unaided eye alone. Although this painting provides striking evidence of having been based on a projected image, and even though eyeglass lenses were produced at least as early as circa 1300, documentation of the first optical instruments to employ refractive optics does not appear until the mid-16th century (Janssen’s compound microscope of 1590 and Galileo’s telescope of 1609 are commonly cited, but the earliest known example is Girolamo Cardamo’s 1550 description of a *camera obscura* incorporating a lens). Demonstration that an optical instrument was in fact in use prior to circa 1550 is thus an object of interest in the context of the history of science as well as the history of art.

Interestingly, as can be seen in Figure 6, in the same region of the painting where the image loses focus, the vanishing point also changes. Had Lotto laid out the pattern geometrically, following, for example, the principles articulated in the 15th century by Leon Battista Alberti,<sup>5</sup> the chance of such a change taking place would have been minimal. However, had he traced the pattern from a projected image, and had he moved the lens in an attempt to refocus after the DOF had been exceeded, such a change would be completely natural.

If we examine the triangular pattern toward the right edge of the table, we discover that the vanishing point changes here as well, at the same depth in the scene that the central feature goes out of focus and the vanishing point changes. However, unlike the central feature, the triangular pattern on the right remains in focus all the way to the back of the table. As discussed below, this can also be easily explained by an optical analysis of the painting. In fact, not only does this work provide convincing scientific evidence that the artist used a lens, there is actually enough information to allow us to cal-

culate its physical properties.

Assuming the width across the shoulders of a typical woman is  $\approx 50$  cm, and measuring the corresponding width in the original painting to be 28 cm, the magnification  $M \approx 0.56$ . For reasons explained in detail below, we believe that intrinsic aberrations of the lens did not allow Lotto to project the entire image onto the canvas at one time. Instead, he was forced to piece it together from several projected “frames,” each of height and width  $\approx 30$ -50 cm. (It should be noted, however, that neither this assumption, nor the precise dimensions, are critical to our analysis of this painting.) Since the visible portion of the tabletop occupies a width of approximately 52 cm on the original painting, this portion corresponds to one projected frame.

To determine the precise focal length of the lens Lotto used would require accurate measurements of his *camera obscura*. However, we can make a reasonable estimate if we assume Lotto’s studio was roughly 3 m deep. Allowing 1 m of that for the table and subjects leaves a 2 m working distance. As Figure 7 shows, if the lens were located 1.5 m from the subjects, the magnification of  $M = 0.56$  would result in an ample 84 cm working distance between it and the canvas. With these values for the lens-subject and lens-canvas distances, and using Equation 5, we find the focal length of the lens was  $f \approx 54$  cm. Interestingly, the diagonal of our assumed range of projected “frames” is in the range 42-70 cm, so such a lens would have provided a “normal” perspective for this frame. Also, whether Lotto used a refractive or a mirror lens, the diopter strength was

$$K_{\text{diopters}} = 100 \text{ cm} / 54 \text{ cm} = 1.86 \text{ diopters}$$

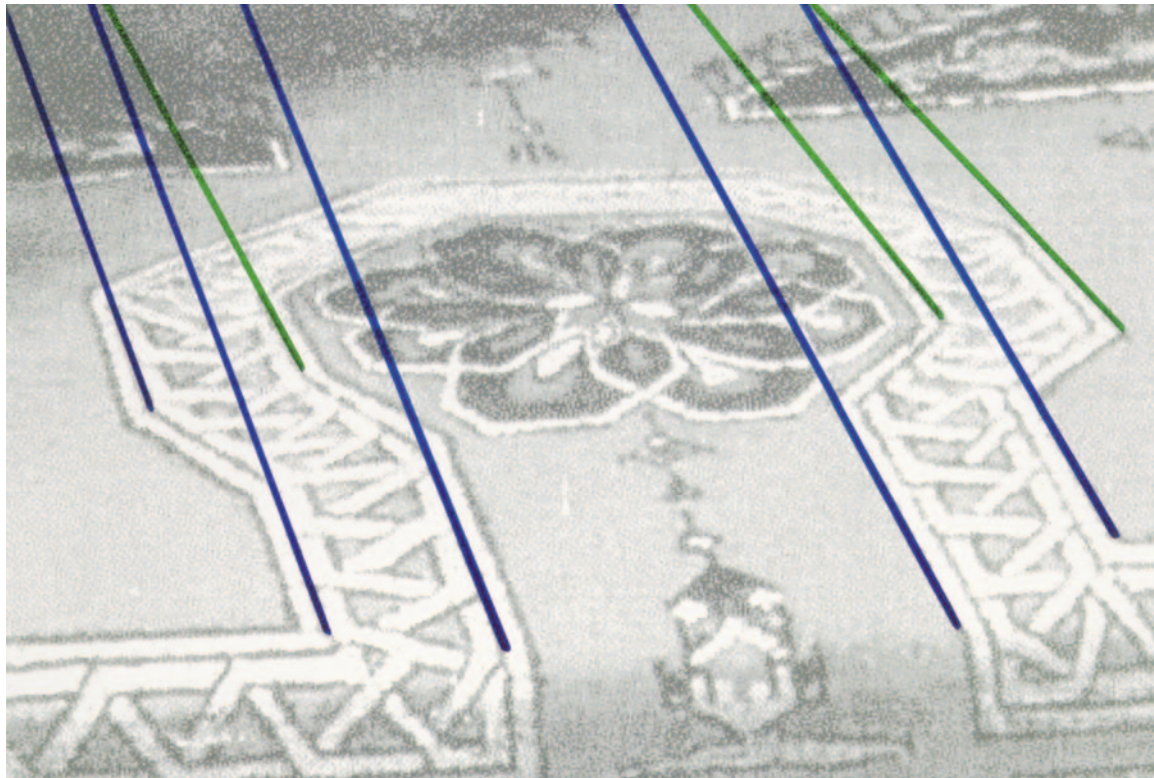
so the curvature of its surface was equivalent to that of a pair of reading glasses.

The triangular pattern in the tablecloth of Figure 5 serves as a built-in fiducial that allows us to determine additional information about the lens. There are seven repeats of the triangular pattern across a 14.48 cm span of the painting at the front of the table, so there is a spacing of 2.07 cm per triangle. Since the magnification is 0.56, this means there was a spacing of 3.70 cm per triangle on the original. The central pattern seems to go “out of focus” at  $\approx 5$ -9 triangles deep into the scene. Since the pattern is at an angle of about  $30^\circ$  with respect to the camera lens, this is a distance of  $(5-9) \times 3.70 \text{ cm} \times \cos 30^\circ = 18.5-33.30 \text{ cm} \times 0.866 \approx 22.5 \pm 6 \text{ cm}$  from the front edge of the table. If we assume that Lotto initially focused on the front edge of the table, we now know he exceeded the DOF of his lens at a distance of  $22.5 \pm 6 \text{ cm}$ .

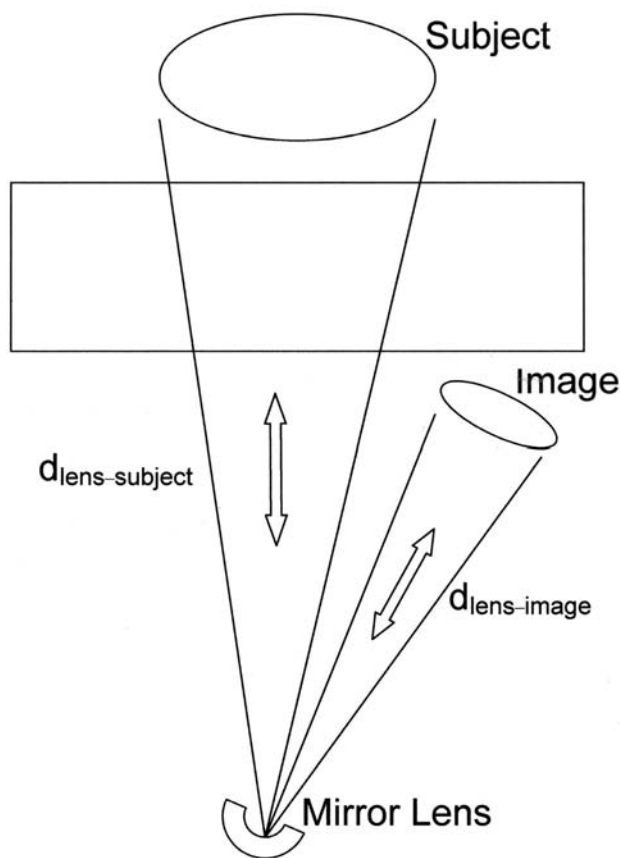
Equation 2 allows us to calculate from this information the physical size of the lens Lotto used. If we assume a circle of confusion on the painting of 2 mm from his simple lens, we find  $f\# \approx 22$ , and thus the diameter of the lens  $D \approx 2.4 \text{ cm}$ . As we have confirmed from our own experiments, a concave mirror of this focal length and diameter projects an image that is bright enough, and sharp enough, for an artist to use when the subject is



**Figure 5.** *Husband and Wife*, Lorenzo Lotto, circa 1523-4. 96 cm × 116 cm. Note how the octagonal pattern in the center of the tablecloth appears to go out of focus as it recedes away from the viewer. Although it may be too small to see as reproduced here, in larger reproductions it is easy to see a change in vanishing point in the pattern at the right of the tablecloth, at the same place where the octagonal pattern goes out of focus.



**Figure 6.** *Husband and Wife* (detail). Two vanishing points are clearly observed, as would happen if a lens were moved in the course of making this painting.



**Figure 7.** A plausible layout for Lorenzo Lotto's studio shown approximately to scale assuming a room of dimensions 3 m × 3 m.

illuminated by strong sunlight.

Finally, let's consider why Lotto painted the central feature out of focus, while the triangular pattern at the right of the table is in focus even though it extends further into the scene. When Lotto exceeded the *DOF* of his lens and was forced to refocus to a point  $\approx 20$  cm further into the scene, the resulting image was measurably reduced in magnification. Since the central feature on the tablecloth was so wide ( $\approx 30$  cm), such a change made it impossible for Lotto to attempt to match the complex geometrical pattern. However, since the triangular pattern at the right edge of the tablecloth was roughly six times narrower, his job there was at least six times easier. Although there isn't space here to describe our complete analysis of this remarkable painting, detailed examination reveals other telltale signs that expose Lotto's struggles to make the geometric patterns on the tablecloth look reasonable as he altered the focus of his lens.

### Other paintings

The Lorenzo Lotto painting is unusual for the wealth of evidence it contains about the artist's use of a lens. Although the majority of paintings we have analyzed lack one or more of the details needed for a similar quantitative analysis, the conclusions we have been able to draw from those that do contain information have been, without exception, consistent with our discoveries. To briefly cite three additional early examples, a Hans Memling painting of 1485 exhibits a change in vanishing point



The National Gallery, London

**Figure 8.** a) *Sibyl*, Dosso Dossi, 1516-20. 68.5 cm × 84 cm. b) *Portrait of a Man*, Giovanni Battista Moroni, 1560-65. 39.7 cm × 47 cm.

that occurs roughly 10 cm into the scene from the front edge of the table, consistent with a lens having been moved to refocus after the *DOF* was exceeded. Similarly, the pattern in the seat back in Anthonis Mor's *Mary Tudor* of 1554 also shows a change in vanishing point. Another example is the tablecloth, and some of the objects on it, in Hans Holbein's *Georg Gisze* of 1532 that show a change in perspective of 10° in the vertical direction. If we make reasonable assumptions about the size of his studio, this is consistent with a lens having been raised (or lowered) by 25 cm part way through completion of the painting. We can only speculate on the reasons Holbein did this, since it is unlikely such a large change in position of the lens could have occurred by accident.

### The lens as a constraint

Some people are upset to learn Renaissance masters of the stature of van Eyck and Holbein used lenses. However, artists certainly did use technical aids in their work—brushes, palettes, plumb lines, grids, and drawing screens are some examples—so a lens is simply one more technical aid we now know some of them made use of. Actually, some art historians find another of our results even more striking, since it implies that for a period of time artists might have had their work *constrained* by a technical aid. There is space to only briefly discuss another type of analysis we have done that suggests this might have been so.

In the course of our work, we noticed that a large fraction of the “photographic quality” portraits of the 15th and 16th century were limited to the head and shoulders of the subjects. So far we have analyzed 12 such portraits that vary in areas of canvasses by over a factor of five, and that were painted by different artists during the period circa 1450 to 1560-5. Two examples from this set of 12 are shown in Figure 8. To compare these paintings we used the subjects' eyes (specifically, the spacing between pupils), appropriately scaled to the sizes of the canvasses and corrected for orientation of the subjects. Surprisingly, in spite of the seeming lack of limitations on the various artists in creating these portraits, the measured interpupil distances turned out to be  $5.86 \pm 0.81$  cm. Since the average interpupil distance of adults is 6.3 cm (normal range 5.3-7.3 cm), this means all 12 portraits were produced at essentially the same magnification of ~90%.

To investigate the implications of this further we did a ray tracing analysis of an optical system configured as in Figure 7, incorporating a mirror lens with  $f = 590$  mm and  $f\# = 3.9, 5.9$  and  $11.8$ . Not surprisingly, aberrations limit the size of acceptable images that can be projected by such a lens. If the horizontal and vertical broadening of sharp features in the projected image are to be kept to less than 20%, we found only an area of  $\sim 30$  cm  $\times$  30 cm on the canvas is useable even when stopped down to  $f\#=11.8$ . If this is relaxed to 40%, at which point only fairly gross features of the subject are still recognizable, the usable area only increases to  $\sim 50$  cm  $\times$  50 cm. While we assumed a perfect spherical surface for our calculations, any manufacturing defects would further reduce the area

of the projected image that had acceptable quality. Significantly, all 12 of the portraits that one of us had previously identified as exhibiting a “photographic quality,” and which we then subsequently analyzed, have their key features (*i.e.*, the subject's head and shoulders) constrained within these areas.

We regard these observations as additional circumstantial evidence substantiating the use of lenses. More significantly, however, is the possible implication that early Renaissance artists paid a price for using lenses. While they could now produce paintings more quickly than before, and with unprecedented realism, it appears intrinsic optical aberrations imposed a constraint on their ability to choose their compositions. Artists quickly developed various ways to overcome this challenge, although not always with perfect success.<sup>3</sup>

### Summary

We have discovered a variety of scientific evidence that strongly supports and extends a theory of painting developed by an artist (David Hockney) based on his visual observations. This work in turn has implications for two other academic disciplines: art history and the history of science. We expect that bringing the properties of the mirror lens, and introducing concepts of image analysis, to the attention of art historians will open new areas of investigation in understanding paintings of the past 600 years. Also, since the principle of the *camera obscura* was discussed in early Chinese and Arabic literature, evidence of the use of optical aids may exist in Eastern and Islamic art as well. Finally, we can't help but note that not only is an understanding of optical science needed for developing key 21st century technologies, it is also fundamental for understanding 15th century art.

### Acknowledgments

We are grateful to Lawrence Weschler of *The New Yorker* and Ultan Guilfoyle of the Solomon R. Guggenheim Museum for their roles in bringing an artist and a scientist together. Also, to Professor Martin Kemp of the University of Oxford for an extended dialogue with David Hockney via fax during the development of his ideas, and to Professors José Sasián and Masud Mansuripur of the University of Arizona for, respectively, performing ray tracing calculations and critically reading this manuscript, and to David Graves for numerous valuable contributions.

### References

1. Brian S. Baigrie, “The Scientific Life of the Camera Obscura,” *Optics & Photonics News*, **11**, 18–21 (February 2000).
2. Early activities are described by Lawrence Weschler in “The Looking Glass,” *The New Yorker*, 64–75 (31 January 2000).
3. David Hockney, *Secret Knowledge: Retracing Six Centuries of Western Art* (Thames and Hudson, London, to be published).
4. Charles E. Engles, *Photography for the Scientist*, (Academic Press, New York, 1968).
5. Martin Kemp, *The Science of Art* (Yale University Press, New Haven, 1990).

David Hockney is an artist with studios in Los Angeles and London. Charles Falco is a professor of optical sciences at the University of Arizona. Correspondence can be sent to falco@u.arizona.edu.